

# A model based on Hirano-Exner equations for two-dimensional transient flows over heterogeneous erodible beds

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## Abstract

In order to study the morphological evolution of river beds composed of heterogeneous material, the interaction among the different grain sizes must be taken into account. In this paper, these equations are combined with the two-dimensional shallow water equations to describe the flow field. The resulting system of equations can be solved in two ways: (i) in a coupled way, solving flow and sediment equations simultaneously at a given time-step or (ii) in an uncoupled manner by first solving the flow field and using the magnitudes obtained at each time-step to update the channel morphology (bed and surface composition). The coupled strategy is preferable when dealing with strong and quick interactions between the flow field, the bed evolution and the different particle sizes present on the bed surface. A number of numerical difficulties arise from solving the fully coupled system of equations. These problems are reduced by means of a weakly-coupled strategy to numerically estimate the wave celerities containing the information of the bed and the grain sizes present on the bed. Hence, a two-dimensional numerical scheme able to simulate in a self-stable way the unsteady morphological evolution of channels formed by cohesionless grain size mixtures is presented. The coupling technique is simplified without decreasing the number of waves involved in the numerical scheme but by simplifying their definitions. The numerical results are satisfactorily tested with synthetic cases and against experimental data.

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## 1. Introduction

River morphodynamics refer to the study of the interactions between the bed and the banks of a river and the flow field. The study of these interactions by means of a numerical model requires a set of equations to describe the flow field (e.g. the shallow water equations [1]), and a conservation equation for the mass of sediment, i.e. the Exner equation [2]. If bedload dominates and the sediment concentration is low (less than 1 %, [3]), the classical Exner equation is enough to determine the morphological changes in a river. This approach has been extensively used for the study of one-dimensional ([4, 5, 6, 7]) and two-dimensional (e.g. [8, 9, 10, 11, 12, 13, 14]) morphodynamic problems.

The Exner equation combined with the two-dimensional shallow water equations has been recently extended to sediment mixtures. In these situations, total sediment transport rates are computed as the sum of the contribution from each grain size. Depending on the proportion of each size fraction and the mode by which bed particles are transported, sediment transport rates are estimated through a bed-load or a suspended-load formula [15, 16, 17, 18, 19]. Additionally, imbalances between the actual and the capacity of sediment transport (non-equilibrium sediment transport models) which arise, among other from surface structuring and sorting, have been considered [15, 16, 19]. Hence, temporal and spatial lag effects between the local hydrodynamic conditions and the sediment load are taken into account [20, 21]. However, as recently noted by [11], the uncertainty on some key parameters associated to non-equilibrium models can lead to significant differences on the results.

When a river bed is composed of sediment mixtures, the general Exner equation for the conservation of mass of sediment is not enough to properly describe the morphodynamic evolution of the river. Under these situations, some other equations are needed to assure the conservation of each grain size present on the bed surface. Since particles on the bed might be exchanged with grain sizes on the substrate, a key issue is to evaluate how particles sort vertically from the surface downwards leading to a vertical stratigraphy of

34 the river bed. Vertical sorting thus depends on the fluxes between the differ-  
35 ent bed layers and the sediment transported on the bed surface. There have  
36 been several attempts to evaluate these vertical exchanges [15, 22, 23, 18, 19].  
37 Among them, the most used widely vertical discretization has been the one  
38 introduced by [24]. In there, Hirano introduced the presence of a sediment  
39 exchange layer, the so called “active layer”. This uppermost layer of the bed  
40 is assumed to concentrate the interactions between the sediment transport  
41 and those fractions of material present on the river bed. Besides, the thick-  
42 ness of the active layer encompasses the fluctuations of the bed elevation at  
43 a given point of the bed [25]. Consequently, this layer acts as a buffer in the  
44 exchanges between the bedload transport and the substrate which provides  
45 a source of sediment to be entrained by the flow, [26, 27, 28].

46  
47 Steady [17, 29] and unsteady [30, 31, 32, 33, 34, 35, 36, 37, 38, 39] active lay-  
48 ers have been assumed by a number of researchers. In both cases, a closure  
49 equation is needed to evaluate its thickness. Since the active layer thick-  
50 ness embraces the fluctuations of the bed [25], a physically-based approach is  
51 needed to link its value to some reference grain size or to some representative  
52 bedform height. Under the hypothesis of one-dimensional and steady flow,  
53 the thickness of the active layer has been usually chosen as multiple of the  
54 characteristic grain-size on the bed [30, 31, 32, 33, 34, 37, 38], which can vary  
55 in time and space.

56  
57 Letting aside the deformation of the bed and the appearance of bedforms  
58 affecting the bed roughness, the temporal variation of the surface composi-  
59 tion implies a subsequent variation of the bed friction. This time-varying  
60 approach of the bed roughness has been traditionally incorporated in one-  
61 dimensional numerical models for mixtures, [31, 32, 33, 34, 36, 37, 38, 39].  
62 However, with the exception of [19] who simulates the evolution of a braided  
63 stream and accounts for the variation of both skin and drag friction, few  
64 two-dimensional numerical models take time-varying roughness into consid-  
65 eration.

66  
67 From the numerical point of view, the coupling/uncoupling of the water flow  
68 equations with those describing the evolution of the bed has attracted the  
69 attention of researchers. [40] displayed that uncoupled strategies were only  
70 valid for a narrow range of hydrodynamic regimes governed by low Froude  
71 numbers, limiting the velocity at which the bed and the flow field interact

72 with one another. Based on De Vriend's approach, [41] developed a cou-  
73 pled numerical model in which a set of approximate solutions for the bed  
74 celerity and sorting celerities, i.e. the speeds at which a perturbation on  
75 the surface fractions propagates along the domain, were proposed. However,  
76 that approximation was obtained assuming quasi-steady flow. Based on Rib-  
77 berink's approximation, [42] graphically estimated the celerities of the system  
78 for unsteady flows and sediment mixtures composed of only two grain sizes.  
79 Additionally, [41] noticed that under certain situations, the Saint-Venant  
80 equations in combination with Hirano's equation lead to an elliptic system  
81 of equations. This elliptic nature is inconvenient for solving unsteady water  
82 flow problems, [29], which are genuinely defined as hyperbolic [43].

83

84 In [44], the Hirano model was mixed with the Exner equation for decreas-  
85 ing the number of interrelationships among variables. This theory has been  
86 widely implemented in steady 1D numerical schemes [32, 34, 37, 38, 39] and  
87 more recently, in a 1D coupled model in [45]. However, this theory has not  
88 been included in a pure two-dimensional unsteady numerical scheme.

89

90 In the present work, an efficient numerical strategy proposed for uniform  
91 grain sizes in [14] is extended to sediment mixtures. For that purpose, a  
92 set of equations to handle the numerical computation of the two-dimensional  
93 flow field and the evolution of the bed and surface texture (Exner equation  
94 and the so-called Hirano model) is introduced. Since the resulting system of  
95 equations is not fully hyperbolic, the bed and sorting celerities may not be  
96 directly computed from the characteristics theory. However, numerical esti-  
97 mations for the wave celerities are provided and to ensure conservation of the  
98 system and to automatically control the numerical stability of the explicit  
99 scheme used to solve the system of equations. Additionally, the formulation  
100 presented herein does not impose any constraint on the magnitude of the  
101 gradients (in the flow field, bed topography and surface texture). Our for-  
102 mulation solves, in a self-stable way, the two-dimensional morphodynamics  
103 using the active layer model in its full extension, i.e. assuming that the time-  
104 varying surface texture affects both the thickness of the active layer and the  
105 bed roughness.

106

107 The work is outlined as follows: Section 2 describes the mathematical model  
108 while in Section 3 the numerical strategy is explained. Section 4 presents the  
109 numerical results obtained, validated with a set of one and two-dimensional

110 test cases and with a two-dimensional experimental test subjected to rapid  
 111 variations of both flow and channel features. In Section 5 conclusions arising  
 112 from the work are described.

113

## 114 **2. Mathematical model**

115 The mathematical model presented for modelling river morphodynamic is  
 116 based on the coupled system of equations formed by the 2D shallow water  
 117 equations to describe the hydrodynamics and the 2D Exner equation, ex-  
 118 tended to multiple grain sizes, to describe the morphological evolution of the  
 119 river bed (elevation and surface grain size distribution or texture).

### 120 *2.1. Hydrodynamic model*

121 Hydrodynamic is formulated by means of the depth averaged shallow water  
 122 equations. Mass and momentum conservation form a system of equations,  
 123 which, in 2D Cartesian coordinates, can be written as follows

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} + \frac{\partial \mathbf{G}(\mathbf{U})}{\partial y} = \mathbf{S}_\tau + \mathbf{S}_b \quad (1)$$

124 where

$$\mathbf{U} = (h, hu, hv)^T \quad (2)$$

125 are the conserved flow variables with  $h$  representing water depth and  $(u, v)$   
 126 the depth averaged components of the velocity vector in the longitudinal  $x$   
 127 and transversal  $y$  coordinates, respectively. The hydrodynamic fluxes  $\mathbf{F}$  and  
 128  $\mathbf{G}$  in (1) are given by

$$\begin{aligned} \mathbf{F} &= \left( hu, hu^2 + \frac{1}{2}gh^2, huv \right)^T \\ \mathbf{G} &= \left( hv, huv, hv^2 + \frac{1}{2}gh^2 \right)^T \end{aligned} \quad (3)$$

129 The term  $\mathbf{S}_\tau$  in (1) accounts for the frictional effects on the bed as

$$\mathbf{S}_\tau = \left( 0, -\frac{\tau_{b,x}}{\rho}, -\frac{\tau_{b,y}}{\rho} \right)^T \quad (4)$$

130 where  $\tau_{b,x}, \tau_{b,y}$  are the bed shear stresses in the  $x$  and  $y$  direction respectively  
 131 and  $\rho$  is the density of the water. Shear stresses have been evaluated by  
 132 means of the Manning's law which in 2D is written as follows

$$\begin{aligned}\tau_{b,x} &= gh \frac{n^2 u \sqrt{u^2 + v^2}}{h^{1/3}} \\ \tau_{b,y} &= gh \frac{n^2 v \sqrt{u^2 + v^2}}{h^{1/3}}\end{aligned}\quad (5)$$

133 where  $n$  is the roughness Manning's coefficient which has to be evaluated  
 134 taking into account the surface texture as it is described in further subsection.  
 135 Finally, the term  $\mathbf{S}_b$  in (1) accounts for the the pressure force along Cartesian  
 136 coordinates  $x$  and  $y$  respectively.

$$\mathbf{S}_b = (0, ghS_{ox}, ghS_{oy})^T \quad (6)$$

137 where  $S_{ox} = \partial z / \partial x$  and  $S_{oy} = \partial z / \partial y$  are the bed slope in the  $x$  and  $y$   
 138 direction.

### 139 2.2. Bed and surface texture evolution model. Hirano's active layer model

140 The evolution of bedload-dominated channels has been traditionally modelled  
 141 by using the classical Exner equation for conservation of mass of sediment  
 142 [2]. This equation, however, does not account for the evolution of the surface  
 143 grain size distribution. Thus, when the bed surface is composed of mixtures of  
 144 grain sizes, particle interactions need to be conservatively accounted for in the  
 145 mathematical model. Temporal and spatial changes in the surface grain size  
 146 distribution can be reproduced by means of the active layer and the sediment  
 147 conservation equations introduced by Hirano [24, 46]. The mathematical  
 148 model proposed by [24] is extended to two-dimensional domains.

149 Exner equation for the conservation of mass of sediment states that the rate  
 150 of change of bed elevation within a control volume is driven by the sediment  
 151 fluxes crossing the boundaries of that volume. In 2D, such equation is written  
 152 as follows

$$\frac{\partial \eta}{\partial t} + \xi \frac{\partial q_{b,x}}{\partial x} + \xi \frac{\partial q_{b,y}}{\partial y} = 0 \quad (7)$$

153 where  $\eta$  is the bed elevation,  $\xi = \frac{1}{1-p_b}$ ,  $p_b$  is the porosity of the sediment  
 154 mixture,  $q_{b,x}$  and  $q_{b,y}$  denote volumetric sediment transport rates per unit  
 155 width along the Cartesian coordinates.

156 Following the methodology proposed by [24], the Exner equation is then  
 157 extended to sediment mixtures assuming that (i) the grain size distribution

158 of the bed surface is divided in  $N$  discrete fractions, (ii)  $F_s$  are the frequencies  
 159 of each grain size on the surface ( subscript  $s$  denotes the particle class  
 160 and ranges from 1 to  $N$ ) and (iii) that a constant density is assumed for  
 161 all sizes, given that all sediment on the channel bed has originated from a  
 162 common source. Each sediment fraction is associated with a characteristic  
 163 grain diameter,  $D_s$ .

164 Each particle size may be transported at different rates. Let us denote  $q_{bs}$   
 165 the fractional sediment transport rate associated to grain size  $s$ , which is  
 166 obtained as the product of the sediment transport capacity that the flow is  
 167 able to mobilize,  $q_{bs}^0$  and the proportion of sediment of that particular grain  
 168 size on the bed surface  $F_s$ .  $q_{bs}^0$  is computed through a closure equation as  
 169 shown below. Hence, the sediment fluxes are written in terms of both flow  
 170 and bed characteristics as,

$$q_{bs} = F_s q_{bs}^0 \quad (8)$$

171 This surface-based formulation[26] assures that, regardless of the magnitude  
 172 of the the fractional sediment transport capacity  $q_{bs}^0$ ,  $q_{bs} = 0$  if the fraction  
 173  $s$  is not present on the bed. Finally, the total sediment transport rate,  $q_b$  is  
 174 obtained as the sum of the sediment fluxes of each grain size  $s$ ,

$$q_b = \sum_{s=1}^N q_{bs} \quad (9)$$

175 Bedload and surface textures vary in time, adjusting to changes in the flow  
 176 field and bed topography. [46] and more recently [44] conceptualize the  
 177 channel bed to be formed by two layers which are defined in Figure 1:(i)  
 178 an uppermost active, exchange or surface layer, the thickness of which,  $L_a$ ,  
 179 extends from the bed surface downwards and (ii) a substrate layer, placed  
 180 underneath the active layer.

