A numerical power spectrum for electroencephalographic processing

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Abstract

The record of the oscillations of the electric potential of the human brain provides useful information about the mind activity at rest and during the achievement of sensory and cognitive processing tasks. It is necessary then the use of appropriate quantitative tools assigning numerical values to the observed variable, in order to define good descriptors of the electroencephalogram, allowing comparisons between different recordings. In this line, we propose a numerical method for the spectral and temporal reconstruction of a brain signal. The convergence of the procedure is analyzed, providing results of the concerned approximation error. In a second part of the text, we use the methodology described to the quantification of the bioelectric variations produced in the brain waves for the execution of a test of attention related to military simulation.

Keywords: Trigonometric Approximation, Power Spectrum, Fast Fourier Transform, Electroencephalogram

MSC 2000: 42A10, 42A16, 62M15, 37M10

1 Introduction

In this paper we propose approximants of spectral type for sampled signals. The method is based on the computation of numerical Fourier coefficients beginning from an experimental variable. The multipliers enable the computation of descriptors such as Activity and Mobility of a time series. The coefficients provide a power spectrum as well, displaying the frequency content of the function. The numerical procedure constitutes an alternative to the classical Fast Fourier Transform algorithm. At the same time, one obtains approximation curves that display the macroscopic cycles underlying the described phenomenon.

We study the convergence of the procedure, proving that for weak conditions (Hölder continuity) the simulated curve tends to the original. The methods are based on classical concepts of analysis and approximation theory. They may be specially useful for the description of movements of stationary or quasi-stationary character, such as electrophysiological records, and all types of phenomena near periodicity.

In a second part of the article, we present an application of the method proposed to electroencephalographic processing. In particular, we inquire about the brain bioelectric patterns produced during an attention task of military simulation character. The study of concentration, as well as its loss, during the execution of jobs performed by the Armed Forces, is a subject of great interest from the neurological, laboral and social point of view. The computational tools currently available allow a deeper insight into the experimental signals, and it is our aim to use them for a better understanding of the brain processes involved in the cognitive and attentional mental activities related to this sector.

2 Finite real sum of Fourier

From here on we assume, without loss of generality, a real compact interval I of length T such that $T = 2\pi$, in order to simplify the calculus. We consider the space of 2π-periodic continuous functions

$$\mathcal{C}(2\pi) = \{f : [-\pi, \pi] \to R : f \quad \text{continuous}, f(-\pi) = f(\pi)\}$$

and the operator

$$\mathcal{J}_m : \mathcal{C}(2\pi) \to \mathcal{C}(2\pi)$$

$$f \to s_m$$

where s_m is the m-th Fourier finite real sum of f defined by (see for instance [1], [2])

$$\mathcal{J}_m(f)(t) = s_m(t) = \frac{a_0}{2} + \sum_{k=1}^{m}(a_k \cos(kt) + b_k \sin(kt))$$

where

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(kt)dt,$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(kt)dt.$$

$\mathcal{J}_m$ is a linear and bounded operator, and the following inequality holds [3]

$$\|\mathcal{J}_m\| \leq A\log(m) \tag{1}$$

In addition $\mathcal{J}_m$ is a projection, $\mathcal{J}_m \circ \mathcal{J}_m = \mathcal{J}_m$.

Fourier expansions make the filtering and processing of a signal easier, omitting or retrieving frequencies in a given band. In principle a Fourier finite sum only assures a good representation of the function in the quadratic mean sense, that is to say, with respect to the norm

$$\|f\|_2 = (\int_I |f|^2 dt)^{1/2}.$$

In the following we deduce some results in order to bound the uniform error committed in the approximation of the function by a finite sum.

For $f \in \mathcal{C}(2\pi)$, let S^* be the trigonometric polynomial of order m such that

$$d_m^*(f) = \|f - S^*\|_\infty = \inf\{\|f - S\|_\infty;\ S \in \tau_m\}$$

where τ_m is the space of trigonometric polynomials of order at most m:

$$\tau_m =< \{1, \sin t, \cos t, \sin 2t, \cos 2t, ..., \sin mt, \cos mt\} >$$

and let us consider the next result due to Lebesgue ([3]).

