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On State Estimation of Timed Petri Nets

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Abstract

This report presents an online algorithm for state estimation of timed choice-free Petri nets. We assume that the net structure and initial marking are known, and that the set of transitions is divided in *observable* and *unobservable* one. Given an observed word and assuming that the time durations associated to the unobservable transitions are unknown, the problem is to estimate the possible states in which the timed net system can be. This work extends the notion of *basis markings* defined for untimed Petri nets considering now the time information. The proposed algorithm deals with three main steps: (1) wait for a new observation and compute the set of basis markings without considering the time; (2) update the set of time equations that contain the time restriction for the unobservable transitions; (3) update the set of basis markings removing the time-inconsistent markings. The extension of the algorithm to general nets is discussed, as well.

Finally, the adaption of the proposed algorithm to distributed system is discussed. A distributed system is composed by a set of timed PN called sites connected by buffers. An agent is assigned for each site and it observes the firing of transitions and performs state estimation algorithm.

Keyword: Petri nets, timed Petri nets, state estimation, observability

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Chapter 1

Introduction

Reconstructing the state of a system from available measurements is a fundamental issue in several applications. State observation can be seen as a self-standing problem, but also as a pre-requisite for solving problems of different nature. This problem has been extensively investigated in time driven systems. On the contrary, despite the attention payed by several authors in the last years, there are relatively few works addressing this topic in discrete and hybrid systems, thus several related problems are still open.

In the case of discrete event systems modeled by Petri nets (PN), different approaches for observability have been recently proposed. In [7] the problem was that of reconstructing the initial marking (assumed only partially known) from the observation of transition firings. In [9] this approach was extended to the observation and control of timed nets. In other works it was assumed that some of the transitions of the net are not observable [4] or undistinguishable [6], thus complicating the observation problem. In [1] the author has studied the possibility of defining the set of markings reached firing a "partially specified" step of transitions using logical formulas, without having to enumerate this set. In [11] the authors have discussed the problem of estimating the marking of a Petri net using a mix of transition firings and place observations.

In this work, we study the problem of state estimation of discrete event systems modeled by timed Petri nets. We assume that the set of transitions is split into two subsets: *observable* and *unobservable*. The firing of the observable transitions can be detected, while the firing of the unobservable transitions cannot and the time durations associated to unobservable transitions are unknown. The basic idea is to extend the notion of *basis markings* to timed nets. The set of *basis markings* is proposed in [8] to characterize the set of *consistent markings*, i.e., the set of possible markings of a PN after an observed word. Knowing the set of basic markings, the set of consistent markings is obtained from the first one by firing the unobservable transitions.

Using some reduction rules, we show how to reduce both the structure and the state space of the unobservable net. The reduction rules merge indistinguishable transitions, in order to reduce the complexity of the state estimation procedure. To reconstruct the marking of the original net it is necessary to determine the markings of the input/output places of merged transitions. These markings can be expressed as the solution of a linear system that expresses their dependence from the marking of the new places.

Assuming that the time durations of the unobservable transitions are not known, we compute together with the set of basis markings a set of *time equations*. This set represents the relation between the observation and time durations of unobservable firing sequences. The set of time equations is used after to reduce the set of basic markings. The online algorithm that we propose estimates the state of a timed PN and is based on the following three main steps: 1) compute the set of basis markings; 2) compute the set of time equations; 3) reduce the set of basis markings according to the set of time equations.

In the context of state estimation of distributed systems, [10] proposed an algorithm for timed Petri net with the assumption that the time durations of transitions are known. We assume that the PN model of each site is a state machine, while the global system is a *Deterministically Synchronized Sequential Processes (DSSP)* system [3]. We discuss the state estimation of timed DSSP with the assumption that all time durations of transitions are unknown.

The following chapters are organized as follows: a background on Petri nets are given in Chapter 2; in Chapter 3 we characterize the time duration of a firing sequences and reduction rules; and, an online algorithm for state estimation of timed PN is introduced in Chapter 4. At last, the discussion of adapting the online algorithm to distributed systems is given in Chapter 5.

Chapter 2

Basic Concepts

In this section, we recall the basic definition of (timed) Petri net system (for a general introduction, see [12]).

