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A pipe network simulation model with dynamic transition between free surface and pressurized flow

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Abstract

Water flow numerical simulation in urban sewer systems is a topic that combines surface flows and pressurized flows in steady and transient situations. A numerical simulation model is developed in this work, capable of solving pipe networks mainly unpressurized, with isolated peaks of pressurization. For this purpose, a reformulation of the mathematical model through the Preissmann slot method is proposed. The numerical model is based on the first order Roe's scheme, in the frame of finite volume methods. The validation has been done by means of several cases with analytic solutions or empirical laboratory data.

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Keywords: Shallow water flow; pressurized flow; pipe network; Preissmann slot; transitory; stationary; finite volume; Roe scheme; upwind; water hammer.

1. Introduction

Numerical simulation of urban sewer systems is characterized by the difficulty of setting a pipe network to simulate both transitory and steady states. The water flow is mainly unpressurized, but the pipes have a limited storage capacity and could be drowned under exceptional conditions if the water level raises quickly. In these situations, the flux is pressurized and another mathematical model should be taken into account in order to solve the problem accurately. A complete sewer system model should be able to solve steady and transient flows under pressurized and unpressurized situations and the transition between both flows (mixed flows). Most of the models developed primarly to study the propagation of hydraulic transients use schemes based on the Method of Characteristics (MOC) to solve both system of equations (e.g. Gray , 1954, Wiggert et al., 1977). This method transform the continuity and momentum partial differential equations into a set of ordinary differential equations that are easier to solve. Although the MOC schemes can handle complex boundary conditions, interpolation is needed in some cases and they are not strictly conservatives. This result in diffusion of the wave fronts that makes waves arrive to the boundaries at a wrong time. Therefore, a new efficient numerical scheme is needed in order to solve accurately the wave front. Mixed flow problems have been addressed in the last decades by several researchers under two main points of view. The first one consists of

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solving separately pressurized and shallow flows. Transitions between both flux types are treated as internal boundary conditions. This kind of models are more complex but capable to simulate sub-atmospheric pressures in the pipe. Other authors (e.g., García-Navarro et al. (1993), León et al. (2009), Kerger et al. (2010)) developed simulations applying the shallow water equations in a slim slot over the pipe (Preissmann slot method). The water level in the slot gives an estimation of the pressure of the pipe. The great advantage of this model is the use of one equation system for solving the complete problem. It has been widely used to simulate local transitions between pressurized/shallow flows but it is not so succesful for abrupt transitions (e.g., Trajkovic et al. (1999)) due to stability problems when the pipe width is replaced for the slot one. This fact results in a big difference between the wave speeds (from $\sim 10 m/s$ in shallow flow to $\sim 1000 m/s$ in pressurized flow). In order to solver this issue, the slot width could be increased, but it implies an accuracy loss in the results, due to the no-mass/momentum conservation. This work is focused on the development of a simulation model capable to solve sewer system networks working mainly under shallow flow situations but exceptionally pressurized, so it is based on the second options previously mentioned. Hence, it is aimed to generalize the shallow water equations by means of the Preissmann slot method. The explicit first order Roe scheme has been used as numerical method for all the simulations. The calculation times are not long so implicit methods or parallel computation have been dismissed. Fluxes and source terms have been treated by means of an upwind scheme (García-Navarro et al. (2000)).

2. Mathematical model

2.1. Shallow water equations

It is generally accepted that unsteady open channel water flows are governed by the 1D shallow water or St. Venant equations. These equations represent mass and momentum conservation along the main direction of the flow and are a good description for most of the pipe-flow-kind problems. They can be written in a conservative form as follows:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + gI_1\right) = gI_2 + gA\left(S_0 - S_f\right) \tag{2}$$

where A is the wetted cross section, Q is the discharge, g is the gravity acceleration, I_1 represents the hydrostatic pressure force term and I_2 accounts for the pressures forces due to channel/pipe width changes:

$$I_1 = \int_0^{h(x,t)} (h-\eta)\, b(x,\eta) d\eta \tag{3}$$

$$I_2 = \int_0^{h(x,t)} (h-\eta)\, \frac{\partial b(x,\eta)}{\partial x} d\eta \tag{4}$$

$$b = \frac{\partial A(x,\eta)}{\partial \eta} \tag{5}$$

The remaining terms, S_0 and S_f, represent the bed slope and the energy grade line (defined in terms of the Manning roughness coefficient), respectively:

$$S_0 = -\frac{\partial z}{\partial x}, \qquad S_f = \frac{Q|Q|n^2}{A^2 R^{4/3}} \tag{6}$$

where $R = A/P$, P being the wetted perimeter. The coordinate system used for the formulation is shown in Fig. 1. It is useful to rewrite the equation system in a vectorial form:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{R} \tag{7}$$

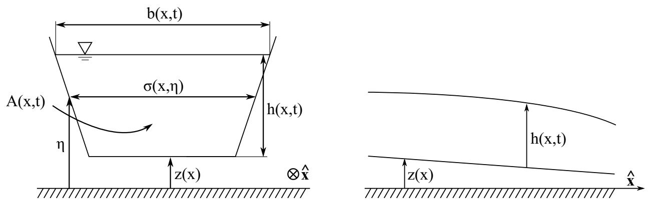

Fig. 1. Coordinate system for shallow water equations.

where

$$\mathbf{U} = (A, Q)^T, \qquad \mathbf{F} = \left(Q, Q^2/A + gI_1\right)^T, \qquad \mathbf{R} = \left(0, gI_2 + gA\left(S_0 - S_f\right)\right)^T \tag{8}$$

In those cases in which $\mathbf{F} = \mathbf{F}(\mathbf{U})$, when $I_2 = 0$, it is possible to rewrite the conservative system by means of the Jacobian matrix of the system:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{J}\frac{\partial \mathbf{U}}{\partial \mathbf{x}} = \mathbf{R}, \qquad \mathbf{J} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} = \begin{pmatrix} 0 & 1 \\ c^2 - u^2 & 2u \end{pmatrix} \tag{9}$$

where c is the wave speed (analogous to the speed of sound in gases), defined as follows:

$$c = \sqrt{g\frac{\partial I_1}{\partial A}} \tag{10}$$

The system matrix can be made diagonal by means of its set of real eigenvalues and eigenvectors, which represent the speed of propagation of the information:

$$\lambda^{1,2} = u \pm c, \qquad \mathbf{e}^{1,2} = (1, u \pm c)^T \tag{11}$$

It is very common to characterize the flow type by means of the Froude number $Fr = u/c$ (analogous to the Mach number in gases). It allows the classification of the flux into three main regimes: subcritical $Fr < 1$, supercritical $Fr > 1$ and critical $Fr = 1$.

2.2. Water-hammer equations

The inmediate response to changes in pipe flow can be taken up by the elastic compressibility of both the fluid and the pipe walls. Unsteady flow in the pipe can be described by the cross-section integrated mass and momentum equations (Chaudhry et al. (1994)):

$$\frac{\partial H}{\partial t} + v\frac{\partial H}{\partial x} + v\sin\theta + \frac{c_{WH}^2}{g}\frac{\partial v}{\partial x} = 0 \tag{12}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial H}{\partial x} + \frac{4\tau_0}{\rho D} = 0 \tag{13}$$

in which $H(x,t)$ is the elevation of the hydraulic grade line, $v(x,t)$ is the local cross-section averaged flow velocity, D is the diameter, τ_0 is the boundary shear, typically estimated by means of a Manning or Darcy-Weissbach friction model. The magnitude c_{WH} represents the speed of an elastic wave in the pipe:

$$c_{WH} = \sqrt{\frac{K/\rho}{1 + \frac{DK}{eE}}} \tag{14}$$

being e the pipe thickness and E, K the elastic modulus of the pipe material and fluid, respectively. By neglecting convective terms, it's possible to get a linear hyperbolic equation system:

$$\frac{\partial H}{\partial t} + \frac{c_{WH}^2}{g}\frac{\partial v}{\partial x} = 0 \tag{15}$$