Theorem 2.1. *There exists a constant M such that, for each $f \in \mathcal{C}(2\pi)$,*

$$\|f - \mathcal{J}_m(f)\|_\infty \leq M d_m^*(f) \log(m), \tag{2}$$

for $m > 1$, where $\mathcal{J}_m(f)$ is the m-th Fourier sum of f.

Theorems of Jackson [1] give upper bounds for $d_m^*(f)$. In particular, a Theorem states that for any function $f \in \mathcal{C}(2\pi)$

$$d_m^*(f) \leq w_f(\frac{\pi}{m+1}), \tag{3}$$

where $w_f(\delta)$ is the modulus of continuity of f, defined as

$$w_f(\delta) = \sup_{|t-t'|\leq\delta} |f(t) - f(t')|.$$

A Hölder continuous function with exponent β ($0 < \beta \leq 1$) satisfies the inequality ([1]):

$$w_f(\delta) \leq k\delta^\beta. \tag{4}$$

In this case, applying Theorem of Lebesgue 2.1, (3) and (4),

$$d_m^*(f) \leq k(\frac{\pi}{m+1})^\beta, \tag{5}$$

the Fourier series $\mathcal{J}_m(f)$ is convergent to f in the uniform sense as well since $\|f - \mathcal{J}_m(f)\|_\infty$ tends to zero when m tends to infinity. This is a particular case of a Dini-Lipschitz condition ($w_f(\delta)\log(\delta) \to 0$ as $\delta \to 0$).

The rate of decay of the Fourier coefficients is measured by the modulus of continuity as well ([6]):

$$|a_k| \leq w_f(\frac{\pi}{k}), \tag{6}$$

$$|b_k| \leq w_f(\frac{\pi}{k}). \tag{7}$$

Now we consider an approximation to the theoretical Fourier sum of a sampled signal. Since f is only known at its samples, we consider the data points $\{(t_n, x_n = f(t_n))\}_{n=0}^N$ and a piecewise linear and continuous function f_0 with vertices on the data, and we use this approximation to compute a Fourier finite sum of the signal. This approach does not require any condition of smoothness of the signal and consequently is useful for a wide range of physical, economic, social and natural variables.

In order to simplify the calculus we assume $t_n - t_{n-1} = h$, for all $n = 1, 2, \dots N$.

The approximation of f by f_0 is bounded by the modulus of continuity ([4], Lemma 3.9):

$$\|f - f_0\|_\infty \leq \omega_f(h). \tag{8}$$

For a continuous function on a compact interval I, $\omega_f(h)$ tends to zero as $h \to 0$. In particular if f is Hölder continuous, we obtain the rate of convergence:

$$\|f - f_0\|_\infty \leq kh^\beta,$$

for a constant β such that $0 < \beta \leq 1$.

Lemma 2.2. *Let f_0 be a piecewise linear and continuous function with vertices $\{(t_n, x_n = f(t_n))\}_{n=0}^N$, $t_n - t_{n-1} = h$ constant and $t_0 = -\pi$, $t_N = \pi$. For m sufficiently large $(m + 1 \geq N/2)$,*

$$w_{f_0}(\frac{\pi}{m+1}) \leq 2w_f(\frac{2\pi}{N}). \tag{9}$$

Proof. The modulus of continuity of f_0 is defined as

$$w_{f_0}(\delta) = \sup_{|t-t'|\leq\delta} |f_0(t) - f_0(t')|$$

Let $t,\ t'$ be such that $|t - t'| \leq \frac{\pi}{m+1}$. For m large enough

$$\frac{\pi}{m+1} \leq h = \frac{2\pi}{N} \tag{10}$$

$(m + 1 \geq N/2)$. If $t,\ t' \in I_n = [t_{n-1}, t_n]$, then

$$f_0(t) - x_n = \frac{(x_n - x_{n-1})}{h}(t - t_n)$$

$$f_0(t') - x_n = \frac{(x_n - x_{n-1})}{h}(t' - t_n)$$

Then:

$$f_0(t) - f_0(t') = \frac{(x_n - x_{n-1})}{h}(t - t')$$

and, using (10),

$$|f_0(t) - f_0(t')| \leq \frac{|x_n - x_{n-1}|}{h}(\frac{\pi}{m+1}) \leq |x_n - x_{n-1}| \leq w_f(h) = w_f(\frac{2\pi}{N}), \tag{11}$$

and the inequality (9) is fulfilled.