2.1 Timed Petri Nets

Definition 2.1. *A PN system is a pair* $\langle \mathcal{N}, \boldsymbol{m}_0 \rangle$*, where* $\mathcal{N} = \langle P, T, \boldsymbol{Pre}, \boldsymbol{Post} \rangle$ *is a net structure with a set of places* P*; a set of transitions* T*; the pre and post incidence matrices* $\boldsymbol{Pre}, \boldsymbol{Post} \in \mathbb{N}_{\geq 0}^{|P|\times|T|}$*; and* $\boldsymbol{m}_0 \in \mathbb{N}_{\geq 0}^{|P|}$ *is the initial marking, where* $|P|$ *is the number of places and* $|T|$ *is the number of transitions.*

The incidence matrix is $\boldsymbol{C} = \boldsymbol{Post} - \boldsymbol{Pre}$. For every node $v \in P \cup T$, the set of its input and output nodes are denoted as ${}^\bullet v$ and $v^\bullet$, respectively. A *directed circuit* of PN is a sequence $p_{i1}t_{i1}p_{i2}t_{i2}\cdots p_{in}t_{in}$, where $p_{ij} \in P, t_{ij} \in T, p_{ij} \in {}^\bullet t_{ij}, t_{ij} \in {}^\bullet p_{i,j+1}$, and $\exists j \neq k, p_{ij} = p_{ik}$ or $t_{ij} = t_{i,j+1}$. A net having no directed circuit is called *acyclic*.

A transition $t \in T$ is enabled at a marking $\boldsymbol{m}$ if and only if $\boldsymbol{m} \geq \boldsymbol{Pre}[\cdot, t]$. If a marking $\boldsymbol{m}'$ is reachable from $\boldsymbol{m}$ by firing a sequence $\sigma = t_{i1}t_{i2}\cdots t_{in}$, where $t_{ij} \in T, j = 1, 2, \ldots, n$: the fundamental state equation can be written as $\boldsymbol{m}' = \boldsymbol{m} + \boldsymbol{C} \cdot \boldsymbol{\sigma}$, where $\boldsymbol{\sigma} \in \mathbb{N}_{\geq 0}^{|T|}$ is the firing count vector of σ; $\boldsymbol{m}[\sigma\rangle$ denotes that σ is firable from $\boldsymbol{m}$, while $\boldsymbol{m}[\sigma\rangle\boldsymbol{m}'$ means the firing of σ drives $\boldsymbol{m}$ to $\boldsymbol{m}'$.

The set of transitions T is partitioned into two sets: T_o and T_u, where T_o is the set of *observable* transitions, whose firing can be detected by an external observer, and T_u is the set of *unobservable* transitions. The firing sequence σ^o is an observable firing sequence, if $t \in \sigma^o$, then $t \in T^o$; σ^u is an unobservable firing sequence, if $t \in \sigma^u$, then $t \in T^u$. An observation function $\lambda : T^* \rightarrow T_o^*$, where T_o^* is the Kleene closure of T_o, extracts a sequence of observable transitions $\lambda(\sigma)$ from σ. Let $\sigma = \sigma_1^u\sigma_1^o\sigma_2^u\sigma_2^o\cdots\sigma_n^u$, then $\lambda(\sigma) = \sigma_1^o\sigma_2^o\cdots\sigma_{n-1}^o$. Observable transitions are represented as white rectangles, while unobservable ones as black rectangles.

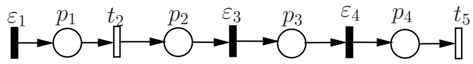

Figure 2.1: Example of $w = \lambda(\sigma)$

Example 2.1. *For the PN in Fig. 2.1, observable transitions are t_2, t_5, and unobservable transitions are $\varepsilon_1, \varepsilon_3, \varepsilon_4$. Let $\sigma = \varepsilon_1 t_2 \varepsilon_3 \varepsilon_4 t_5$, then the observed word of σ is $w = \lambda(\sigma) = t_2 t_5$.*

Definition 2.2. *A timed PN system is a triple $\langle \mathcal{N}, \boldsymbol{\theta}, \boldsymbol{m}_0 \rangle$, where $\langle \mathcal{N}, \boldsymbol{m}_0 \rangle$ is a PN system and $\boldsymbol{\theta} \in \mathbb{R}_{\geq 0}^{|T|}$ is the time vector that associates to each transition t_j a constant time delay, $\theta_j = \boldsymbol{\theta}[t_j]$.*

The time duration of a transition is deterministic, i.e., if a transition t_j is enabled at time τ, t_j is fired at $\tau + \theta[j]$. The single server semantic is used, which means a transition cannot be enabled simultaneously more than once.