181 The active layer accounts for the average uppermost bed layer that con-  
 182 tributes to sediment transport. Therefore, all particles entrained into bedload  
 183 are supplied from the active layer. Under these conditions, the probability  
 184 of a particle to be entrained per unit time is constant and equal to 1, [25].  
 185 This implies that the probability for substrate particles to be entrainment is  
 186 zero. Thus, substrate texture does not affect sediment transport rates and  
 187 their texture. The vertical discrete fraction distribution of each  $s$  grain size  
 188 in the bed  $f_s$ , presents a discontinuity depending on the vertical position,  $z$

$$f_s = \begin{cases} F_s(x, y, z, t) & \text{if } \eta - L_a < z < \eta \\ f_{ss}(x, y, z) & \text{if } z < \eta - L_a \end{cases} \quad (10)$$

189 where  $f_{ss}$  are the fractions of the substrate which do not vary in time but  
 190 may vary in the longitudinal and transverse directions, i.e.  $x$  and  $y$  Cartesian  
 191 coordinates. Of particular interest is the variation of  $f_{ss}$  along the vertical  
 192 coordinate which represents the stratigraphy of the bed at any given point.  
 193 The non-time dependency of  $f_{ss}$  is true when the river bed at a particular  
 194 location undergoes a single phase of aggradation or degradation. Multiple  
 195 stages of bed aggradation/degradation change the stratigraphy of the bed  
 196 over time. For the sake of simplicity, the present formulation does not account  
 197 for such temporal changes. Additionally, bedload transport rates associated  
 198 with each grain size, which are allowed to evolve over time, are denoted as  
 199  $f_{bs} = q_{bs}/q_b$ .

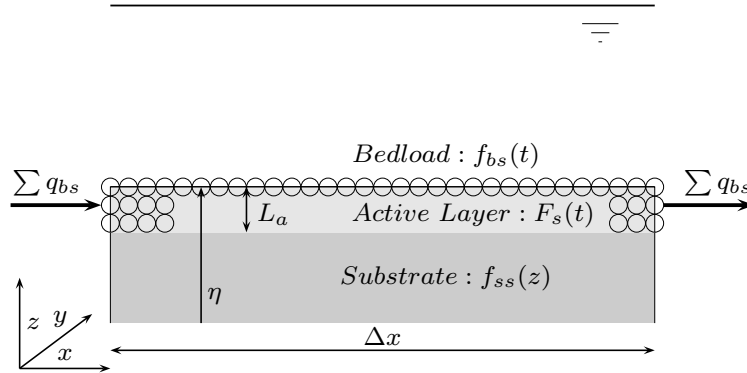


Figure 1: Two bed layer model

200 All sediment fractions described above must be conserved. Therefore the  
 201 following constraints must be fulfilled

$$\sum_{s=1}^N F_s = 1 \quad \sum_{s=1}^N f_{bs} = 1 \quad \sum_{s=1}^N f_{ss} = 1 \quad (11)$$

202 The equation for the conservation of mass of each discrete grain size fraction  
 203  $V_s$  present on the bed is derived next. The application of the Reynolds  
 204 transport theorem and the mass balance equation on an arbitrary control



205 volume  $\Omega$  yields the general integral equation for the conservation of sediment  
 206 for the  $s$ -th particle size

$$\frac{dV_s}{dt} = \frac{d}{dt} \int_{\Omega} \rho(x, t) d\Omega = 0 \Leftrightarrow \frac{\partial}{\partial t} \int_{\Omega} \rho d\Omega + \oint_{\Gamma} \rho \mathbf{u}_r \mathbf{n} d\Gamma = 0 \quad (12)$$

207 where  $V_s$  is the fractional mass of sediment within the control volume  $\Omega$ ,  $\Gamma$   
 208 represents mobile and fixed boundaries of the control volume the mass fluxes  
 209 flow across,  $\mathbf{u}_r$  is the relative velocity between the flow velocity and the speed  
 210 at which the boundary  $\Gamma$  moves, and  $\mathbf{n}$  is the outward unit vector normal to  
 211  $\Gamma$ .

212 The time evolution of  $V_s$  in equation (12) is computed taking into account the  
 213 fractional mass of sediment within the control volume, represented by the first  
 214 volume integral in the right hand-side equation (12), and the sediment fluxes  
 215 through the boundaries of the domain, accounted by the second contour  
 216 integral in equation (12). The fractional mass is evaluated as

$$V_s = \int_{\Omega} \rho d\Omega = \int_0^{\eta} \int_{A(x,y)} \rho_s f_s (1 - p_b) dA dz \quad (13)$$

217 where  $A(x, y)$  is the area across which sediment moves vertically. For this  
 218 case  $A(x, y) = \Delta x \Delta y$ .  $\rho_s$  is the density of the sediment. Then, equation (13)  
 219 becomes

$$V_s = \int_0^{\eta} \rho_s f_s (1 - p_b) \Delta y \Delta x dz \quad (14)$$

220 The integral in (14) is split in two parts to account for the vertical disconti-  
 221 nuity of the bed stratigraphy  $f_s$  in (10), which has been sketched in Figure  
 222 1

$$V_s = \rho_s (1 - p_b) \Delta y \Delta x \left[ \underbrace{\int_0^{\eta-L_a} f_{ss} dz}_{\text{Substrate}} + \underbrace{\int_{\eta-L_a}^{\eta} F_s dz}_{\text{Active Layer}} \right] \quad (15)$$

223 As stated earlier in (10) the substrate fraction  $f_{ss}$  may depend on the vertical  
 224 coordinate  $z$  on stratified beds. The second integral in (15) involving the  
 225 active layer texture  $F_s$  represents the fractional volume of sediment within  
 226 this layer. As stated above,  $F_s$  is assumed to be constant within the active

227 layer,  $L_a$ . Bearing these hypotheses in mind, the rate of change of the volume  
 228 of sediment  $V_s$  in (15) is

$$\frac{\partial V_s}{\partial t} = \rho_s (1 - p_b) \Delta y \Delta x \frac{\partial}{\partial t} \left[ \int_0^{\eta-L_a} f_{ss} dz + F_i L_a \right] \quad (16)$$

229 The temporal evolution of the mass of sediment  $V_s$ , i.e. the integral in the  
 230 right hand-side of (16) can be rewritten by applying the Leibnitz' rule

$$\frac{\partial V_s}{\partial t} = \rho_s (1 - p_p) \Delta y \Delta x \left[ f_{ss(z=\eta-L_a)} \frac{\partial}{\partial t} (\eta - L_a) + \frac{\partial}{\partial t} F_s L_a \right] \quad (17)$$

231 The term  $f_{ss(z=\eta-L_a)}$  represents the fractional exchange of material between  
 232 the active layer and the substrate, hereafter denoted  $f_{es}$ . This term depends  
 233 on whether bed aggrades or degrades [30, 47]

$$f_{es} = \begin{cases} f_{ss} & \text{if } \frac{\partial \eta}{\partial t} < 0 \\ \alpha F_s + (1 - \alpha) f_{bs} & \text{if } \frac{\partial \eta}{\partial t} > 0 \end{cases} \quad (18)$$

234 where  $\alpha$  is a parameter ranging between 0 and 1 and that needs to be cali-  
 235 brated. Substrate fractions are considered if degradation occurs. Conversely,  
 236 a linear combination of the surface and bedload transport textures is taken  
 237 in case of aggradation.

238 The net flux of sediment across the boundaries of the control domain (contour  
 239 volume in the right hand-side of (12)) is denoted as  $\Psi_s$  and it is computed  
 240 as

$$\Psi_s = \oint_{\Gamma} \rho \mathbf{u}_r \mathbf{n} d\Gamma = \rho_s \Delta y (q_{bs,x+\Delta x} - q_{bs,x}) + \rho_s \Delta x (q_{bs,y+\Delta y} - q_{bs,y}) \quad (19)$$

241 Finally, gathering equation (17) and (19), the sediment mass balance for the  
 242 fraction  $s$  in (12) can be expressed as

$$(1 - p_b) \left[ f_{es} \frac{\partial}{\partial t} (\eta - L_a) + \frac{\partial}{\partial t} (F_s L_a) \right] = - \frac{\partial q_{bs,x}}{\partial x} - \frac{\partial q_{bs,y}}{\partial y} \quad (20)$$

243 If the bed is composed of uniform material, fractions  $f_{es}$  and  $F_s$  are constant  
 244 in time and equal to 1. Substituting these values in (20) yields the classical  
 245 Exner equation for uniform grain sizes [44].

246 *2.2.1. Weak-hyperbolicity strategy*

247 The way by which equation (20) is solved is the original contribution of this  
248 work. A detailed description of the proposed method is outlined below.

249  
250  $F_s L_a$  is the conserved variable in (20). This term expresses the fractional  
251 volume of sediment within the active layer. The time variation of this variable  
252 is balanced by  $\Psi_s^v$  and  $\Psi_s^h$  (Figure 2). Rearranging terms, equation (20) can  
253 be rewritten as

$$\frac{\partial}{\partial t} (F_s L_a) = - \underbrace{\frac{1}{1-p_b} \left( \frac{\partial q_{bs,x}}{\partial x} + \frac{\partial q_{bs,y}}{\partial y} \right)}_{\Psi_s^v} - \underbrace{f_{es} \frac{\partial}{\partial t} (\eta - L_a)}_{\Psi_s^h} \quad (21)$$

254 Mathematical advantages arise from expressing the rate of change of the sur-  
255 face fractions as in (21). Namely, although (21) is not a hyperbolic equation,  
256 a wave speed at which perturbation of the surface texture propagates  $\lambda_{F_s}$   
257 can be estimated through the sediment fluxes as follows,

$$\frac{\partial}{\partial t} (F_s L_a) + \xi \left( \frac{\partial q_{bs,x}}{\partial x} + \frac{\partial q_{bs,y}}{\partial y} \right) = -f_{es} \frac{\partial}{\partial t} (\eta - L_a) \quad (22)$$

$$\lambda_{F_s} \approx \xi \frac{\partial \mathbf{q}_{bs}}{\partial F_s L_a} = \xi \frac{\partial \mathbf{q}_{bs}^0 F_s}{\partial (F_s L_a)} \quad (23)$$

258 where  $\mathbf{q}_{bs}^0 = (q_{bs,x}^0, q_{bs,y}^0)$ , being  $q_{bs,x}^0$  the sediment discharge in the longitudinal  
259 direction and  $q_{bs,y}^0$  the sediment discharge in the transversal direction. This  
260 wave speed was first estimated for uniform material in [14]. Here, this idea  
261 has been extended to heterogeneous sediment. The wave speed provides  
262 information of the celerity at which the surface texture changes, hereafter  
263 referred to as sorting celerity. This sorting celerity needs to be retained to  
264 ensure numerical stability of the solver, [48, 14], and also for the upwinding  
265 technique considered in the next section.

266 *2.2.2. Closure equations*

267  $4 + N$  equations need to be solved. 3 equations arise from the hydrodynamic  
268 model, i.e. mass and momentum conservation equations, system (1), and the  
269 rest  $1 + N$  equations are derived from the morphodynamic model (the classical  
270 Exner and Hirano equations, (7) and (22) to update the bed elevation and  
271 the surface fractions respectively). The number of dependent variables is

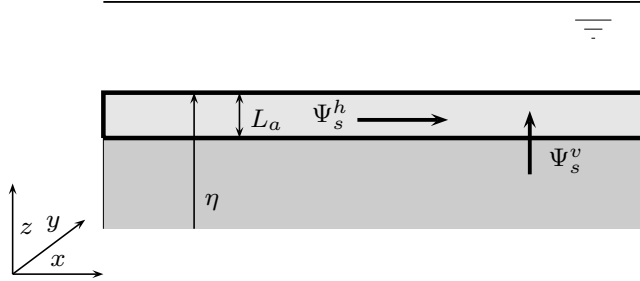


Figure 2: Mass conservation in active layer

272 thus equal to  $4 + N$ : (i) the flow depth  $h$ , (ii) the depth-averaged velocities  
 273 in Cartesian coordinates  $u$  and  $v$ , (iii) the bed elevation  $\eta$  and (iv) the surface  
 274 fractions on the bed surface  $F_s$ .

275 However, 3 complementary equations are needed to describe the bed rough-  
 276 ness (expressed in terms of the Manning's coefficient  $n$  in (5)), sediment  
 277 transport rates  $q_{bx}$ ,  $q_{by}$  in (7) and the thickness of the active layer  $L_a$  in (22).

278

279 *Bed roughness*

280 Bed shear stresses are computed by means of the Manning's equation, which  
 281 takes the roughness of the bed  $n$  into account. This roughness, which is  
 282 associated with the texture of the bed, is computed by using the Manning-  
 283 Strickler formula [49]:

$$n = \frac{1}{26} D_{90}^{1/6} \quad (24)$$

284 where  $D_{90}$  is the 90 percentile of the bed, i.e. the grain size of the surface  
 285 texture such that 90% of the bed is finer. Equation (24) implies that when  
 286 the bed surface is composed of sediment mixtures, the surface texture (i)  
 287 may be non-uniform across the domain and (ii) it may vary in time at a  
 288 given location (10). Therefore, under these conditions,  $n$  is not constant.  
 289 Conversely, it varies according to the temporal and spatial evolution of  $D_{90}$ .