If $t,\ t'$, such that $|t-t'| \leq \frac{\pi}{m+1}$, belong to consecutive intervals, $t < t_n < t'$, applying (11)

$$|f_0(t) - f_0(t')| \leq |f_0(t) - f_0(t_n)| + |f_0(t_n) - f_0(t')| \leq 2w_f(\frac{2\pi}{N}).$$

□

Proposition 2.3. *Let f_0 be as in the previous Lemma. Then,*

$$\|f_0 - \mathcal{J}_m(f_0)\|_\infty \leq 2Mw_f(\frac{2\pi}{N})\log(m).$$

Proof. According to Theorem 2.1,

$$\|f_0 - \mathcal{J}_m(f_0)\|_\infty \leq Md_m^*(f_0)\log(m),$$

Applying the Jackson's Theorem (3) and Lemma 2.2,

$$d_m^*(f_0) \leq w_{f_0}(\frac{\pi}{m+1}) \leq 2w_f(\frac{2\pi}{N}),$$

and the result is obtained. □

We have now a Theorem of approximation error.

Theorem 2.4. *Let $f \in \mathcal{C}(2\pi)$ be the function providing the data $\{(t_n, x_n)\}_{n=0}^N$. For m sufficiently large*

$$\|f - \mathcal{J}_m(f_0)\|_\infty \leq w_f(\frac{2\pi}{N})(1 + 2M\log(m)).$$

Proof. The result is a consequence of (8) and the previous Proposition, bearing in mind the inequality

$$\|f - \mathcal{J}_m(f_0)\|_\infty \leq \|f - f_0\|_\infty + \|f_0 - \mathcal{J}_m(f_0)\|_\infty.$$

□

Consequence 2.5. *If the original function f is Hölder continuous, and choosing m, N suitably, the convergence of the procedure is ensured, when the sampling step h tends to zero. For instance, we may take $m = [N/2]$.*

The next result bounds the discretization error in the Fourier coefficients.

Proposition 2.6. *Let f_0 be the broken-line interpolant of f with respect to data $\{(t_n, x_n)\}_{n=0}^N$, where $h = t_n - t_{n-1}$. If $\overline{a}_k$, $\overline{b}_k$ are the approximate Fourier coefficients,*

$$|a_k - \overline{a}_k| \leq 2w_f(h),$$

$$|b_k - \overline{b}_k| \leq 2w_f(h).$$

Proof. Let us consider that, according to their definition

$$|a_k - \overline{a}_k| \leq \frac{1}{\pi}\int_{-\pi}^{\pi} |f(t) - f_0(t)|dt,$$

and the inequality (8) gives the result. □

Consequence 2.7. *For a Hölder continuous function with exponent β, the rate of convergence of the approximation of the Fourier coefficients is $\mathcal{O}(h^\beta)$.*

Consequence 2.8. *The following upper bound of the approximate Fourier coefficients is obtained:*

$$|\overline{a}_k| \leq w_f(\frac{\pi}{k}) + 2w_f(\frac{2\pi}{N}) \leq 3w_f(\delta_{kN}),$$

$$|\overline{b}_k| \leq w_f(\frac{\pi}{k}) + 2w_f(\frac{2\pi}{N}) \leq 3w_f(\delta_{kN}),$$

where $\delta_{kN} = \max\{\pi/k, 2\pi/N\}$.

These inequalities are due to the previous Proposition, the monotony of w_f and (6), (7).

3 Application to electroencephalographic processing

The methods proposed above have been applied to the quantification of electroencephalographic signals for the study of attention in activities carried by the members of the Spanish Armed Forces. Our goal is to study the attention/concentration of the military personnel in simulated situations that would require alertness and a possible assessment of the lack of the concentration/attention at any given time.