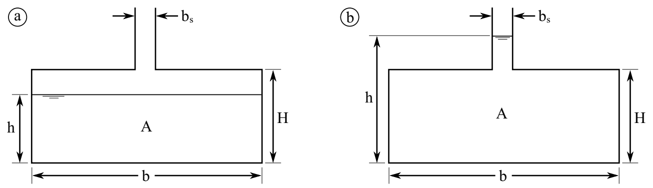

Fig. 2. (a) Pure shallow flow (b) Pressurized pipe.

$$\frac{\partial v}{\partial t} + g\frac{\partial H}{\partial x} + \frac{4\tau_0}{\rho D} = 0 \tag{16}$$

These equations conform a hyperbolic system, analogous to the shallow water equations, considering the pressure and the velocity as the conserved variables.

3. Preissmann slot model

The Preissmann slot approach assumes that the top of the pipe or closed channel is connected to a hypothetical narrow slot, open to the atmosphere, so the shallow water equations can be applied including this slot (see Fig. 2). The slot width is ideally chosen equaling the speed of gravity waves in the slot to the water hammer wavespeed, so the water level in the slot is equal to the pressure head level. This model is based on the previously remarked similarity between the wave equations which describe free surface/pressurized flows. The water hammer flow comes from the capacity of the pipe system to change the area and fluid density, so forcing the equivalence between both models requires that the slot stores as much fluid as the pipe would by means of a change in area and fluid density. The pressure term and the wavespeed for both situations are:

$$A \leq bH \longrightarrow h = \frac{A}{b}, \qquad I_1 = \frac{A^2}{2b}, \qquad c = \sqrt{g\frac{\partial I_1}{\partial A}} = \sqrt{g\frac{A}{b}} \tag{17}$$

$$A > bH \longrightarrow h = H + \frac{A - bH}{b_s}, \qquad I_1 = bH\left(\frac{A - bH}{b_s} + \frac{H}{2}\right) + \frac{(A - bH)^2}{2b_s}, \qquad c = \sqrt{g\frac{\partial I_1}{\partial A}} = \sqrt{g\frac{A}{b_s}} \tag{18}$$

The ideal choice for the slot width results in:

$$c_{WH} = c \Rightarrow b_s = g\frac{A_f}{c_{WH}^2} \tag{19}$$

in which A_f is the full pipe cross-section.

4. Finite volume numerical model

4.1. Explicit first order Roe scheme

Roe scheme is based on a local linearization of the conserved variables and fluxes:

$$\delta\mathbf{F} = \tilde{\mathbf{J}}\delta\mathbf{U} \tag{20}$$

It is necessary to build an approximate Jacobian matrix $\tilde{\mathbf{J}}$ whose eigenvalues and eigenvectors satisfy:

$$\delta\mathbf{U}_{i+1/2} = \mathbf{U}_{i+1} - \mathbf{U}_i = \sum_{k=1}^{2}(\tilde{\alpha}_k\tilde{\mathbf{e}}_k)_{i+1/2} \tag{21}$$

$$\delta\mathbf{F}_{i+1/2} = \mathbf{F}_{i+1} - \mathbf{F}_i = \tilde{\mathbf{J}}_{i+1/2}\delta\mathbf{U}_{i+1/2} = \sum_{k=1}^{2}\left(\tilde{\lambda}_k\tilde{\alpha}_k\tilde{\mathbf{e}}_k\right)_{i+1/2} = \delta\mathbf{F}^+_{i+1/2} + \delta\mathbf{F}^-_{i+1/2} \tag{22}$$