We make the following assumptions:

(A1) the initial marking and net structure are known;

(A2) the unobservable induced subnet ($PN_u = \langle P, T_u, \boldsymbol{Pre_u}, \boldsymbol{Post}_u \rangle$)is acyclic, where $\boldsymbol{Pre}_u$ and $\boldsymbol{Post}_u$ are pre and post incidence matrices constrained by T_u;

(A3) The time durations of observable transitions are known, while the time durations of unobservable transitions are unknown.

The second assumption implies that there are not spurious solutions in the unobservable subnet, i.e., all markings, solution of the state equation are reachable. Therefore, the set of basis markings can be characterized using the state equation.

Even if the initial marking is known, because of the partial observation, the state of timed PN's cannot be determined by the observation. To characterize the possible set of markings we use a subset of it, which is called the set of *basis markings*. Knowing the set of basis markings, the consistent markings, which are the possible markings in the net system, can be obtained by simply firing the unobservable transitions from the basis markings.

Definition 2.3. *[8] Given a marking $\boldsymbol{m}$ and an observable transition $t \in T_o$, we define the set of* explanations *of t at $\boldsymbol{m}$ as $\Sigma(\boldsymbol{m}, t) = \{\sigma \in T_u^* | \boldsymbol{m}[\sigma\rangle \boldsymbol{m'}, \boldsymbol{m'} \geq \boldsymbol{Pre}[\cdot, t]\}$.*

The set of minimal explanations *of t at $\boldsymbol{m}$ as $\Sigma_{min}(\boldsymbol{m}, t) = \{\sigma \in \Sigma(\boldsymbol{m}, t) | \nexists \sigma' \in \Sigma(\boldsymbol{m}, t) : \boldsymbol{\sigma'} \lneqq \boldsymbol{\sigma}\}$, where $\boldsymbol{\sigma'} \lneqq \boldsymbol{\sigma}$ means that for every t, $\boldsymbol{\sigma}'[t] \lneqq \boldsymbol{\sigma}[t]$ and there exists t such that $\boldsymbol{\sigma}'[t] < \boldsymbol{\sigma}[t]$.*

2.2 Basis Marking

In the following, the set of basis markings without time is introduced. The set of basis markings of observation w is denoted by $\mathcal{M}_b(w)$.

Definition 2.4. *The set of basis markings of observation $w = vt$ is defined as $\mathcal{M}_b(w) = \{\boldsymbol{m} \in \mathbb{N}_{\geq 0}^{|P|} | \forall \boldsymbol{m}' \in \mathcal{M}_b(v) : \forall \sigma \in \Sigma_{min}(\boldsymbol{m}', t), \boldsymbol{m}'[\sigma t\rangle \boldsymbol{m}\}$. For empty word ϵ, $\mathcal{M}_b(\epsilon) = \{\boldsymbol{m}_0\}$.*

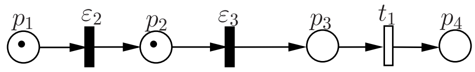

Figure 2.2: Example of the set of basis markings

Example 2.2. *Let us consider the PN in Fig. 2.2 with $\boldsymbol{m}_0 = [1, 1, 0, 0]^T$. The unobservable transitions are ε_2 and ε_3, while the observable transition is t_1. Assume t_1 has been observed.*

The set of basis markings before any observation is $\mathcal{M}_b(\epsilon) = \{\boldsymbol{m}_0\}$, where ϵ is the empty word. When $w = t_1$ is observed, the set of explanations is $\Sigma(\boldsymbol{m}_0, w) = \{\sigma_1, \sigma_2\}$, where $\sigma_1 = \varepsilon_3, \sigma_2 = \varepsilon_2\varepsilon_3$. Therefore, the set of minimal explanations is $\Sigma_{min}(\boldsymbol{m}_0, w) = \{\sigma_1\}$. By firing $\sigma_1 t_1$, the marking $\boldsymbol{m}_1 = [1, 0, 0, 1]^T$ is obtained and the new set of basis marking is $\mathcal{M}_b(t_1) = \{\boldsymbol{m}_1\}$.