290

291 *Bed load sediment transport capacity formula*

292 The fractional bedload transport capacity  $q_{bs}^0$  can be obtained by using prob-  
 293 ability laws [50, 2] or by means of empirically fitted expressions to experi-  
 294 mental data (e.g. [51, 52, 53]). The modulus of the sediment transport rate,  
 295  $\mathbf{q}_{bs}^0 = (q_{bs,x}^0, q_{bs,y}^0)$  is

$$|\mathbf{q}_{bs}^0| = \sqrt{(q_{bs,x}^0)^2 + (q_{bs,y}^0)^2} \quad (25)$$

296 Bedload transport rates are usually expressed in dimensionless form by the  
297 Einstein number

$$\Phi_s = \frac{|\mathbf{q}_{bs}^0|}{\sqrt{g(r-1)D_s^3}} \quad (26)$$

298 where  $r = \rho_s/\rho_w$  is the ratio between sediment ( $\rho_s$ ) and water ( $\rho$ ) densities.  
299 Dimensionless sediment transport rates are usually expressed in terms of the  
300 dimensionless shear stress or Shields number as

$$\theta_s = \frac{|\mathbf{T}_b|}{g(\rho_s - \rho_w)D_s} \quad (27)$$

301 where  $\mathbf{T}_b = (\tau_{b,x}, \tau_{b,y})$  is the shear stress at the bed obtained assuming steady  
302 flow through the Manning's coefficient. This allows expressing  $|\mathbf{T}_b|$  as

$$|\mathbf{T}_b| = \sqrt{\tau_{b,x}^2 + \tau_{b,y}^2} \quad (28)$$

303 which, with the aid of (5), leads to the following expression for the Shields  
304 number:

$$\theta_s = \frac{n^2}{(s-1)D_s h^{1/3}}(u^2 + v^2) = \frac{n^2}{(s-1)D_s h^{1/3}}|\mathbf{u}|^2 \quad (29)$$

305 Fractional bedload transport rates are calculated using the sediment trans-  
306 port capacity formula derived by [54]. This equation, based on the difference  
307 between the acting dimensionless bed shear stress  $\theta_s$  and the dimensionless  
308 critical shear stress for the onset of motion  $\theta_{cs}$  associated with the  $s$ th grain  
309 size, is expressed as

$$\Phi_s = 17(\theta_s - \theta_{cs})(\sqrt{\theta_s} - \sqrt{\theta_{cs}}) \quad (30)$$

310  $\theta_{cs}$  associated with the grain size  $s$  is obtained by using the hiding/exposure  
311 function proposed by [55] as

$$\frac{\theta_{cs}}{\theta_{c50}} = \begin{cases} 0.843 \left(\frac{D_s}{D_{50}}\right)^{-1} & \frac{D_s}{D_{50}} \leq 0.4 \\ \left(\frac{\log 19}{\log(19 \frac{D_s}{D_{50}})}\right)^2 & \frac{D_s}{D_{50}} > 0.4 \end{cases} \quad (31)$$

312 where  $\theta_{c50}$  and  $D_{50}$  are the dimensionless critical shear stress and grain size  
 313 associated with the median diameter of surface texture, respectively.

314

315 *Active layer*

316 The definition of the active layer model requires a closure relation to de-  
 317 scribe its thickness. A constant value of  $L_a$  has been commonly assumed.  
 318 However, as stated by [41], this approach deeply affects the bed celerity. In  
 319 one-dimensional numerical models,  $L_a$  is usually associated with a charac-  
 320 teristic length of the river bed (e.g. some reference sediment diameter in  
 321 the plane bed case, the dune height in case of the appearance of bedforms).  
 322 Since the active layer thickness accounts for the fluctuations of the bed ele-  
 323 vation,  $D_{90}$  is usually taken as the reference grain size to which its thickness  
 324 is related. Hence, the thickness of the active layer is expressed as

$$L_a = KD_{90} \quad (32)$$

325 where  $K$ , ranges between 1 and 3 [30, 31, 32, 33, 34, 36, 37, 38, 39].

### 326 **3. Numerical scheme**

#### 327 *3.1. Hydrodynamic numerical scheme*

328 The system of equations in (1) is integrated using the Gauss theorem in a  
 329 grid cell  $\Omega_L$ .  $\mathbf{n}$  denotes the outward vector to the cell edge  $\Omega_L$  and  $\mathbf{E}_\mathbf{n} =$   
 330  $\mathbf{F}n_x + \mathbf{G}n_y$ .

$$\frac{\partial}{\partial t} \int_{\Omega_L} \mathbf{U} d\Omega + \oint_{\partial\Omega_L} \mathbf{E}_\mathbf{n} dl = \int_{\Omega_L} (\mathbf{S}_\tau + \mathbf{S}_b) d\Omega \quad (33)$$

331 The second integral in (33) can be explicitly obtained as a sum over the cell  
 332 edges,

$$\frac{\partial}{\partial t} \int_{\Omega_L} \mathbf{U} d\Omega + \sum_{k=1}^{NE} \int_{l_k} \mathbf{E}_{\mathbf{n}_k} dl_k = \int_{\Omega_L} (\mathbf{S}_\tau + \mathbf{S}_b) d\Omega \quad (34)$$

333 where  $dl_k$  is the length of each edge of a cell and  $NE$  is the number of edges  
 334 of a grid cell  $\Omega_L$ . The values of the conserved variables inside the cells,  $\mathbf{U}_L^n$ ,  
 335 are assumed piecewise constant, i.e. averaged within each cell  $A_L$ . Thus, an  
 336 uniform value at each cell  $A_L$  is obtained

$$\mathbf{U}_L^n = \frac{1}{A_L} \int_{\Omega_L} \mathbf{U}(x, y, t^n) d\Omega \quad (35)$$

337 Substituting the piecewise assumption, (35), in (34), this latter equation is  
338 written as

$$\frac{\partial}{\partial t} \int_{\Omega_L} \mathbf{U} d\Omega + \sum_{k=1}^{NE} (\mathbf{E}_n)_k l_k = \sum_{k=1}^{NE} \mathbf{S}_{\tau n} l_k + \sum_{k=1}^{NE} \mathbf{S}_{bn} l_k \quad (36)$$

339 where  $l_k$  is the length of each edge of a cell and  $\mathbf{S}_{bn}$  and  $\mathbf{S}_{\tau n}$  are the integrals  
340 of the friction and bed slope terms [9].

341 The numerical scheme to solve (36) is constructed by means of an approx-  
342 imate Jacobian matrix  $\tilde{\mathbf{J}}_{n,k}$  at each edge  $k$  between neighbouring cells and  
343 defined through the normal fluxes between these adjacent cells  $\mathbf{E}_n$

$$(\delta \mathbf{E}_n)_k = \tilde{\mathbf{J}}_{n,k} \delta \mathbf{U}_k \quad (37)$$

344 where  $\delta(\mathbf{E}_n)_k = (\mathbf{E}_R - \mathbf{E}_L)_{n,k}$ ,  $\delta \mathbf{U}_k = \mathbf{U}_R - \mathbf{U}_L$ , and  $\mathbf{U}_L$  and  $\mathbf{U}_R$  are the  
345 initial values of the conserved variables at adjacent cells  $L$  and  $R$ .

346 From this approximate Jacobian matrix a set of three real eigenvalues  $\tilde{\lambda}_k^m$  and  
347 eigenvectors  $\tilde{\mathbf{e}}_k^m$  are obtained. From this definition, it is possible to define two  
348 approximate matrices  $\tilde{\mathbf{P}} = (\tilde{\mathbf{e}}^1, \tilde{\mathbf{e}}^2, \tilde{\mathbf{e}}^3)$  and  $\tilde{\mathbf{P}}^{-1}$  which allow to diagonalize the  
349 Jacobian matrix,

$$\tilde{\mathbf{J}}_{n,k} = \tilde{\mathbf{P}}_k \tilde{\Lambda}_k \tilde{\mathbf{P}}_k^{-1} \quad (38)$$

350 being  $\tilde{\Lambda}_k$  the matrix which contains the eigenvalues in the diagonal. In  
351 addition, the vector of conserved variables,  $\mathbf{U}$  is then split through the matrix  
352 of eigenvectors,  $\tilde{\mathbf{P}}$ , as

$$\delta \mathbf{U}_k = \tilde{\mathbf{P}}_k \tilde{\mathbf{A}}_k = \sum_{m=1}^3 (\tilde{\alpha} \tilde{\mathbf{e}})_k^m \quad (39)$$

353 The source terms in (36) are also projected onto the matrix of eigenvectors,  
354  $\tilde{\mathbf{P}}$  to guarantee the exact equilibrium between fluxes and source terms,

$$(\mathbf{S}_{bn}, \mathbf{S}_{\tau n})_k = \tilde{\mathbf{P}}_k \tilde{\mathbf{B}}_k = \sum_{m=1}^3 (\tilde{\beta} \tilde{\mathbf{e}})_k^m \quad (40)$$

355 Based on the above information the volume integral in the cell at time  $t^{n+1}$   
 356 is expressed as

$$\mathbf{U}_L^{n+1} = \mathbf{U}_L^n - \sum_{k=1}^{NE} \sum_{m=1}^3 (\tilde{\lambda}^- \tilde{\alpha} - \tilde{\beta}^-)^m \tilde{\mathbf{e}}_k^m l_k \frac{\Delta t}{A_L} \quad (41)$$

357 The superscript minus in (41) implies that only the incoming waves are con-  
 358 sidered for updating the values of each cell, defining  $\lambda^- = \frac{1}{2}(\lambda - |\lambda|)$ . Split-  
 359 ting the fluxes as shown in equations (39)-(40) allows no special treatment at  
 360 the boundary cells: the conserved variables  $\mathbf{U}$  are updated in time by means  
 361 of the incoming information which is averaged at each edge. Complete details  
 362 can be found in [9]

363 To avoid numerical instabilities, time step  $\Delta t$  has to be reduced sufficiently  
 364 so that there are no interactions of waves between neighbouring cells. The  
 365 Courant dimensionless number, CFL is used to control the stability of the  
 366 numerical scheme

$$CFL = \frac{\Delta t^{hydro}}{\Delta t^{\tilde{\lambda}}} \quad \text{where} \quad \Delta t^{\tilde{\lambda}} = \frac{\min(\chi_L, \chi_R)}{\max |\tilde{\lambda}^m|} \quad (42)$$

367 where the superscript  $m$  ranges from 1 to 3, according to the three equations  
 368 (1) for the hydrodynamic part.  $\chi$  is the relevant distance for numerical  
 369 stability which, in a two-dimensional model, must consider the area of the  
 370 adjacent cells  $L$  and  $R$  and the length of the shared  $k$  edges  $l_k$  [56],

$$\chi_L = \frac{A_L}{\max_{k=1, NE} l_k} \quad (43)$$

371 Equation (42) allows choosing an appropriate  $\Delta t$  such that it always falls  
 372 within the stability region as

$$\Delta t^{hydro} \leq CFL \Delta t^{\tilde{\lambda}} \quad (44)$$

373 with  $CFL=1$  in the case of 1D configurations and  $CFL \leq 1/2$  in the case of  
 374 triangular unstructured grids.

### 375 3.2. Morphodynamic numerical scheme

#### 376 3.2.1. Bed elevation updating

377 Following [14], sediment conservation equation (7) needs to be integrated in  
 378 a grid cell  $\Omega_L$



$$\frac{\partial}{\partial t} \int_{\Omega_L} \eta d\Omega + \oint_{\Gamma} \xi(\mathbf{q}_{bn}) d\Gamma = 0 \quad (45)$$

379 where it is worth recalling that  $\mathbf{q}_{bn} = (q_{b,x}n_x + q_{b,y}n_y)$  and also  $q_{b,x} =$   
 380  $\sum_{s=1}^N q_{bs,x}$ ,  $q_{b,y} = \sum_{s=1}^N q_{bs,y}$  where  $N$  is the number of sediment grain sizes  
 381 contained in the mixture.