3.1 Subjects

The EEG (electroencephalographic) signals were collected in the Hospital General de la Defensa de Zaragoza from a group of Armed Forces people who voluntarily offered to cooperate, composed of Caballeros Cadetes Alumnos, Officers and Subofficial of the Academia General Militar de Zaragoza and from the Hospital General de la Defensa de Zaragoza. EEGs were recorded at rest and during the performance of a tactical simulation exercise consisting of a tank driving using a videogame.

For each subject, the following signals of 3 minutes long were collected:

- EEG at rest with closed eyes.
- EEG while a tank driving is simulated using an iPad (j1).

The EEG signal from each subject was recorded with a digital computer using ambulatory software *Compumedics Limited Profusion EEG 4*. The record was monopolar (with Ag/AgCl electrodes) and 16 channels of the international 10-20 Jasper system referenced Cz were used. The test was conducted in a dimly lit, quiet room, at constant temperature and in an electrically shielded room.

In this study we present the results of an analysis of the frontal channels F7 and F8, occipital O1 and O2 and temporal T5 and T6 of the EEG signals at rest with eyes closed and while performing the task j1 of 13 subjects.

3.2 Methods

We obtained the Fourier coefficients of one minute of the electroencephalographic signals at rest with closed eyes and during the simulation of a military task (j1). The signal was sampled at 256 points per second and analyzed the third minute of each record.

We have computed the Hjorth parameters Activity and Mobility ([5]) for both types of EEGs. The Activity is a measurement of the power (and thus the magnitude) of the signal and the Mobility is an indicator of angular frequency.

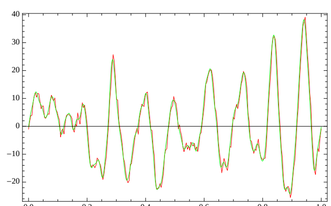

Figure 1: In red one second of the initial signal of the O2 channel from an EEG at rest with closed eyes, in green the reconstructed signal

Furtherly, we performed a test to check if the signals admit a model of colored (or $1/f$) noise, where

$$S(\omega) \simeq k\omega^{-exp},$$

and $S(\omega)$ is the power spectrum. The correlation obtained would allow us to infer the nature of the EEG variable.

We computed an average frequency as well, as mean of the frequencies weighted by the amplitudes obtained by means of the FFT algorithm.

3.2.1 Statistical tests

Once the different EEG quantification parameters were computed, a comprehensive statistical study of the results obtained was performed.

First an exploratory data analysis was performed, making line diagram, Box-Plot graphics, etc. Later on a Shapiro-Wilk's test was implemented in order to verify the hypothesis of normality of the data.

To study the significant differences between the samples with closed eyes and in the tank driving simulation for each quantifier, a T-parametric test for related samples was executed. This test contrasts the hypothesis that both types of EEG (closed eyes and j1) at every electrode (F7, F8, O1, O2, T5, T6) have the same mean (i.e., the average difference between tasks is zero). The statistic used was the T of Student.

To test the results and bring more consistency to the findings, two additional non-parametric tests were implemented:

- The Wilcoxon test for paired samples contrasts that both types of EEG in any electrode have the same distribution function (i.e., no significant differences between measurements). Primarily it computes the magnitude of the differences between pairs of measurements at each individual with closed eyes and during the task j1. The Z is used as statistical test.

- The sign test for related samples contrasts that both types of EEG in any electrode have the same distribution function. Primarily it computes the differences between the pairs and classifies the positive and negative in order to check the equality of distributions. In this case, unlike the above, the magnitude of the differences is irrelevant.

4 Results and statistics

Several quantifiers of the EEG signal were analyzed: Hjorth parameters (Activity and Mobility), exponent of colored noise and FFT mean frequency. The results are shown in corresponding subsections describing the group averaged values of the indices.

4.1 Hjorth parameters

Means in each channel for Activity and Mobility are shown in a self-explanatory table, collecting the averages of the values on the group.

	ACTIVITY		**MOBILITY**	
	Closed eyes	Task j1	Closed eyes	Task j1
F7	171.441	291.124	78.794	65.879
F8	165.918	504.180	79.315	68.016
O1	270.083	243.937	74.249	96.667
O2	240.045	204.159	72.788	100.425
T5	180.891	192.937	73.135	97.910
T6	203.801	173.456	73.524	96.841

Activity shows that mean differences more significant between the two tasks occur in F7 (119.679) and F8 (-338.262) for j1 task. Variability and heterogeneity of the measurements are also higher in F7 and F8.