For a marking $\boldsymbol{m}$ in the set of basis markings, there exists σ such that $\boldsymbol{m}_0[\sigma\rangle\boldsymbol{m}$. The sequence σ is composed by the observable transitions and unobservable firing sequences, which are minimal explanations. In order to represent the firing sequences that drive the marking from $\boldsymbol{m}_0$ to $\boldsymbol{m}$, based on the set of minimal explanation, we present the set of minimal firing sequences.

Definition 2.5. *Given a marking $\boldsymbol{m}$ and an observed word $w = t_{i1}t_{i2} \cdots t_{i,n-1}t_{in}$, we define the set of* firing sequences consistent with w *as* $\Gamma(\boldsymbol{m}, w) = \{\sigma \in T^* | \sigma = \sigma_1^u t_{i1} \sigma_2^u t_{i2} \cdots t_{i,n-1} \sigma_n^u t_{in}, \boldsymbol{m}_0[\sigma\rangle\boldsymbol{m}\}$.

Based on $\Gamma(\boldsymbol{m}, w)$, we define the set of minimal firing sequences *as $\Gamma_{min}(\boldsymbol{m}, w) \subseteq \Gamma(\boldsymbol{m}, w)$, that $\sigma_j^u, j = 1, \ldots, n$ is a minimal explanation of corresponding marking and observation.*

Definition 2.6. *The set of basis markings at time τ of a timed Petri net is defined as $\mathcal{M}_b(w, \tau) = \{\boldsymbol{m} \in \mathcal{M}_b(w) | \exists \sigma \in \Gamma_{min}(\boldsymbol{m}, w), \sigma = \sigma' t, \lambda(\sigma t) = w, t \text{ is observed at } \tau\}$.*

The firing sequences consistent with w defines the firing sequences whose observation word is w and lead the system to the marking $\boldsymbol{m}$.

Chapter 3

Time Duration of Firing Sequence and Reduction Rules

3.1 Time Duration of Firing Sequence

In order to estimate the state of a timed PN, it is important to know the time duration of a firing sequence. In this chapter, we define and analyze such time duration.

Let us consider a firing sequence $\sigma = t_1 t_2 \cdots t_n$, and τ_1 and τ_n are the time instant when t_1 and t_n are fired, respectively. The time duration of σ is denoted by $\iota(\sigma)$ and it is defined as the time duration from the enabling of t_1 to the firing of t_n:

$$\iota(\sigma) = \tau_n - (\tau_1 - \theta_1). \tag{3.1}$$

Proposition 3.1. *Let $\sigma = t_1 t_2 \cdots t_n$, the following equation is satisfied:*

$$max\{\theta_1, \ldots, \theta_n\} \leq \iota(\sigma) \leq \sum_{i=1}^{n} \theta_i. \tag{3.2}$$

If one and only one transition from σ is enabled at each time instant, then

$$\iota(\sigma) = \sum_{i=1}^{n} \theta_i \tag{3.3}$$

Proof. If there exists overlapping of time durations, the time duration of the firing sequence is less than the sum of the time durations of all transitions (3.2). If there is no overlapping, then (3.3) holds. □

The previous proposition can be generalized to sequences that can be partitioned into subsequences. For example, if $\sigma = \sigma_1 \sigma_2 \cdots \sigma_n$ and at each time moment, the enabled transitions belong to one and only one subsequence σ_i, then:

$$\iota(\sigma) = \iota(\sigma_1) + \iota(\sigma_2) + \cdots + \iota(\sigma_n). \tag{3.4}$$

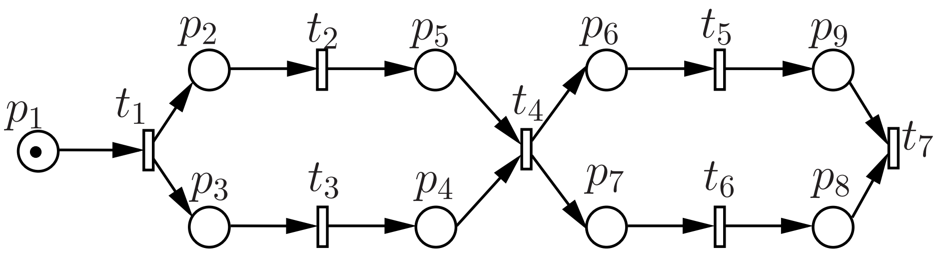

Figure 3.1: Example of $\iota(\sigma) = \iota(\sigma_1) + \iota(\sigma_2) + \cdots + \iota(\sigma_n)$