The $\tilde{\alpha}_k$ coefficients represent the variable variation coordinates in the Jacobian matrix basis. The eigenvalues and eigenvectors are expressed in terms of the average flux and wave speeds:

$$\tilde{\lambda}_k = (\tilde{u} \pm \tilde{c}), \qquad \tilde{\mathbf{e}}_k = (\tilde{u} \pm \tilde{c})^T \tag{23}$$

where

$$\tilde{u}_{i+1/2} = \frac{Q_{i+1}\sqrt{A_i} + Q_i\sqrt{A_{i+1}}}{\sqrt{A_i A_{i+1}}\left(\sqrt{A_{i+1}} + \sqrt{A_i}\right)}, \qquad \tilde{c}_{i+1/2} = \sqrt{\frac{g}{2}\left[\left(\frac{A}{b}\right)_i + \left(\frac{A}{b}\right)_{i+1}\right]} \tag{24}$$

The source terms of the equation system 7 are also discretizated using an upwind scheme (García-Navarro et al. (2000)):

$$\left(\tilde{\mathbf{R}}\delta x\right)_{i+1/2} = \left(\sum_{k+}\tilde{\beta}_k\tilde{\mathbf{e}}_k\right)_{i+1/2} + \left(\sum_{k-}\tilde{\beta}_k\tilde{\mathbf{e}}_k\right)_{i+1/2} \tag{25}$$

in which $\tilde{\beta}_k$ coefficients take the role of $\tilde{\alpha}_k$ ones for the fluxes. Then, the complete discretization of the system becomes:

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\delta t}{\delta x}\left[\left(\sum_{k+}\left(\tilde{\lambda}_k\tilde{\alpha}_k - \tilde{\beta}_k\right)\tilde{\mathbf{e}}_k\right)_{i+1/2} + \left(\sum_{k-}\left(\tilde{\lambda}_k\tilde{\alpha}_k - \tilde{\beta}_k\right)\tilde{\mathbf{e}}_k\right)_{i+1/2}\right] \tag{26}$$

4.2. Boundary conditions and stability conditions

In order to solve a numerical problem, it is necessary to impose some physical boundary conditions (BC) at the domain limits. The number of boundary conditions depends on the flow regime (subcritical or supercritical), so there are four posibilities for a 1D numerical problem (see Table 1).

Table 1. Boundary conditions.

Flow regime and boundary	Number of physical BC to impose
Upstream subcritical flow	1
Upstream supercritical flow	2
Downstream subcritical flow	1
Downstream supercritical flow	0

In the cases where a pipe junction is present, additional inner boundary conditions are necessary in order to solve the problem. Fig. 3 shows an example of a 3 pipe junction.
The water level equality condition is imposed at the junction:

$$h_2(1) = h_3(1) = h_1\left(I^1_{MAX}\right) \tag{27}$$

Discharge continuity condition is formulated depending on the flow regime:

$$Q_1\left(I^1_{MAX}\right) = Q_2(1) + Q_3(1), \qquad (subcritical\,junction) \tag{28}$$

$$Q_2(1) = Q_3(1) = \frac{1}{2}Q_1\left(I^1_{MAX}\right), \qquad (supercritical\,junction) \tag{29}$$

In a general case, considering N pipes:

$$h_1 = h_2 = \cdots = h_N \quad , \quad \sum_{i=1}^{N} Q_i = 0 \tag{30}$$

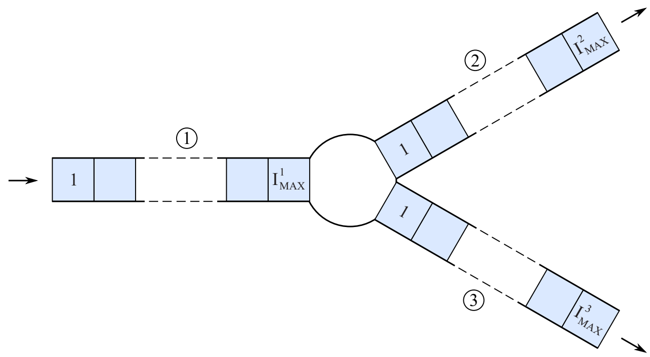

Fig. 3. Example of pipe junction.