382 Using Gauss theorem, assuming a piecewise representation of the variable  $\eta$   
 383 and noting that the second integral can be expressed as the sum of fluxes  
 384 across the edges of the cell  $\Omega_L$ ,

$$\frac{\partial}{\partial t} \int_{\Omega_L} \eta d\Omega + \sum_{k=1}^{NE} \xi \int \mathbf{q}_{bn_k} dl_k = 0 \quad (46)$$

385 Then, the Godunov first order method is built through a flux scheme, con-  
 386 sidering outgoing and incoming fluxes through the edges of the cell. Hence  
 387 the bed elevation  $\eta$  is updated as

$$\eta_L^{n+1} = \eta_L^n - \sum_{k=1}^{NE} \xi q_{bn,k}^* \frac{\Delta t l_k}{A_L} \quad (47)$$

388 where total sediment fluxes across each edge are written as the sum of the  
 389 fractional bedload transport rate  $s$ ,

$$q_{bn,k}^* = \sum_{s=1}^N q_{bsn,k}^* \quad (48)$$

390 The fractional bedload transport rates  $q_{bsn,k}^*$  are computed following the up-  
 391 wind philosophy, i.e., taking the values from left or right side according the  
 392 sign of the celerity,

$$q_{bsn,k}^* = \begin{cases} q_{bsn,L} & \text{if } \tilde{\lambda}_{bsn,k} > 0 \\ q_{bsn,R} & \text{if } \tilde{\lambda}_{bsn,k} < 0 \end{cases} \quad \text{with } \begin{cases} q_{bsn,L} = (q_{bs,x}n_x + q_{bs,y}n_y)_L \\ q_{bsn,R} = (q_{bs,x}n_x + q_{bs,y}n_y)_R \end{cases} \quad (49)$$

393 being  $q_{bsn,L}$  and  $q_{bsn,R}$  the bedload transport rates associated with the  $s$  grain  
 394 size across neighbouring cells ( $L$ ,  $R$ ).  $\tilde{\lambda}_{bsn,k}$  is the numerical bed celerity, i.e.  
 395 the speed at which changes in the bed propagate along the domain, estimated  
 396 as

$$\tilde{\lambda}_{\text{bsn},k} \approx \frac{\delta(\xi q_{\text{bsn},k})}{\delta\eta} \quad (50)$$

397 where  $\delta q_{\text{bsn},k} = (q_{\text{bsn},R}^0 F_{s,R} - q_{\text{bsn},L}^0 F_{s,L})$  is the normal sediment transport  
 398 flux across the  $k$  edge and  $F_{s,R}$  and  $F_{s,L}$  are the content of the grain size  $s$   
 399 on the bed surface in cells  $R$  and  $L$  respectively.

400 At this stage, the stability criterion has to be revisited to include the esti-  
 401 mations of bed celerities defined in (50). Hence, the time step limitation for  
 402 the bedload transport is imposed as

$$\Delta t^{\text{bed}} = CFL \Delta t^{\tilde{\lambda}_b} \quad \text{where} \quad \Delta t^{\tilde{\lambda}_b} = \frac{\min(\chi_L, \chi_R)}{|\tilde{\lambda}_{\text{bsn},k}|} \quad (51)$$

403 with  $CFL=1$  in the case of 1D configurations and  $CFL \leq 1/2$  in the case of  
 404 triangular unstructured grids.

405 Considering both time restrictions, hydrodynamic and morphodynamic wave  
 406 speeds, the following stability criterion is defined so that numerical stability  
 407 is numerically ensured

$$\Delta t = \min(\Delta t^{\text{bed}}, \Delta t^{\text{hydro}}) \quad (52)$$

408 A detailed description of both hydrodynamic and morphodynamic numerical  
 409 schemes can be found in [14].

### 410 3.2.2. Surface grain size fraction update

411 The following physically-based, self-stable numerical scheme is introduced.  
 412 We start this mathematical development from (21)

$$\frac{\partial}{\partial t} (F_s L_a) = -\frac{1}{1-p_b} \left( \frac{\partial q_{\text{bs},x}}{\partial x} + \frac{\partial q_{\text{bs},y}}{\partial y} \right) - f_{es} \frac{\partial}{\partial t} (\eta - L_a) \quad (53)$$

413 Equation (21) is integrated following the same steps used above: (i) integra-  
 414 tion over a grid cell  $\Omega_L$ , (ii) applying the Gauss theorem and (iii) assuming a  
 415 piecewise representation of the conserved variables. Thus, the Godunov first  
 416 order method is built through a flux scheme. Therefore, the surface fraction  
 417  $s$  at cell  $L$  is updated as follows,

$$(F_{s_L} L_a)^{n+1} = (F_{s_L} L_a)^n + \Delta t \left[ \sum_{k=1}^{NE} \xi (-q_{\text{bsn},k}^*) \frac{l_k}{A_L} - f_{esL} \frac{\Delta(\eta - L_a)}{\Delta t} \right] \quad (54)$$

418 where the last term associated with  $(\eta - L_a)$  is computed for simplicity  
 419 implicitly, i. e. evaluating that term as a source term using known quantities  
 420 from the present and past time levels [57, 33, 34, 38]. Additionally, the flux  
 421 associated with the fractional transport rate  $q_{bsn,k}^*$  between the cells  $L$  and  
 422  $R$  is evaluated following an upwind technique as follows,

$$q_{bsn,k}^* = \begin{cases} q_{bsn,L} & \text{if } \tilde{\lambda}_{F_s,k} > 0 \\ q_{bsn,R} & \text{if } \tilde{\lambda}_{F_s,k} < 0 \end{cases} \quad \text{with} \quad \begin{cases} q_{bsn,L} = (q_{bs,x}n_x + q_{bs,y}n_y)_L \\ q_{bsn,R} = (q_{bs,x}n_x + q_{bs,y}n_y)_R \end{cases} \quad (55)$$

423 where the sorting celerities  $\tilde{\lambda}_{F_s,k}$  are defined as

$$\tilde{\lambda}_{F_s,k} \approx \frac{\delta(\xi q_{bsn,k})}{\delta(F_s L_a)} \quad (56)$$

424 where  $\delta q_{bsn,k} = (q_{bsn,R}^0 F_{s,R} - q_{bsn,L}^0 F_{s,L})$  is the normal flux of the sediment  
 425 transport rate across the edge  $k$  and  $\delta F_s L_a = (F_{s,R} L_{a,R} - F_{s,L} L_{a,L})$ . Addi-  
 426 tionally, since mass conservation must be satisfied, the following condition

$$\sum_{s=1}^N (F_s L_a) = L_a \quad (57)$$

427 must be fulfilled.  $\tilde{\lambda}_{F_s,k}$  in equation (56) estimates a numerical sorting celerity  
 428 for each grain size. Therefore, the numerical stability criterion defined by  
 429 (51) and (52) must include this wave speed. Consequently, the time step  
 430 restriction for the sorting wave is fixed as

$$\Delta t^{fraction} = CFL \Delta t^{\tilde{\lambda}_{F_s}} \quad \text{where} \quad \Delta t^{\tilde{\lambda}_{F_s}} = \frac{\min(\chi_L, \chi_R)}{|\tilde{\lambda}_{F_s}|} \quad (58)$$

431 With this new constraint, the time step that governs the stability of the  
 432 numerical scheme proposed to solve the system of equations formed by (1),  
 433 (7) and (21) is obtained as

$$\Delta t = \min(\Delta t^{fraction}, \Delta t^{bed}, \Delta t^{hydro}) \quad (59)$$

434 This new stability criterion prevents instabilities of the numerical scheme.  
 435 The performance of the numerical outcomes are presented in the next section.

#### 436 4. Examples of application: test cases

437 The weak hyperbolicity strategy outlined in the previous section is now ap-  
438 plied to several test cases. The first two test cases aim to analyze the ability  
439 of the numerical model to reach equilibrium conditions under aggradational  
440 and degradational scenarios. The results of these numerical experiments are  
441 compared to a widely used and well established one-dimensional model [39].  
442 The propagation of the surface sediment sorting along the domain is ana-  
443 lyzed. The third test case is focused on a two dimensional dam break. The  
444 last numerical run presents the comparison of a set of experiments on dam  
445 removal with the presence of mixtures. The sediment transport capacity for-  
446 mula proposed by Ashida-Michiue [54] has been used in all simulations. The  
447 thickness of the active layer is computed as one time the  $D_{90}$ , i.e.  $K = 1$ ,  
448 equation (32).

##### 449 4.1. One dimensional synthetic tests

450 In a feed experiment in a flume, i.e. when both water discharge and sedi-  
451 ment feed rate are introduced in the channel at constant rates, bed slope and  
452 surface grain size distribution gradually adjusts to a steady state in which  
453 bedload transport rate and its texture at every single point of the channel  
454 match those of the feed [58]. Under these conditions, the ability of the nu-  
455 merical scheme to reach a steady state can be easily evaluated. Equation  
456 (18) states that the bed response imposes different vertical fluxes of sedi-  
457 ment depending on whether the bed aggrades or degrades. Therefore, in  
458 order to examine the performance of the numerical model under all possible  
459 conditions, two general aggradation and degradation test cases are analyzed.

460  
461 First, we need to determine the equilibrium conditions for a given initial  
462 setup. This occurs when the bedload transport rate and texture are such that  
463 the bed does not aggrade or degrade. To do so, we need to apply the selected  
464 sediment transport capacity formula for the given initial conditions. If the  
465 equilibrium feed rate is then modified but its grain size distribution is fixed, a  
466 new equilibrium will be achieved after a process of aggradation/degradation.  
467 All the test cases considered herein disturb the initial equilibrium condition  
468 by arbitrarily increasing/reducing the feed rate by 35% (see sections below).  
469 In order to prevent spurious numerical instabilities arising from the jump  
470 in the longitudinal distribution of the sediment transport rates between the  
471 channel inlet and the cross-sections downstream once the feed is modified,

472 sediment feed rate is introduced by means of an upstream boundary condi-  
 473 tion function that gradually adjust the initial sediment transport rate to the  
 474 desired one, i.e. 35% higher or lower than the one at initial conditions. This  
 475 function takes the form of a sinusoidal function as follows

$$q_b^{inlet}(t) = q_b^{eq} \sin\left(\frac{\pi}{4} \pm \frac{t}{4996.132}\right) \frac{1}{0.707} \quad (60)$$

476 where  $q_b^{inlet}$  is the feed rate (bedload transport at the inlet),  $q_b^{eq}$  is the bedload  
 477 transport at equilibrium. To achieve the  $\pm 35\%$ , a lag time of 3924 s is defined  
 478 ( $\sim 4996.132 \cdot \pi/4$ ). Feed rate is constant and equal to  $q_b^{eq}$  hereafter.

479 All simulations are based on a straight rectangular 6 m-long, 1 m-wide chan-  
 480 nel and a bed slope  $S_o=0.002$  m/m. A steady stage is considered as an initial  
 481 configuration for all runs. In order to strengthen the accuracy and the qual-  
 482 ity of the numerical predictions, three different grain size distributions have  
 483 been considered under constant water discharge for all cases ( $Q = 0.2$  m<sup>3</sup>/s).  
 484 The vertical texture of the bed is considered equal to the surface texture.  
 485 Conversely, due to the different surface textures used in the tests, feed rate  
 486 and texture at the inlet slightly vary for each run. Boundary conditions (feed  
 487 rate and grain size distribution at the inlet and water surface elevation at the  
 488 outlet) are different for each test. Recall that the bed roughness (and hence  
 489 the water depth and the sediment transport rates) changes according to the  
 490 grain size distribution of the surface (24). Water depth at the outlet results  
 491 from a steady state calculation reached with the same initial configuration  
 492 (slope and surface texture) but under fixed-bed conditions. This ensures that  
 493 the initial condition for the mobile bed calculations is uniform and steady.  
 494 No fixed bed elevation is imposed at the channel outlet. This implies that  
 495 the bed at this location evolves in time until a new equilibrium is achieved.  
 496 However, since the water depth at the outlet has been held constant through-  
 497 out the simulations, this condition is equivalent to that in which the change  
 498 in the bed is constrained and a fixed water surface elevation at this station  
 499 is imposed. The mesh size for all tests is  $\Delta x=0.10$  m.

500

#### 501 *4.1.1. One-dimensional degrading tests*

##### 502 **Test 1. Texture 1: One grain size.**

503

504 A uniform size distribution with  $D_s=2.828$  mm is considered (Table 1). Ini-  
 505 tial and boundary conditions for this test case can be found in Table 1.

test case	N	Grain sizes $D_s$ (mm)				Grain size fractions $F_s$ (-)				$D_g$ (mm)	$q_b^{eq}$ ( $m^2/s$ )	$q_{b,feed}$ ( $m^2/s$ )	$\eta_{w,o}$ (m)
1	1	2.828				1				2.828	$4.7810 \cdot 10^{-5}$	$3.1079 \cdot 10^{-5}$	0.1921
2	2	1.834	2.181	0.5	0.5	0.5	0.5	0.5	2.000	$4.5717 \cdot 10^{-5}$	$2.9716 \cdot 10^{-5}$	0.1894	
3	4	1.541	1.830	2.182	2.593	0.25	0.25	0.25	0.25	2.000	$4.7667 \cdot 10^{-5}$	$3.0984 \cdot 10^{-5}$	0.1921

Table 1: Initial and boundary conditions for all one-dimensional test cases:  $D_s$ ,  $F_s$  grain size distribution of the feed and the bed surface,  $D_g$ : mean geometric grain size of the feed and the bed surface,  $q_b^{eq}$  sediment transport rate in equilibrium with the initial conditions,  $q_{b,feed}$ : feed rate,  $\eta_{w,o}$ : water surface elevation at the outlet.

506 Figure 3a shows evidences that little changes in the bed are observed after  
507 approximately  $t = 10800$  s. Figures 3b and c compare the water discharge  
508 and sediment transport rates entering and leaving the flume respectively.  
509 Figure 3b demonstrates that the model is conservative as far as the mass  
510 of water is concerned. As pointed out in Figure 3c, the initial imbalance of  
511 sediment causes the bed to degrade until shortly after  $t = 10800$  s when the  
512 mass of sediment going out matches that entering the flume. This means  
513 that the new equilibrium conditions have been achieved.

514 In order to study the convergence of the numerical scheme, different mesh  
515 sizes have been considered. The bed elevation at different positions and with  
516 several mesh sizes together with the sediment rate at the inlet is plotted  
517 in Figure 4. As the number of cells involved in the calculation increases,  
518 differences among results provided by the numerical model decrease. The  
519 resulting equilibrium slope for each mesh size is listed in Table 2. Bearing in  
520 mind that the reference equilibrium slope is  $0.00167$  m/m, Table 2 illustrates  
521 how the convergence approaches 1 (we use a first order numerical scheme) as  
522 the cell size of the mesh  $\Delta x$  gets finer.