Example 3.1. *Let us consider the PN in Fig. 3.1 with* $\boldsymbol{m}_0 = [1, 0, 0, 0, 0, 0, 0, 0, 0]^T$ *and* $\boldsymbol{\theta} = [1, 2, 3, 1, 2, 3, 1]^T$. *Since it is a deterministic PN, the following observed word is obtained* $w = t_1t_2t_3t_4t_5t_6t_7$ *at the following time instants* $1, 3, 4, 5, 7, 8, 9$.

Let us write w *as* $w = \sigma = \sigma_1\sigma_2\sigma_3\sigma_4\sigma_5$, *with* $\sigma_1 = t_1, \sigma_2 = t_2t_3, \sigma_3 = t_4, \sigma_4 = t_5t_6, \sigma_5 = t_7$. *According to* (3.1), *the time durations are* $\iota(\sigma) = 9, \iota(\sigma_1) = 1, \iota(\sigma_2) = 3, \iota(\sigma_3) = 1, \iota(\sigma_4) = 3, \iota(\sigma_5) = 1$. *Since the condition in*(3.4) *is satisfied,*

$$\begin{aligned} \iota(\sigma) &= \iota(\sigma_1) + \iota(\sigma_2) + \iota(\sigma_3) + \iota(\sigma_4) + \iota(\sigma_5) \\ &= 1 + 3 + 1 + 3 + 1 = 9. \end{aligned}$$

3.2 Reduction Rules

The firing of unobservable transitions cannot be distinguished by observation. In order to reduce the state space of the unobservable subnet, reductions can be used. In this section, based on [2], reduction rules are applied to ordinary timed Petri net systems. The rules should be applied before the state estimation algorithm.

The first reduction rule is shown in Fig. 3.2(a), $\varepsilon_1, \cdots, \varepsilon_{n-1}$ are unobservable and, $|p_1{}^\bullet| = 1$; $|{}^\bullet p_i| = |p_i{}^\bullet| = |{}^\bullet\varepsilon_j| = |\varepsilon_j{}^\bullet| = 1, i = 2, \ldots, n-1, j = 1, \ldots, n-1$. The unobservable firing sequence $\varepsilon_1\varepsilon_2\cdots\varepsilon_{n-1}$ moves a token from p_1 to p_n and can be merged into one transition $\varepsilon_{1,n-1}$, such that, in the reduced net,

- $\boldsymbol{m}[p_{1,n-1}] = \sum_{i=1}^{n-1} \boldsymbol{m}[p_i]$,
- $\theta_{1,n-1} = \sum_{i=1}^{n-1} \theta_i$.

Fig. 3.2(b) illustrates the second reduction rule, $\varepsilon_1, \cdots, \varepsilon_{n+1}$ are unobservable transitions and $|p_i{}^\bullet| = 1, i = 1, \ldots, n$; $|{}^\bullet p_{n+1}| = n$ and $|p_{n+1}{}^\bullet| = 1$. The unobservable firing sequence $\varepsilon_1\varepsilon_{n+1}$ $(\ldots,\ \varepsilon_n\varepsilon_{n+1})$ moves a token from p_1 $(\ldots,\ p_n)$ to p_{n+2}. Therefore, ε_1 and ε_{n+1} $(\ldots,\ \varepsilon_n$ and $\varepsilon_{n+1})$ can be merged into one transition $\varepsilon_{1,n+1}$ $(\ldots,\ \varepsilon_{n,n+1})$, such that,