In some cases, a storage well junction is used, so the boundary conditions are modified as follows:

$$h_1 = h_2 = \cdots = h_N = H_{well} \quad , \quad \sum_{i=1}^{N} Q_i = A_{well}\frac{dH_{well}}{dt} \tag{31}$$

where A_{well} and H_{well} are the well top area and depth, respectively. The presented method is explicit, so it requires a control on the time step in order to avoid instabilities. Hence the Courant-Friedrichs-Lewy (CFL) condition is applied in terms of the CFL number:

$$\Delta t_{max} = \frac{\Delta x}{max\,(|u| + c)}, \qquad CFL = \frac{\Delta t}{\Delta t_{max}} \leq 1 \tag{32}$$

5. Test cases

The model is applied to several analytic or experimental test cases in order to evaluate its accuracy.

5.1. Steady state over a bump

Following Murillo et al. (2012) a frictionless rectangular *25m x 1m* prismatic channel is assumed with the variable bed level$z\,(8 \leq x \leq 12) = 0.2 - 0.05(x-10)^2$ and initial conditions given by $h(x,0) = 0.5 - z(x)$ and $u(x,0) = 0$. The test cases simulated are summarized in Table 2.

Table 2. Steady state over a bump.

Test	Upstream $Q\left(m^3/s\right)$	Downstream $h\,(m)$
#1	0.18	0.33
#2	1.53	0.66 (sub)

Fig. 4 (a) shows the formation of hydraulic jump that connects both subcritical and supercritical regimes. In the second case, shown in Fig. 4 (b) the connection is made without any shock wave and it can be proved (and numerically checked) that this transition takes place in the highest part of the bump.

5.2. Dam-break

Dam-break is a classical example of non-linear flow with shocks, used to test the accuracy and conservation of the numerical scheme, by comparing with its analytical solution. In the case presented, an initial discontinuity of $1m$: $0.5m$ ratio with no friction was considered. Fig. 5 shows the good agreement between numerical and analytical solution at the given time.

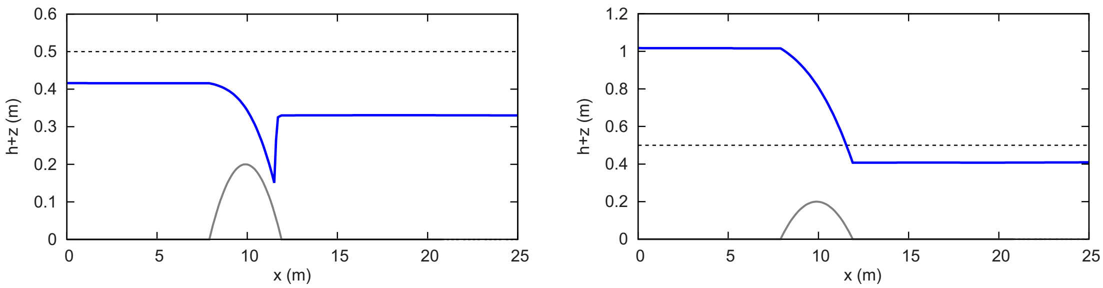

Fig. 4. Steady state over a bump. Initial state (dashed line). Bed level (grey). Numerical solution (blue): (a) Test #1; (b) Test #2.

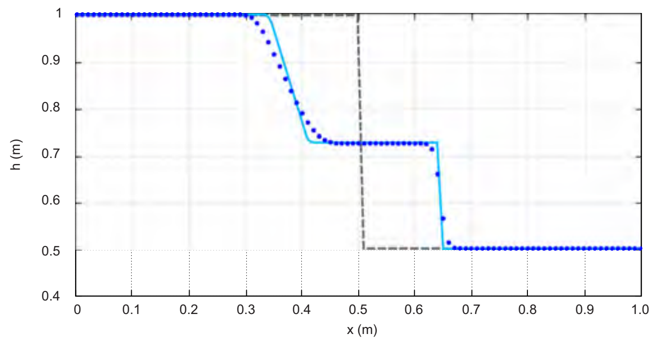

Fig. 5. Dam-break test case. Initial state (dashed line). Numerical results (blue dots). Analytical solution (cyan line).