Mesh size	$S_o$ (m/m)	Convergence
$\Delta x = 0.2$ m	0.00160117	-
$\Delta x = 0.1$ m	0.00165882	0.88632
$\Delta x = 0.05$ m	0.00166058	0.95866
$\Delta x = 0.025$ m	0.00166872	0.98991
$\Delta x = 0.0125$ m	0.00167011	0.99497

Table 2: Degradation case. 1 Fraction. Summary of the final bed slope convergence with different mesh sizes.

## 523 Test 2. Texture 2: two grain sizes.

524

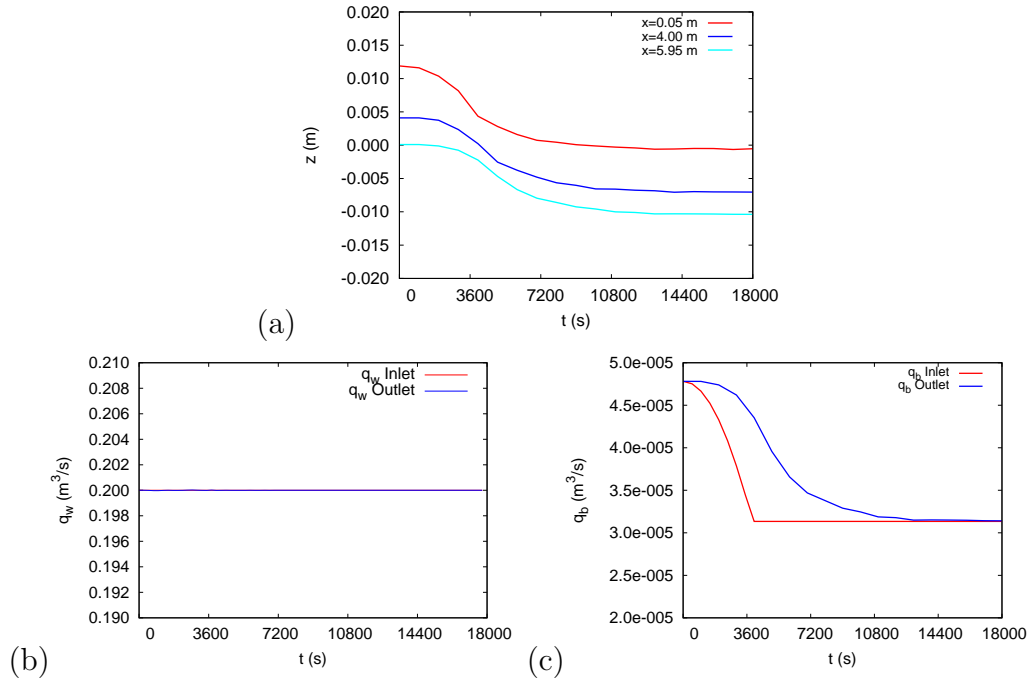


Figure 3: Bed level evolution in time at several points within the channel (a) and water (b) and solid discharge (c) in time. Degradation case. 1 Fraction.

525 In the second test case, the grain size distribution is composed of two equally  
 526 distributed grain fractions (Table 1). Initial and boundary conditions for the  
 527 test are listed in Table 1.

528 The temporal imbalance in the sediment transport rate at the inlet and the  
 529 outlet is shown in Figure 5a,b. Figure 5c presents the evolution of the bed  
 530 elevation at four stations along the channel. The temporal evolution of the  
 531 two surface fractions at the same four channel stations is illustrated in figures  
 532 5d and 5e. These figures show how the fine and the coarse fraction decrease  
 533 and increase respectively so that the geometric mean size of the bed surface  
 534 gradually coarsens until approximately  $t = 10000$  s. The small inset panels in  
 535 Figures 5 illustrate how the surface grain distribution gradually adjust along  
 536 the channel starting from the uppermost stations. The same trend is sug-  
 537 gested in the evolution of the bed elevation in Figure 5c. Figure 5g displays  
 538 the evolution of the time step associated with each wave speed (hydrody-  
 539 namic, bed and sorting). Initially, the time step for the numerical simulation  
 540 is controlled by the sorting celerities. The surface adjustment of the finer

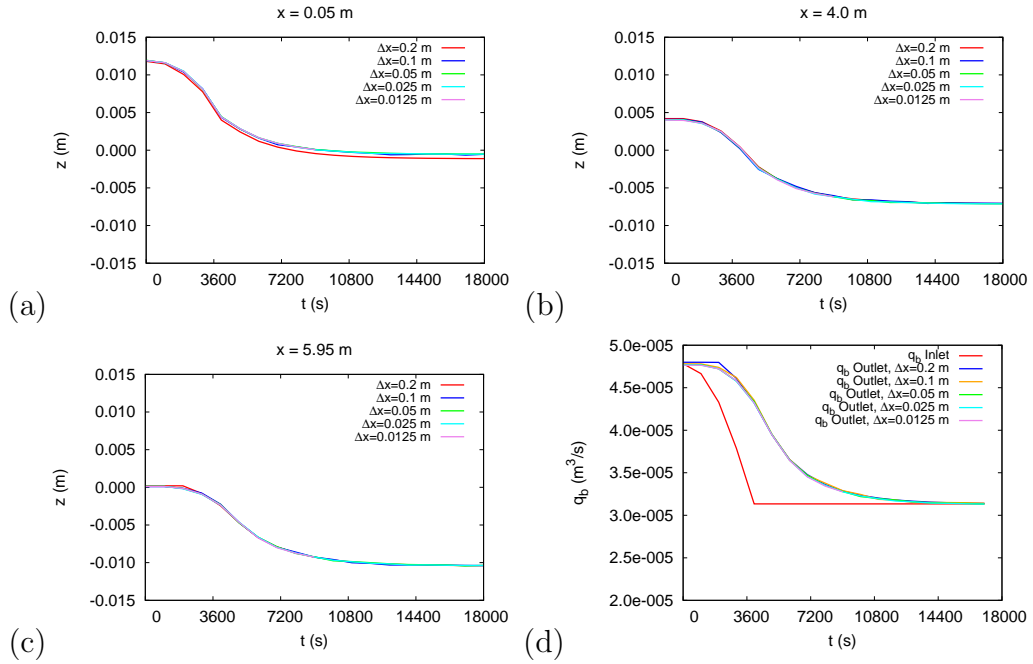


Figure 4: Degradation case. 1 Fraction. Convergence analysis using different mesh sizes.(a)-(c): Temporal evolution of the bed elevation at different channel stations. (d) Temporal evolution of the sediment transport rate at the channel outlet.

541 fraction is produced much faster than the bed and the hydrodynamic adjust-  
 542 ments. This leads to a smaller time steps until the bed gradually adjusts.  
 543 Note that the time step during the adjustment period of time,  $t < 15000s$ ,  
 544 is either controlled by the bed or the sorting celerities.

545

### 546 **Test 3. Texture 3: Four grain sizes.**

547

548 This test case consist of a sediment mixture composed of four sediment grain  
 549 sizes (Table 1).

550 Figure 6 illustrates similar results to those presented in Figure 5. As ex-  
 551 pected, the bed surface gradually coarsens (Figure 6d-h) at the same time as  
 552 the channel degrades -Figure (6c)- in the transient degradational adjustment  
 553 of the channel. The new steady state is reached between  $t = 10000 s$  and  $t$   
 554  $= 15000 s$ . As with the previous test, time to adjustment increases with the  
 555 distance downstream. This is noticeable in the bed elevation panel (Figure  
 556 6c) but especially in the surface grain size distribution. Further, the adjust-



557 ment rates of the bed surface (measured by the slope of curves in the small  
558 inset panels) gradually decline in the downstream direction.

#### 559 *4.1.2. One-dimensional aggrading tests*

560 The second test case is analogous to the first one but now the new equilibrium  
561 profile is attained after a transient aggradational process. To achieve this the  
562 initial sediment transport capacity obtained for each grain size using the same  
563 initial conditions is increased by 35%. All other variables remain the same as  
564 for degradation tests. For the sake of brevity only the results obtained with  
565 2 and 4 fractions are reported (tests 1 and 3 in the previous section and in  
566 Table 1).

567 Figures 7 and 8 illustrate the results of the numerical aggradation experi-  
568 ments when the sediment mixture is composed of two and four grain sizes  
569 respectively. These two figures show, from a qualitative point of view, the  
570 same results: (i) mass of water is fully conserved right after the commence-  
571 ment of the experiment, (ii) sediment transport rate at the outlet is initially  
572 lower than the feed rate and gradually increases until attaining equilibrium  
573 conditions at the same time (iii) bed surface coarsens initially in the transient  
574 aggradational adjustment of the channel, then it fines as a consequence of  
575 the imposed sediment discharge and finally it achieves an equilibrium stage.  
576 Finally, Table 3 compares the computed values obtained with the new numer-  
577 ical model with the reference values obtained with a widely used numerical  
578 model [39].

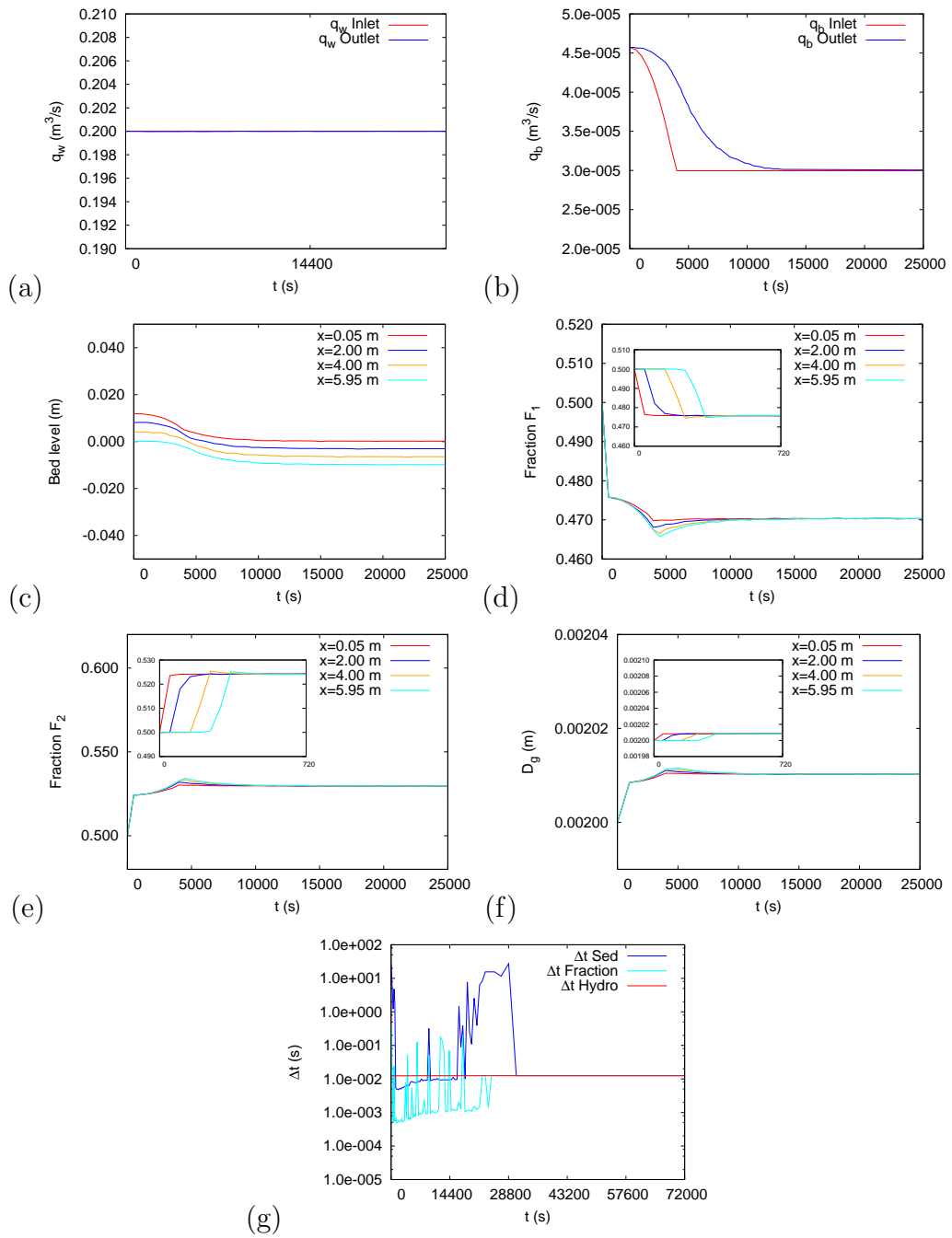


Figure 5: Degradation case. 2 Fractions. Temporal evolution for (a) water discharge, (b) sediment discharge, (c) bed elevation, (d) fraction  $F_1$ , (e) fraction  $F_2$ , (f) geometric diameter and (g) timestep