- $\boldsymbol{m}[p_{i,n+1}] = \boldsymbol{m}[p_i] + \boldsymbol{m}[p_{n+1}]$,

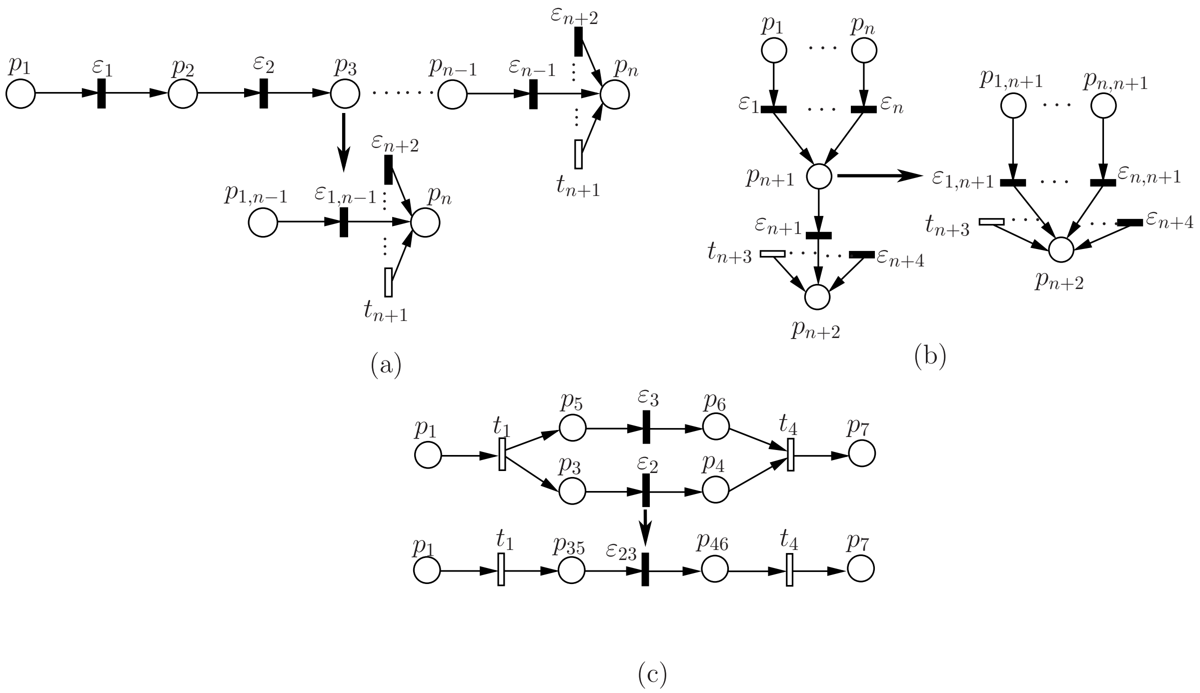

Figure 3.2: Illustration of the reduction rules

- $\theta_{i,n+1} = \theta_i + \theta_{n+1}$.

The third reduction rule is presented in Fig. 3.2(c), unobservable transitions ε_2 and ε_3 cannot be distinguished in the firing sequence $t_1\varepsilon_2\varepsilon_3t_4$ or $t_1\varepsilon_3\varepsilon_2t_4$. Therefore, ε_2 and ε_3 can be merged into one transition ε_{23}, such that, the time duration is $\theta_{23} = max\{\theta_2, \theta_3\}$. The marking of the reduced net satisfies:

- $\boldsymbol{m}[p_{35}] = \boldsymbol{m}[p_3] + \boldsymbol{m}[p_5], \boldsymbol{m}[p_{46}] = 0$.
- $\boldsymbol{m}[p_{35}] = 0, \boldsymbol{m}[p_{46}] = \boldsymbol{m}[p_4] + \boldsymbol{m}[p_6]$.

Chapter 4

State estimation of choice-free nets

The state estimation algorithm proposed here contains three steps: (1) the set of basis markings is computed without considering time; (2) the set of time equations is obtained; (3) the set of basis markings is reduced based on the time information.

4.1 Computation of basis markings

The set of basis markings at time $\tau = 0$ is $\mathcal{M}_b(\epsilon, 0) = \{\boldsymbol{m}_0\}$. Let us assume that the current set of basis markings at time τ is $\mathcal{M}_b(w, \tau)$, where w is the actual observation. When the firing of a new transition t_j is observed at time τ_j, the following operations should be performed in order to compute $\mathcal{M}_b(wt_j, \tau_j)$.