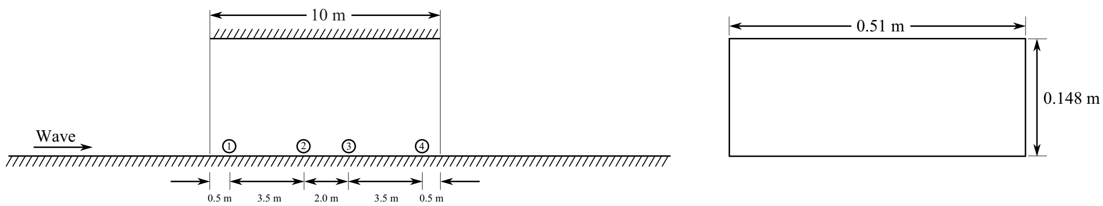

Fig. 6. Wiggert experimental setup.

5.3. *Wiggert test case*

The experimental case designed by Wiggert (Wiggert (1972)) and widely numerically reproduced (e.g. Kerger et al. (2010), Bourdarias et al. (2007)) is a horizontal *30 m* long and *0.51 m* wide flume. In the middle part a *10 m* roof is placed, setting up a closed rectangular pipe *0.148 m* in height (see Fig. 6). The Manning roughness coefficient is $0.01m^{-1/3}s$. A water level of *0.128 m* and no discharge are considered as initial conditions. Then a wave coming from the left causes the pressurization of the pipe. The imposed downstream boundary conditions are the same values measured by Wiggert (Fig. 7).

Fig. 8 shows the numerical results for the four gauges but the experimental comparison is only done for the second one, due to data availability issues. An overall good agreement is observed. Only small numerical oscillations are observed.

6. Pipe networks

In this section a simple looped pipe network (see Fig. 9 (a)) is presented (Wixcey (1990)). Each pipe is closed, squared, *1 m* wide and *100 m* long. The bed slopes are $S_{0,1} = 0.002$, $S_{0,2} = S_{0,3} = 0.001$, $S_{0,4} = 0.0$, $S_{0,5} = S_{0,6} = 0.001$, $S_{0,7} = 0.002$ and the Manning friction coefficient *n=0.01*. A first calculation was done in order to obtain a

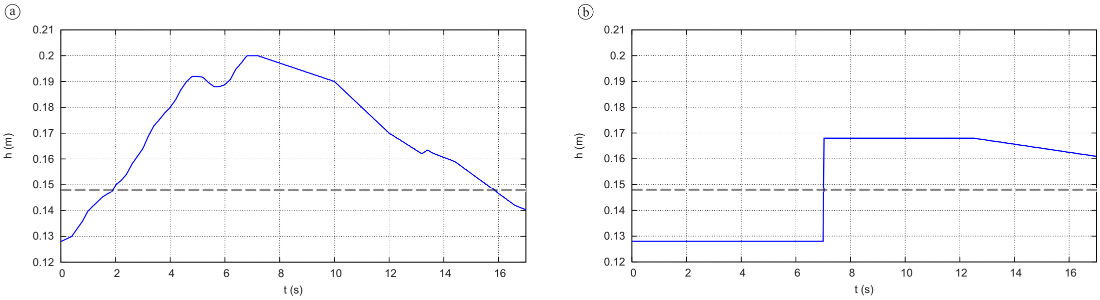

Fig. 7. Upstrem (a) and downstream (b) pressured head level.