Observing the results for Mobility, a significant increase was seen in the means of that parameter when comparing the EEG at rest with closed eyes to the simulation task, in the channels corresponding to the occipital and temporal areas (O1, O2, T5, T6). This is explained by the activation of the temporal region of the brain (primary auditory cortex) due to the sound of the simulator, and the primary visual area processing the task.

The obtained data were analyzed with the statistical methods mentioned previously. The most appropriate test to see if there are significant differences between measurements with closed eyes and the simulation task j1 is the T-parametric test for related samples, but we also employed two non-parametric tests to contrast results and bring more consistency to the findings.

The application of the statistical methods to the values obtained for the Activity provided the following results:

- The parametric test was found unsuitable because the variables failed the assumption of normality (Shapiro-Wilk test), necessary to use this test.

- The non-parametric Wilcoxon test for related samples pointed to significant differences only in the F7 and F8 channels with p-values (significances) of less than 0.05. Specifically we found for F7 ($Z = -1.956, \mathrm{sig} = 0.05$) and for F8 ($Z = -2.341, \mathrm{sig} = 0.019$).

- The non-parametric sign test for related samples obtained the same differences as in the previous case, specifically F7 (sig = 0.012) and F8 (sig = 0.003).

The application of the statistical methods to the values obtained for the parameter Mobility provided the following results:

- $T-$test for related samples indicated significant differences in all channels except F7 with $p-$values (significances) of less than 0.05. Specifically F8 ($T = 2.395, \mathrm{sig} = 0.034$), O1 ($T = -3.014, \mathrm{sig} = 0.011$), O2 ($T = -4.857, \mathrm{sig} = 0.0003$), T5 ($T = -4.121, \mathrm{sig} = 0.001$) and T6 ($T = -5.160, \mathrm{sig} = 0.0002$). In all cases except at F7 and F8, the higher average was obtained in the task j1, with much larger differences in the occipital and temporal areas.

- With the non-parametric Wilcoxon test for paired samples, significant differences in all channels were observed except F7 with $p-$values (significances) of less than 0.05. These results agree with those obtained in the $T-$test.

- The non-parametric sign test for related samples showed significant differences in all channels except F7 and F8, with $p-$values (significances) of less than 0.05, specifically sig= 0.022.

4.2 Exponent of colored noise

The averages obtained in each channel for the exponent are shown in a self-explanatory table.

	EXPONENT	
	Closed eyes	Task j1
F7	1.496	1.579
F8	1.461	1.534
O1	1.702	1.173
O2	1.680	1.100
T5	1.661	1.122
T6	1.675	1.136

In this case, a lower exponent reflects a higher complexity of the signal (in a fractal sense). However, we have a qualified opinion on this parameter, as the correlations obtained in its computation are not high and the test on the nature of the signal is not conclusive.

It is noted that the most important differences in the means between the measurements with closed eyes and task j1 occur in O1, O2, T5 and T6, being higher in eyes closed. Variability and heterogeneity of the measurements are higher in those channels.

After performing the statistical tests previously listed the following results were obtained:

- The data fitted the normality hypothesis (Shapiro-Wilk test), so a T parametric test for related samples was applied. The results proved significant differences in the channels analyzed in occipital and temporal areas with $p-$values (significances) of less than 0.05. Specifically O1 ($T = 4.493, \mathrm{sig} = 0.0007$), O2 ($T = 6.682, \mathrm{sig} = 0.00003$), T5 ($T = 6.933, \mathrm{sig} = 0.00002$) and T6 ($T = 7.558, \mathrm{sig} = 0.000001$). The highest averages were found in the EEG at rest in occipitals.

- In the non-parametric Wilcoxon test for related samples the results showed significant differences in O1, O2, T5 and T6 channels with p-values (significances) of less than 0.05. These results agree with those obtained in T test.

- The non-parametric sign test for related samples showed significant differences in the O1, O2, T5 and T6 channels with p-values (significances) of less than 0.05. These results agree with those obtained in the T test and Wilcoxon test.