1. Let $\mathcal{M}_b(wt_j, \tau_j) = \emptyset$,
2. For each $\boldsymbol{m} \in \mathcal{M}_b(w, \tau)$,
   (a) compute $\Sigma_{min}(\boldsymbol{m}, t_j)$,
   (b) let $\mathcal{M}' = \{\boldsymbol{m}' | \boldsymbol{m}[\sigma t_j\rangle \boldsymbol{m}', \sigma \in \Sigma_{min}(\boldsymbol{m}, t_j)\}$,
   (c) let $\mathcal{M}_b(wt_j, \tau_j) = \mathcal{M}_b(wt_j, \tau_j) \cup \mathcal{M}'$.

For each basis marking $\boldsymbol{m}$ of the previous set, the set of minimal explanations is computed in $\Sigma_{min}(\boldsymbol{m}, t_j)$. Therefore, when t_j is observed after the firing of the minimal explanations of $\Sigma_{min}(\boldsymbol{m}, t_j)$ from $\boldsymbol{m}$, the new set of basic markings is obtained.

Example 4.1. *Let us consider the PN's in Fig. 4.1 with* $\theta_1 = 1$ *and* $\boldsymbol{m}_0 = [1, 1, 1, 0, 0]^T$. *The set of minimal firing sequences for the empty word is* $\Gamma_{min}(\boldsymbol{m}_0, \epsilon) = \emptyset$, *and the set of basis marking at time* 0 *is* $\mathcal{M}_b(\epsilon, 0) = \{\boldsymbol{m}_0\}$.

If $w = t_1$ *is observed at time* 4, $\mathcal{M}_b(t_1, 4)$ *is computed as follows:*

1. $\mathcal{M}_b(t_1, 4) = \emptyset$;

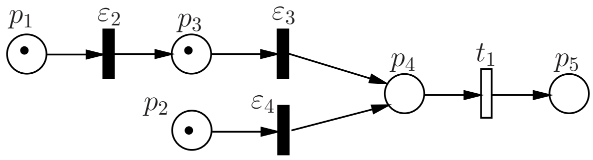

Figure 4.1: PN system used in Example 4.1

2. $\Sigma_{min}(\boldsymbol{m}_0, t_1) = \{\varepsilon_3, \varepsilon_4\}$*;*

3. $\mathcal{M}' = \{\boldsymbol{m}_1 = [1,0,1,0,1]^T, \boldsymbol{m}_2 = [1,1,0,0,1]^T\}$*, where* $\boldsymbol{m}_0[\varepsilon_4 t_1\rangle \boldsymbol{m}_1$*,* $\boldsymbol{m}_0[\varepsilon_3 t_1\rangle \boldsymbol{m}_2$*;*

4. $\mathcal{M}_b(t_1, 4) = \{\boldsymbol{m}_1, \boldsymbol{m}_2\}$*.*

The sets of minimal firing sequences are $\Gamma_{min}(\boldsymbol{m}_1, w) = \{\varepsilon_4 t_1\}, \Gamma_{min}(\boldsymbol{m}_2, w) = \{\varepsilon_3 t_1\}$*.*

4.2 Computation of the set of time equations

The set of basis markings in the previous section is computed without considering any time consideration. Assuming that the time durations associated to the unobservable transitions are not known, in this section we provide a procedure to obtain a set of equations to characterize all possible time durations associated to these unobservable transitions. It will be shown also how this set of time equations can be used to remove those time-inconsistent markings from the set of basis markings.

Let us assume that the time instant at which t_j was observed is τ_j, while the current set of basis markings is $\mathcal{M}_b(wt_j, \tau_j)$. To each set of basis markings we associate a *set of time equations*. These equations are obtained as the union of different equations. Let $\Gamma = \bigcup_{\boldsymbol{m} \in \mathcal{M}_b(wt_j, \tau_j)} \Gamma_{min}(\boldsymbol{m}, wt_j)$ be the set of all minimal firing sequences of all basis markings. The following time equation o_{τ_j} is obtained:

$$\forall \sigma \in \Gamma, min\{\iota(\sigma)\} = \tau_j.$$

Example 4.2. *In Example 4.1, the set of basis markings at time* 4 *has been computed. The set of minimal firing sequences are* $\Gamma_{min}(\boldsymbol{m}_1, t_1) = \{\varepsilon_4 t_1\}$ *and* $\Gamma_{min}(\boldsymbol{m}_2, t_1) = \{\varepsilon_3 t_1\}$*. Therefore,* $\Gamma = \{\varepsilon_3 t_1, \varepsilon_4 t_1\}$ *and the time equation is*

$$o_4 : \min\{\iota(\varepsilon_3 t_1), \iota(\varepsilon_4 t_1)\} = 4.$$

This has the following intepretation: because t_1 *has been fired at* 4 *and since for its firing,* ε_3 *or* ε_4 *should fire the firing delay of at least one of the following sequences* $\varepsilon_3 t_1$ *and* $\varepsilon_4 t_1$ *should be* 4*.*