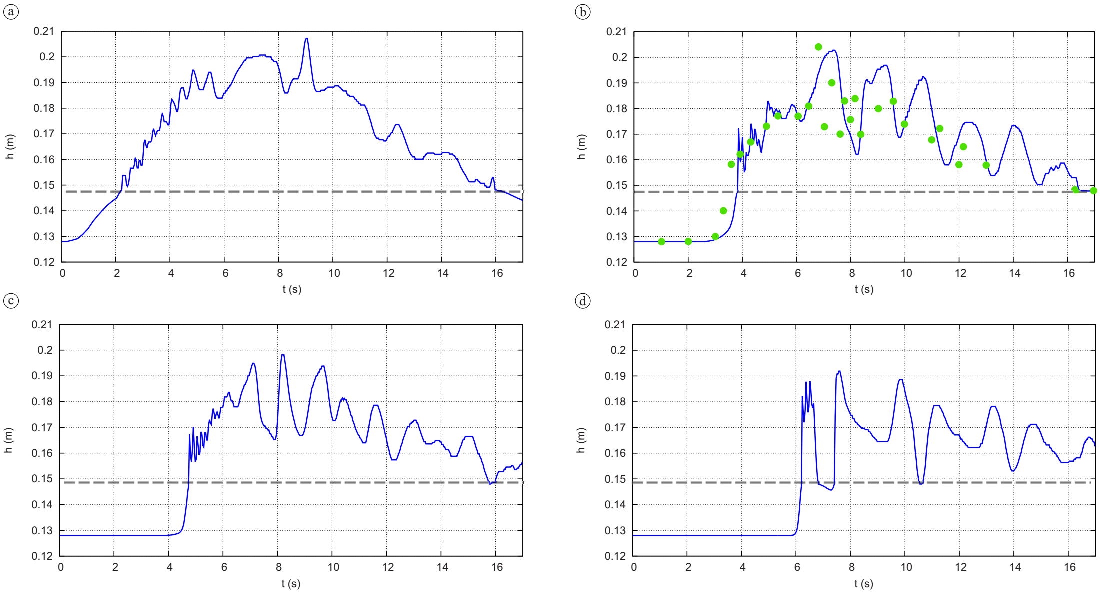

Fig. 8. Comparision between numerical results (blue) and experimental data (green dots) for the Wiggert test case.

steady state from the following conditions:

$$Q_1(i) = Q_7(i) = 0.1m^3/s, \qquad Q_2(i) = Q_3(i) = Q_5(i) = Q_6(i) = 0.05m^3/s, \qquad Q_4(i) = 0 \tag{33}$$

$$h_1(i) = h_2(i) = h_3(i) = h_4(i) = h_5(i) = h_6(i) = h_7(i) = 0.2m, \qquad Q_1(1) = 0.1m^3/s \tag{34}$$

Junctions J_1 and J_2 are treated as normal confluences and a storage well of $5m^2$ top surface is assumed in junctions W_1 and W_2. The steady state is now used as an initial condition for a second calculation. A triangular function of peak discharge Q_{MAX} and a period of *600 s* (Fig. 9 (b)) is imposed at the beginning of the pipe 1. The minimum discharge is $0.1m^3/s$. Two different situations were studied. In the first one the peak discharge is fixed to $Q_{MAX} = 2.0m^3/s$, so the flow remains unpressurized all over the pipe system. In the second case, the maximum discharge is raised to $Q_{MAX} = 3.0m^3/s$, so pressurization occurs in some points of pipe 1. Fig. 10 shows the results for the pressurized case. The results in pipes 3 and 6 have been omitted because of symmetry reasons. The discharge at the center of the pipe 4 is constrantly zero and equal in magnitude but opposite in sign in the symmetric points.