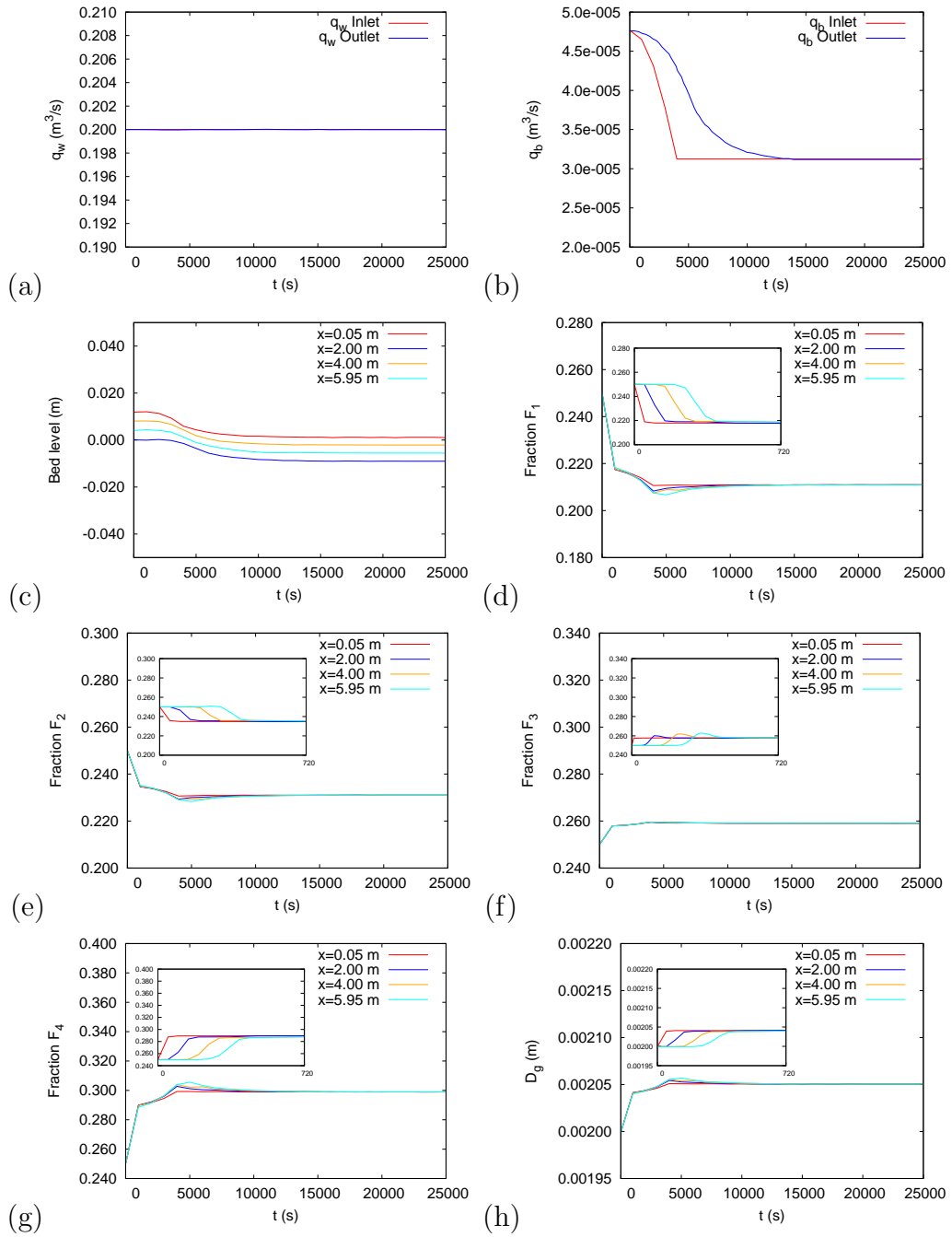


Figure 6: Degradation case. 4 Fractions. Temporal evolution for (a) water discharge, (b) sediment discharge, (c) bed level, (d) fraction  $F_1$ , (e) fraction  $F_2$ , (f) fraction  $F_3$ , (g) fraction  $F_4$  and (h) geometric diameter.

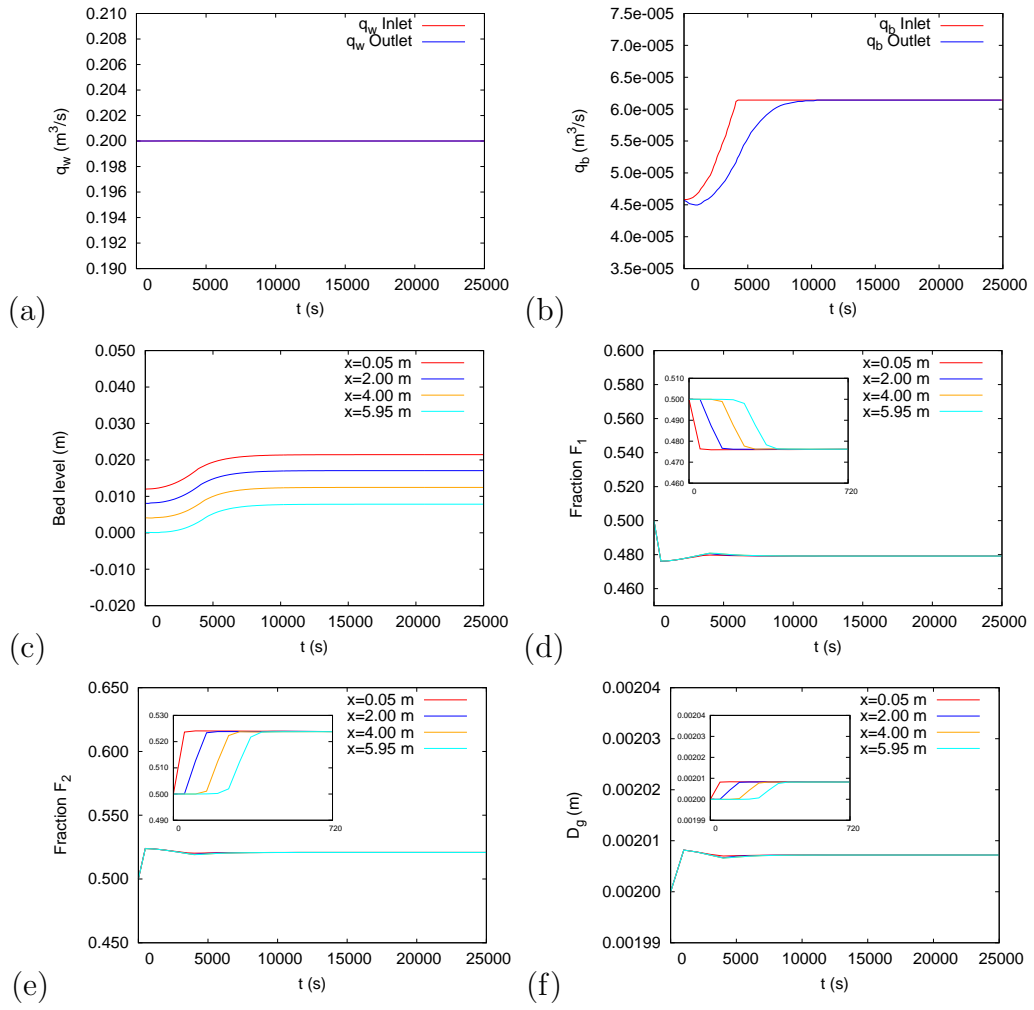


Figure 7: Aggradation case. 2 Fractions. Temporal evolution for (a) water discharge, (b) sediment discharge, (c) bed level, (d) fraction  $F_1$ , (e) fraction  $F_2$  and (f) geometric diameter.

Degradation	$S_{o,eq}^{Reference}(m/m)$	$S_{o,eq}^{computed}(m/m)$	$D_{g,eq}^{Reference}(m)$	$D_{g,eq}^{computed}(m)$
1 Fraction	0.001670	0.001658	0.002001	0.001999
2 Fractions	0.001667	0.001663	0.002030	0.002010
4 Fractions	0.001670	0.001697	0.002050	0.002012
Aggradation				
2 Fractions	0.002300	0.002650	0.002007	0.002007
4 Fractions	0.002290	0.002300	0.002035	0.002035

Table 3: Degrading and aggrading test cases. Comparison between the reference and computed values.

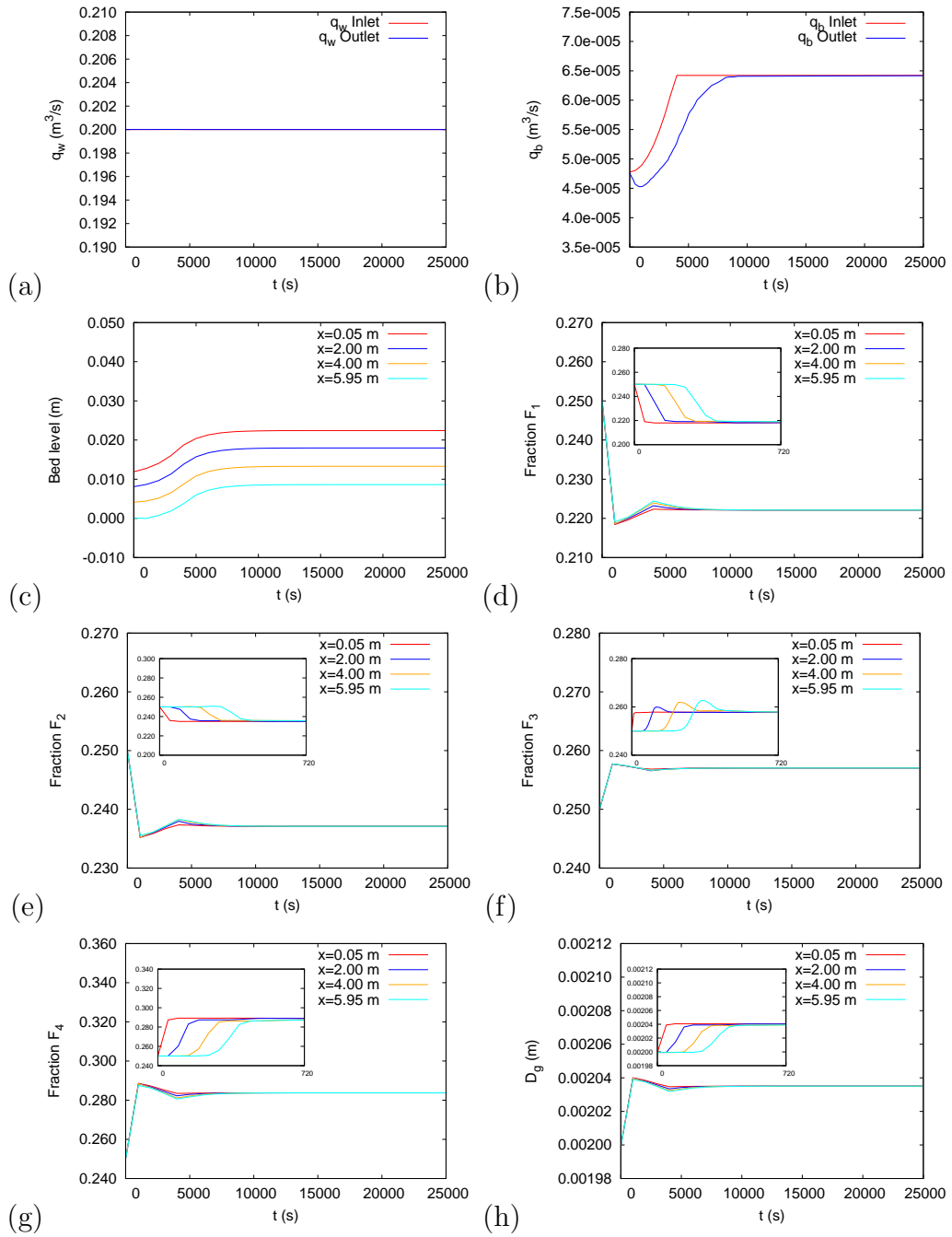
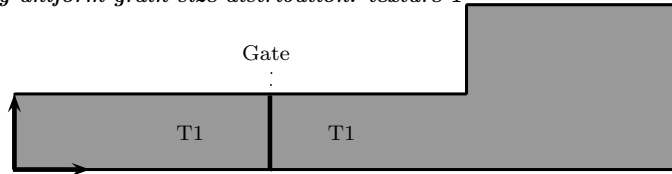


Figure 8: Aggradation case. 4 Fractions. Temporal evolution for (a) water discharge, (b) sediment discharge, (c) bed level, (d) fraction  $F_1$ , (e) fraction  $F_2$ , (f) fraction  $F_3$ , (g) fraction  $F_4$  and (h) geometric diameter.

579 *4.2. Two-dimensional asymmetric dam break*

580 This numerical experiment has been considered for testing the performance  
 581 of the numerical model under more realistic scenarios where important gra-  
 582 dients of the computed variables (bed elevation, flow field and grain size  
 583 distribution) are observed. This test case consists of an asymmetrical dam  
 584 break over a dry and erodible bed (Figure 9). The asymmetry arises from  
 585 the sudden expansion on the left bank of the channel which causes a local  
 586 erosion around the corner and consequently, sediment deposition downwards.  
 587 This test is used to examine the behaviour of the numerical model under a  
 588 highly rapid flow in a two-dimensional geometry. The initial water depth  
 589 condition has been set 0.25 m upstream the gate, which is located in the  
 590 middle (Figure 9). An unstructured mesh with a cell size of  $0.01m^2$  has been  
 591 considered and the CFL has been imposed equal to 0.5.

*Case DB-1: spatially uniform grain size distribution: texture 1*



*Case DB-2: spatially non-uniform grain size distribution: texture 1 + texture 2*

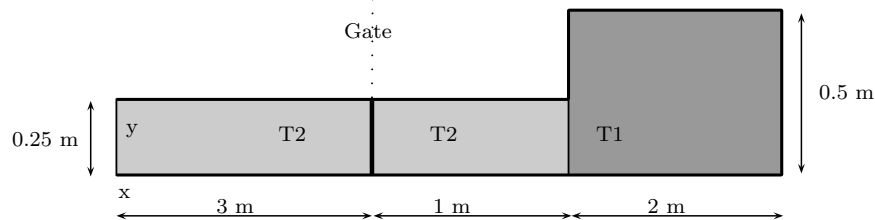


Figure 9: Sketch of the two-dimensional asymmetric dam break: plan view of the spatially uniform grain size distribution Case DB-1 (top) and of the spatially non-uniform grain size distribution Case DB-2 (bottom). T1 and T2 represent the two initial surface textures used in the numerical simulations (Table 4).