4.3 Mean frequency

The averaged values obtained for the mean frequency for each channel and task are shown in a self-explanatory table.

	MEAN FREQUENCY	
	Closed eyes	Task j1
F7	15.700	13.922
F8	15.875	14.381
O1	12.557	16.319
O2	12.864	16.797
T5	13.353	18.155
T6	12.973	17.356

One can observe from the numbers that the mean frequency increases considerably in the job j1 for the occipital and temporal areas.

The statistical analysis on the data yielded the following results:

- In the $T-$test for related samples the results show significant differences in all channels except F8 with p-values (significances) of less than 0.05.

  In the temporal and occipital area the p-values obtained are substantially lower than 0.05. Specifically O1 ($T = -5.9254, \mathrm{sig} = 0.00007$), O2 ($T = -6.599, \mathrm{sig} = 0.00003$), T5 ($T = -7.208, \mathrm{sig} = 0.00001$) and T6 ($T = -8.008, \mathrm{sig} = 0.000002$). The averages are greater in the j1 task.

  In the F7 electrode the p-value obtained is very close to the significance level prefixed (0.05), being ($T = 2.283, \mathrm{sig} = 0.046$).

  In the line graphs below we can observe that the differences in F7 are not as clear as in T6:

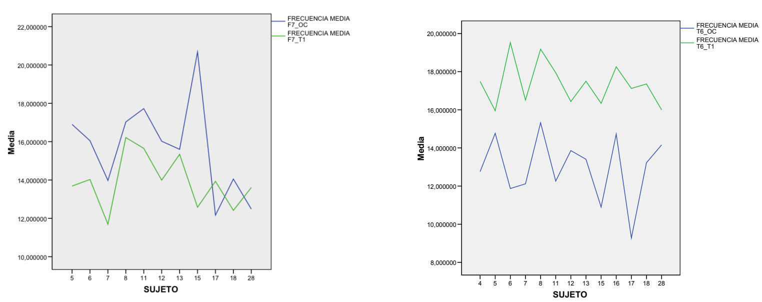

Figure 2: Line graphs of the mean frequency in F7 and T6

In the Box-Plot in Figure 3 the mean frequency values are shown at rest and performing the simulation job for each of the electrodes studied.

In F7 and F8 channels the Box-Plot are very similar, while in O1, O2, T5 and T6 the Box-Plot of the simulation task displays higher values compared to the closed eyes EEG.

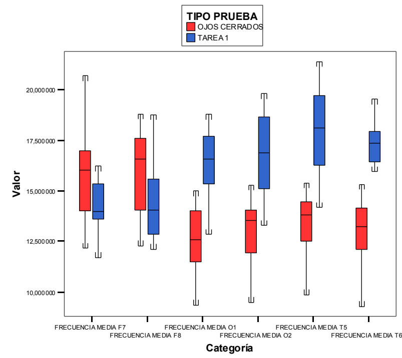

Figure 3: Box-Plot of the mean frequency in both types of recording for each electrode analyzed

Non-parametric tests may help to decide if the difference is considered significant or not in F7.

- The application of the Wilcoxon test for paired samples provided differences in all channels except F8 with p-values (significances) of less than 0.05.
- The sign test for related samples showed variations in O1, O2, T5 and T6, with p-values (significances) of less than 0.05 (in all cases sig = 0.0002).

5 Conclusions

It can be concluded that the parameters of quantification of EEG used so far to analyze the EEG signals collected to the members of the Armed Forces show significantly the activation of the occipital and temporal areas involved in tank driving simulation.

The results show an increase in the Mobility and mean frequency, that reaches clearly the beta band for the sensorial processing sites. That is to say, the spectral content of the signal is moved to higher frequencies, and this fact is accompanied by a reduction of the amplitude of the signal in O1, O2 and T6, but not reaching significance.

Concerning the results obtained for the Activity, only the non-parametric tests indicate conclusively a significant increase in measurements F7 and F8 channels during the test j1.

The results obtained in the exponent, considering both types of tests, indicate conclusively a significant decrease of the measurements in the O1, O2, T5 and T6 channels during test j1. The values corresponding to closed eyes are higher than in task j1 significantly on these leads, pointing to a lower complexity.

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