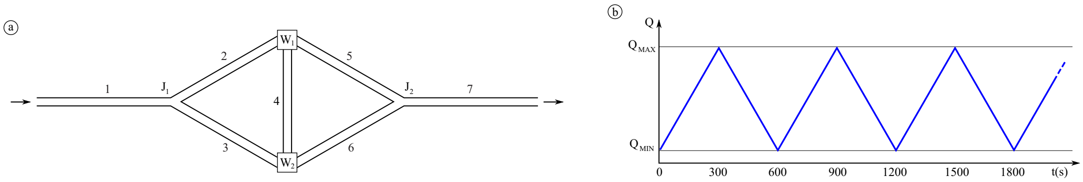

Fig. 9. (a) Scheme of the looped pipe network; (b) Triangular inlet hydrograph in the pipe 1.

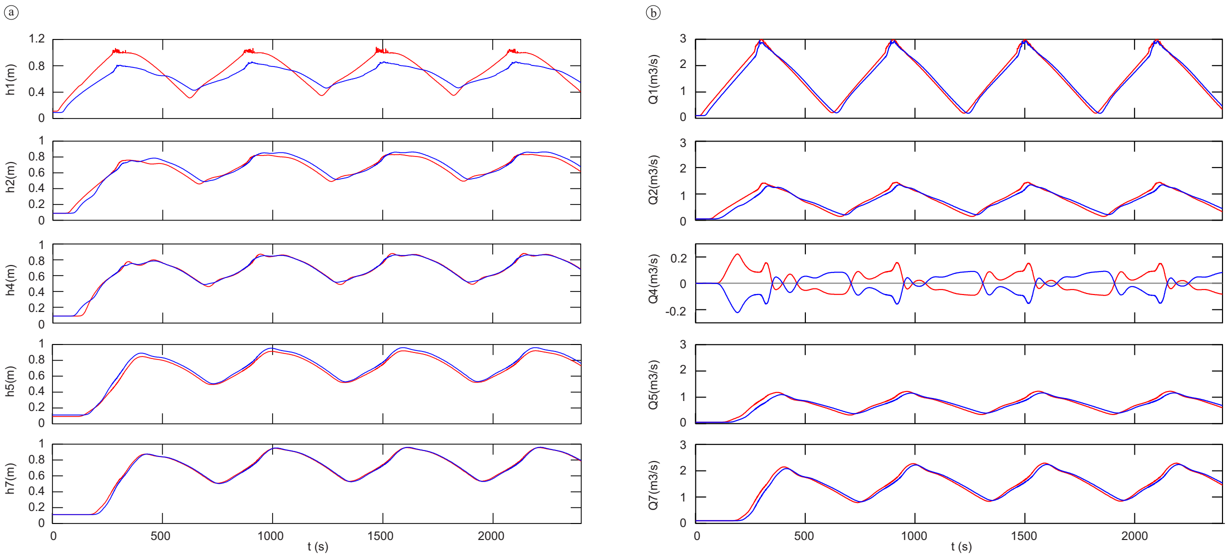

Fig. 10. Time histories at grid points *i=N/2* (red) and *i=N* (blue) for punctually pressurized flow: (a) Water depth; (b) Discharge. The discharge for pipe 4 is recorded at locations *i=1* (red), *i=N/2* (grey) and *i=N* (blue).

7. Conclusions

The present work leads to the conclusion of the reasonably good applicability of the Preissmann slot model for an estimation of the pressure values in transitory situations between both shallow and pressurized flows. In this case, a small deviation of the ideal value of the slot width introduces an error in the mass and momentum conservation. However, it is clear that wider slots improves the numerical model stability, so it is remarkably important to find an agreement between both facts, in benefit of the numerical results accuracy. The model takes advanteges of the similarity between the shallow water and pressurized flow equations and provides a way to simulate pressurized pipes treating the system as a open channel in the slot. This fact result in a easier computer implementation of the model because it avoids the numerical complexities that would be required to model separately the pressurized/free-surface portions by water-hammer/shallow-water equations. The use of an explicit scheme implies a limitation on the time step. This limiting condition increases the calculation time in the pressurized cases, due to the small width of the slot. It is possible to adapt the method to more realistic systems, like pipe networks which can be punctually pressurized. In these cases, additional internal boundary conditions are necessary in order to represent pipe junctions or storage wells.

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