592 Two distinct cases have been considered (Figure 9). In the first case, hereafter  
 593 referred to as Case DB-1, a grain size distribution formed by four grain classes  
 594 T1 ( $D_g = 5.837 mm$ ) constitutes the initial surface texture throughout the  
 595 domain. The second case, called Case DB-2, considers two different regions  
 596 as far as the initial surface texture is concerned: the surface texture on

597 the 4 m-long channel (both upstream and downstream of the dam, Figure  
598 9) consists of a mixture T2 formed by four grain classes with  $D_g = 1.884$   
599  $mm$ ; the surface texture in region 2, which encompasses the last 2 m of the  
600 domain, i.e. the expansion of the channel, is composed by the previous grain  
601 size distribution T1. Neither water discharge nor feed rate is introduced  
602 from the channel inlet. Therefore, all morphological adjustments occurred  
603 after the dam breaks are driven by the difference in water surface elevation  
604 upstream and downstream of the dam.

	$D_{s1} = 1.095mm$	$D_{s2} = 2.121mm$	$D_{s3} = 4.242mm$	$D_{s4} = 7.745mm$	
<i>Texture</i>	$F_1$	$F_2$	$F_3$	$F_4$	$D_g(mm)$
T1	0.05	0.05	0.20	0.7	5.837
T2	0.20	0.78	0.02	0.0	1.884

Table 4: Grain size distributions used in the two numerical experiments.

605 Figures 10 and 11 present the evolution of the bed elevation and the finest  
606 grain fraction respectively (corresponding to  $D_{s1} = 1.095 mm$  and represent-  
607 ing 20% and 5% of the mixtures T1 and T2). Left and right panels in each  
608 Figure represent the homogeneous and heterogeneous initial surface textures,  
609 respectively.

610 Figure 10 shows different bed elevation patterns depending on the initial grain  
611 size distribution of the surface (Figure 9). A deeper scour hole around the  
612 corner where channel expands to the left is observed when an heterogeneous  
613 initial bed surface is considered. By mass conservation, the depositional zone  
614 extending from the upper left corner to the lower right corner within the  
615 expansion zone is larger when the initial surface texture is composed of two  
616 different grain sizes. The finer sediment mixture along the upstream channel,  
617 intrinsically more mobile than the coarse mixture, is mainly responsible for  
618 the increase in the scour hole. The higher mobility of the finest grain texture  
619 is presented in Figure 11, in which the finest grain class, associated with  
620  $D_{s1} = 1.095 mm$  is taken as a proxy. This figure shows how this grain size  
621 is washed away over the expansion when the incoming channel is composed  
622 of a finer mixture (right panels). On the contrary, when the entire domain  
623 is composed of a coarse mixture (left panels), the finest fractions only are  
624 present in the uppermost region of the depositional zone.

625 The grain size distribution of the surface can be summarized by the geometric  
626 mean diameter,  $D_g$ . Figure 12, which illustrates the spatial distribution of  
627  $D_g$  for the two initial conditions outlined in Figure 9, confirms the results



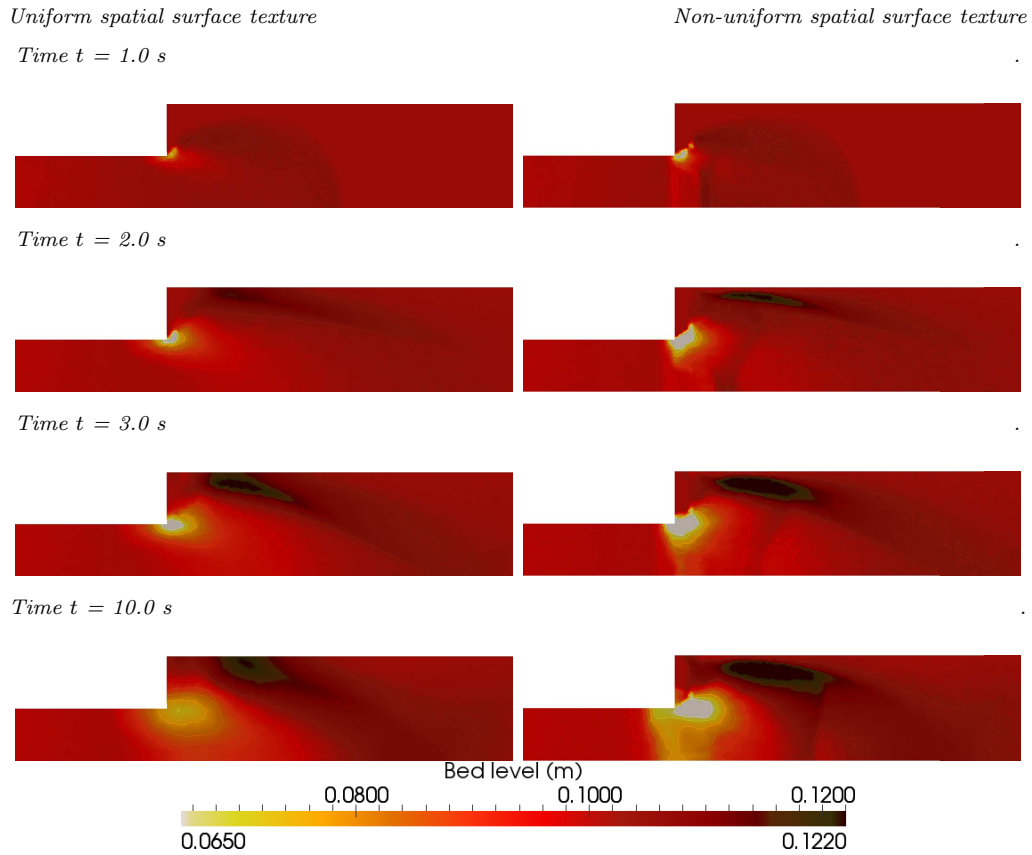


Figure 10: Spatial distribution of the bed elevation at times  $t = 1.0, 2.0, 3.0$  and  $10.0$  s. Left and right figures illustrate the results obtained with a uniformly and non-uniformly distributed initial surface texture, i.e. Case DB-1 and Case DB-2.

628 presented in Figure 11. The finer material initially placed along the incoming  
 629 channel (texture T1) is transported down to the expansion area contributing  
 630 to the fining of this area in case DB-2 (non-uniform initial texture) (Figure 12  
 631 right). This material is transported as a convection-like perturbation across  
 632 the wider section of the domain. Downstream of the edge of the wave of fine  
 633 sediment that is translated across the expansion section, an elongated patch  
 634 of coarse material expands. As time passes after the dam break, the patch  
 635 size increases. This material is likely to have been dragged by the wave of  
 636 fine material transported from the upstream narrow channel. The last surface  
 637 distribution obtained at  $t = 10$  s illustrates a decline in the extension of the  
 638 coarse patch. This figure seems to show that the coarse material is likely

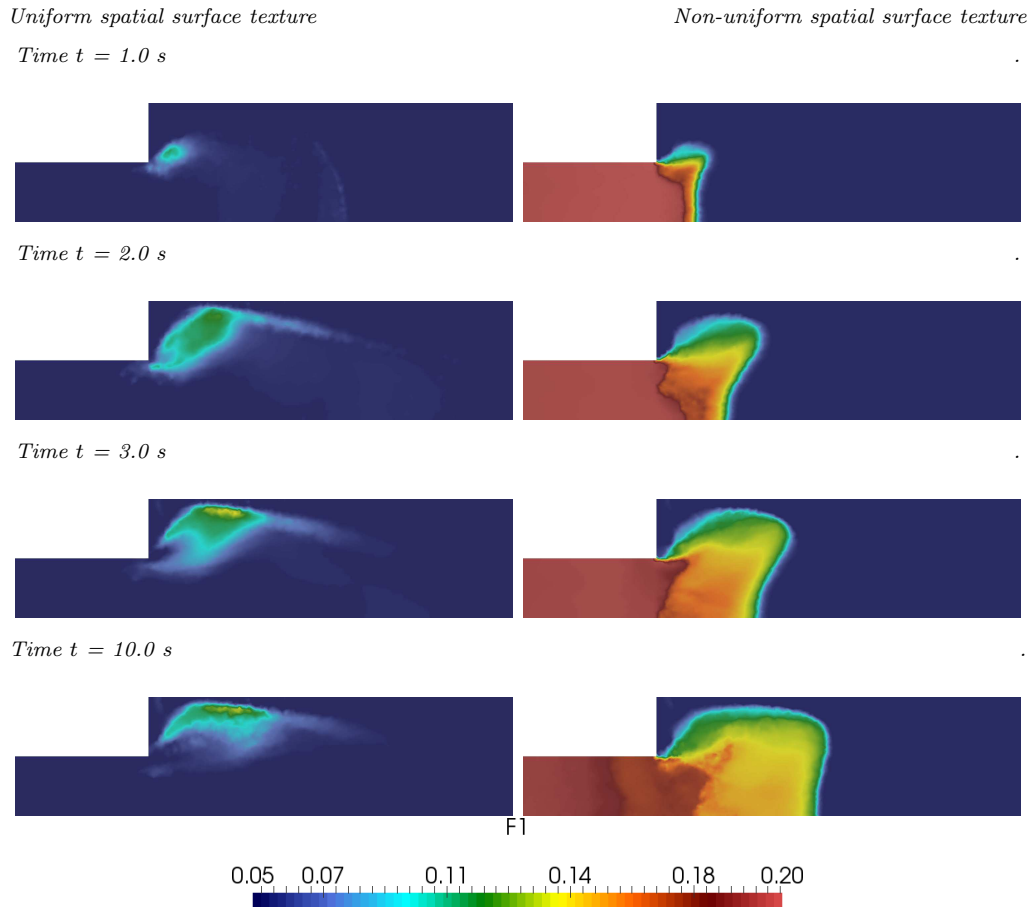


Figure 11: Spatial distribution of the finest fraction  $F1$  at times  $t = 1.0, 2.0, 3.0$  and  $10.0$  s. Left and right figures illustrate the results obtained with a uniformly and non-uniformly distributed initial surface texture, i.e. Case DB-1 and Case DB-2.

639 to have been buried by the fine sediment transported from the upstream  
 640 narrow channel. However, since the numerical model does not store the  
 641 vertical stratigraphy of the bed, this point cannot be fully demonstrated.  
 642 Left panels of the Figures 12 show the spatial distribution of  $D_g$  when the  
 643 uniform texture T2 extends throughout the domain (Figure 9). Two major  
 644 features are noticeable from these plots: (i) the coarsening of the area around  
 645 the sudden expansion and (ii) the fining of the upper zone of the expansion  
 646 area. These two zones are well correlated with the areas where degradation  
 647 and aggradation take place respectively (Figures 10). The finer fractions of  
 648 material entrained from the degradation areas are transported and deposited

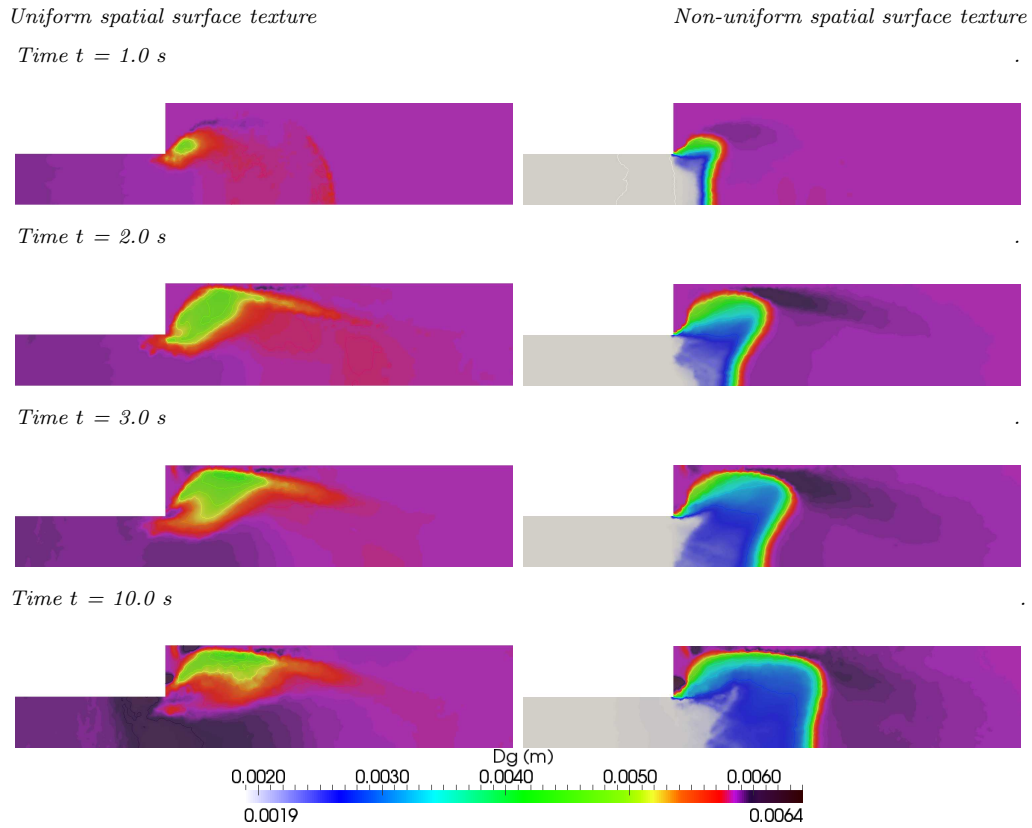


Figure 12: Spatial distribution of the mean geometric diameter at times  $t = 1.0, 2.0, 3.0$  and  $10.0$  s. Left and right and considering the same grain size distribution within all the channel (left) and with a different spatial grain size distribution (right)

649 on the areas where the bed aggrades. Figure 12 presents the expected results  
 650 as far as the evolution of  $D_g$  are concerned.

651 Results presented in Figure 11 show how different and significant channel  
 652 adjustments are driven by changes in the spatial distribution of the surface  
 653 texture. It is worth noting that in the both numerical experiments, the shape  
 654 of the scour hole and the depositional zone is qualitatively the same as the  
 655 one formed when the sediment of the bed is composed of uniform sediment  
 656 [59].

657 The influence of sediment mixtures on the bed surface in highly rapid and  
 658 variable flows such as dam breaks, is shown in Figure 13. This figure illus-  
 659 trates the bed elevation after dam break in which a uniformly distributed  
 660 material expands throughout the domain. However, unlike the simulation

661 Case DB-1, where the initial distribution was a sediment mixture of four  
662 different grain classes (Table 4), this material is composed of only one grain  
663 size of  $D = 5.837 \text{ mm}$ , i.e. equal to  $D_g$  in the simulation Case DB-1. All  
664 other parameters of the numerical experiment are the same as the ones pre-  
665 viously used. Thus, Figure 13 needs to be compared with the left-hand side  
666 panels of Figure 10. Results at  $t = 2 \text{ s}$  and  $10 \text{ s}$  are shown in Figure 13.  
667 Changes in the bed elevation when a uniform material is used as initial sur-  
668 face texture are less pronounced than when a sediment mixture is considered  
669 (Figure 10): less erosion and less aggradation are noticed with uniform sedi-  
670 ment. This response arises from the lack of hiding/exposure effects when the  
671 material is uniform. An increase of sediment transport rates occurs when a  
672 sediment of mixture is considered. This occurs because of the presence of fine  
673 grain particles on the bed. This fine material, inherently more mobile than  
674 the coarse fractions, enhances the mobility of the coarse material, increasing  
675 bedload transport rates [60]. These effects are taken into account by the  
676 hiding/exposure function, which when the bed surface is composed of mul-  
677 tiple grain sizes, affects the critical shear stress for the initiation of motion  
678 for each grain size. Figure 13 demonstrates the importance of considering  
679 multiple grain sizes as far as river morphodynamics are concerned.

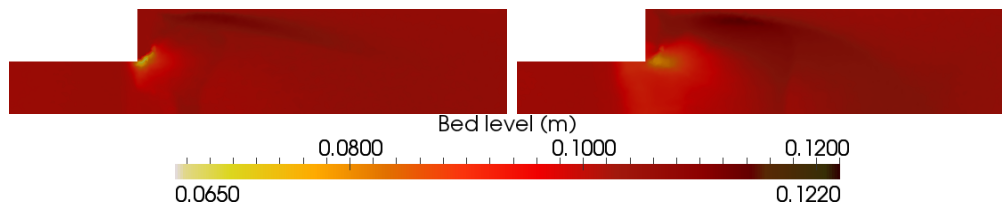


Figure 13: Bed level evolution at times  $t = 2.0$  (left) and  $10.0 \text{ s}$  (right) using Ashida-Michue formula and considering uniform grain size with  $D_s = 5.837 \text{ mm}$

#### 680 4.3. Comparison with experiments: Dam removal

681 The last set of comparisons of the numerical model is carried out by testing  
682 its performance against some experiments on dam removal. The bed was  
683 composed of a poorly sorted mixture ( $D_g = 3.39 \text{ mm}$ ,  $\sigma_g = 1.8$ ). The dam  
684 was composed of three slats of  $4 \text{ cm}$  each that were sequentially removed.  
685 Thus, the total height of the dam was  $12 \text{ cm}$ . After the dam was removed,  
686 a channel was formed within the upstream deposit. This channel rapidly  
687 eroded and the erosions migrated upstream. The width of the upstream

688 eroding channel narrowed during rapid stages of bed degradation and slowly  
 689 widened once bed degradation rates decreased. The experiment used in this  
 690 test compares the channel evolution (elevation and width). Water discharge  
 691 was set constant to  $0.002 \text{ m}^3/\text{s}$ . The experiment was conducted under no  
 692 feed conditions. These experiments are particularly useful for the purpose  
 693 of testing the numerical model because: (i) the bed surface was composed  
 694 of a mixture of sediment, (ii) flow was supercritical, bed changes were pro-  
 695 duced very rapidly (thus, significant gradients in the computed variables are  
 696 expected) and (iii) channel width changes introduce important two dimen-  
 697 sional processes. The experimental results presented herein represent the  
 698 evolution of the channel after the second slat is removed. More details of the  
 699 experiments can be found in [38]. An unstructured mesh with a cell size of  
 700  $0.01\text{m}^2$  has been considered and the CFL has been imposed equal to 0.5.  
 701 Figure 14 illustrates the evolution of the bed elevation along the center of  
 702 the channel at four different times during the run. The numerical model  
 703 underpredicts the erosion observed during the experiment, i.e. the erosion  
 704 along the channel progresses upstream faster in the experiments than it is  
 705 predicted by the numerical model. This might be due to an underprediction  
 706 of the sediment transport rate, given by the Ashida-Michiue formula, that  
 707 leads to an excess of surface bed coarsening that ends up limiting channel  
 708 erosion.

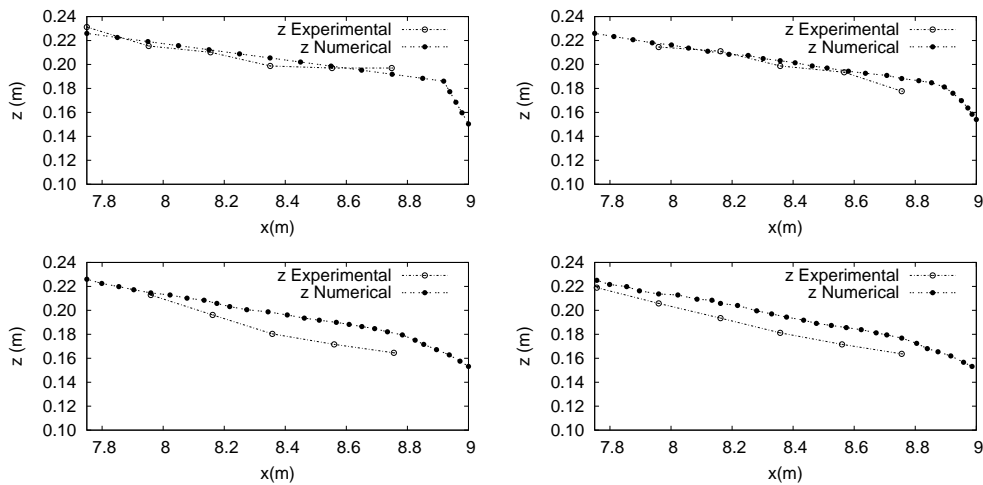


Figure 14: Numerical results and experimental data at times  $t = 0, 10, 50$  and  $90\text{s}$  using Ashida-Michiue: measured bed level surface and computed bed level surface

709 Figure 15 shows the evolution of the channel width at three stations along  
 710 the channel. Numerical results nicely reproduce the complex two-dimensional  
 711 evolution of the channel width: magnitude and timing of the narrowing and  
 712 widening of the cross-sections are well modelled. The final width of the  
 713 channel at the end of the experiment is also reproduced. The numerical  
 714 model does not account for sudden slides and slumps. This is the reason that  
 715 the sudden increases of the channel width, caused by lateral mass movement  
 716 from the sidewalls, cannot be reproduced [38].

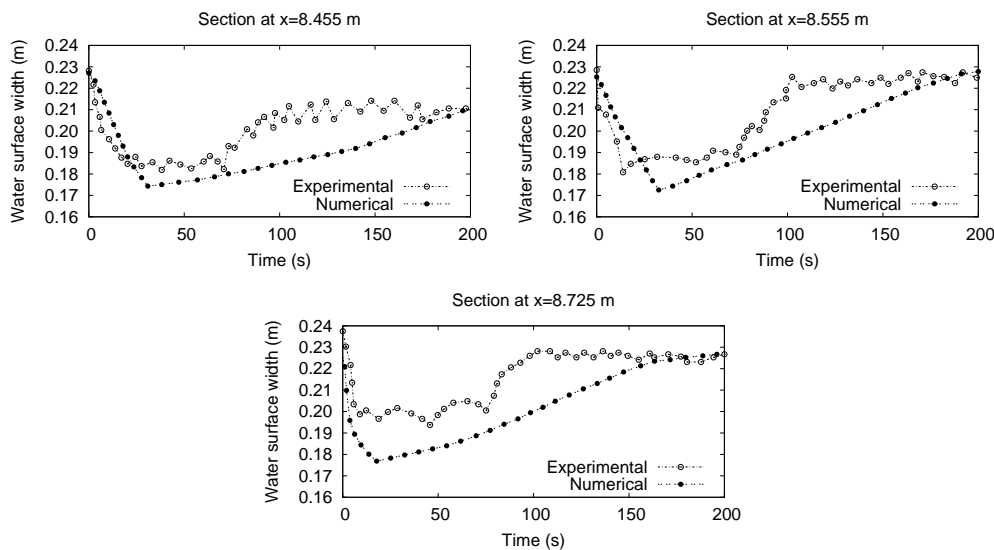


Figure 15: Numerical results and experimental data at times  $t = 0, 10, 50$  and  $90$ s using Ashida-Michiue formula: measured water surface width and computed water surface width

717 No surface samples were extracted from the bed during the experiment. How-  
 718 ever, given the characteristics of the experiments (strong degradation, con-  
 719 stant discharge and no sediment feed), some surface coarsening is expected.  
 720 Figure 16 presents the expected results: (i) surface texture coarsens, (ii) the  
 721 farther upstream the station from the dam the higher the coarsening and  
 722 (iii) the longer it takes to start coarsening. The mean geometric diameter  
 723  $D_g$  is coarsened in both stations as bed degradation proceeds. The station  
 724 which is located farther from the dam ( $x=7.6$  m) suffers a bigger coarsening  
 725 process. This arises because upstream, bed degradation rates decrease with  
 726 time. Hence, the finest grain sizes of the surface are winnowed while the  
 727 coarsest fractions remain in place. This finest material is transported down-

728 stream, causing a smaller coarsening process at station  $x=8.0$  m. Results  
729 presented in Figure 16 follow a similar trend as other numerical predictions  
730 of the experiment [38] and they are in agreement with channel bed coarsening  
731 after dam removal observed in field cases [61, 62].

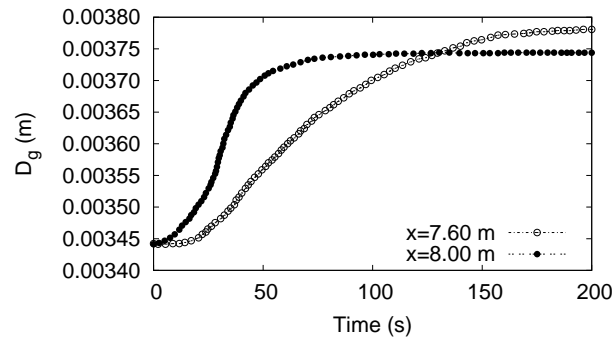


Figure 16: Numerical results of the temporal evolution of the geometric mean diameter at two stations located in the centre of the incising channel.

## 732 **5. Conclusions**

733 A novel two-dimensional numerical finite volume scheme for water flows over  
734 erodible beds considering non-uniform grain sizes has been developed. The  
735 numerical model includes the Shallow water equations to describe the flow  
736 field with the Exner equation for the bed evolution and the Hirano equations  
737 to describe the surface grain size distributions by means of a weak-coupled  
738 strategy. Thanks to this methodology, a self-stable explicit scheme is devel-  
739 oped and no tuning parameters are required for controlling the stability of  
740 the model by means of the CFL condition.

741 The first test cases considered for evaluating the model are based on synthetic  
742 aggradation/degradation test cases. These test cases are useful for verifying  
743 the correct integration of the fluxes in order to obtain a final equilibrium  
744 stage. The numerical model is able to gradually achieve new equilibrium  
745 conditions once the initial configuration is disturbed. Successful results are  
746 obtained regardless of whether this new equilibrium condition is achieved  
747 through an aggradational or degradational transient process.

748 The second test case considered is a genuinely 2D transient problem where  
749 the computed variables display large gradients. The key point is the presence  
750 of a sudden enlargement which causes a notable local scour and sediment  
751 deposition downwards in the expanded area. Several initial texture condi-  
752 tions have been considered and the numerical scheme has provided self-stable  
753 results for the flow, bed level and the sediment fractions on the bed surface.  
754 Significant differences are observed in the results depending on whether the  
755 initial surface is composed of uniform material or a sediment mixture. These  
756 results evidence the importance of mixtures in river morphodynamics.

757 Lastly, the numerical model is compared with a set of experiments on dam  
758 removal. The numerical model is able to predict the general trend of degra-  
759 dation, changes in channel width and surface adjustments observed during  
760 the experiment.

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