If t_1 is observed again at time 6, the sets of minimal explanations are $\Sigma_{min}(\boldsymbol{m}_1, t_1) = \{\varepsilon_3\}$, $\Sigma_{min}(\boldsymbol{m}_2, t_1) = \{\varepsilon_4, \varepsilon_2\varepsilon_3\}$, implying the set of basis markings is $\mathcal{M}_b(t_1t_1, 6) = \{\boldsymbol{m}_3, \boldsymbol{m}_4\}$, where $\boldsymbol{m}_3 = [1, 0, 0, 0, 2]^T, \boldsymbol{m}_4 = [0, 1, 0, 0, 2]^T$, and the sets of firing sequences consistent with $w = t_1t_1$ are $\Gamma_{min}(\boldsymbol{m}_3, t_1) = \{\varepsilon_4 t_1 \varepsilon_3 t_1\}$ and $\Gamma_{min}(\boldsymbol{m}_4, t_1) = \{\varepsilon_3 t_1 \varepsilon_4 t_1$, $\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1\}$, while the corresponding time equation is

$$o_6 : \min \{\iota(\varepsilon_4 t_1 \varepsilon_3 t_1), \iota(\varepsilon_3 t_1 \varepsilon_4 t_1), \iota(\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1)\} = 6.$$

Let us analyze o_6. First of all, according to the definition of the time duration of a sequence, $\iota(\varepsilon_4 t_1 \varepsilon_3 t_1)$ and $\iota(\varepsilon_3 t_1 \varepsilon_4 t_1)$ provides the same information. The time durations of the two firing sequences are the same. Hence one of them can be removed from o_6. Removing for example the second one, we obtain

$$o_6 : \min \{\iota(\varepsilon_4 t_1 \varepsilon_3 t_1), \iota(\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1)\} = 6.$$

According to o_4, $\theta_3 \geq 4 - \theta_1 = 3$. We will show that in o_6, $\iota(\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1) > 6$ hence it is never the one that gives the minimum and can be removed.

$$\iota(\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1) \geq \theta_3 + \theta_3 + \theta_1 = 2\theta_3 + \theta_1 \geq 7$$

Therefore, $\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1$ is inconsistent with the time information. It can be deleted from o_6, so $o_6 = \iota(\varepsilon_4 t_1 \varepsilon_3 t_1) = 6$, and the corresponding basis marking should be removed, i.e., $\mathcal{M}_b(t_1t_1, 6) = \{\boldsymbol{m}_3 = [1, 0, 0, 0, 2]^T\}$.

As it was illustrated by the previous example, some basis markings are time inconsistent with the observation. On the other hand, some time equations that are obtained can be redundant.

In order to remove an element $\iota(\sigma_j)$ from a minimum function o_j the following procedure can be used: (i) let $\sigma_j = \sigma_j^1 \sigma_j^2 \ldots \sigma_j^r$ such that (3.4) is satisfied, i.e., the time duration of σ_j is the sum of time durations of the subsequences: $\iota(\sigma_j) = \iota(\sigma_j^1) + \iota(\sigma_j^2) + \ldots, + \iota(\sigma_j^r)$; (ii) find $\sigma_{k,l}^i, i = 1, \ldots$ in O such that they are subsequences of $\sigma_j^l, l = 1, \ldots, r$; according to (3.2), $\iota(\sigma_j^l) \geq \iota(\sigma_{k,l}^i), \forall i$; (iii) if $\sum_i \iota(\sigma_{k,l}^i) > \tau_j$, where τ_j is the time instant when o_j is computed, $\iota(\sigma_j)$ should be removed from o_j.

Proposition 4.1. *Let O be the current set of time equations, where*

$$O = \left\{ \begin{array}{c} min\{\iota(\sigma_{1,1}), \iota(\sigma_{1,2}), \ldots, \iota(\sigma_{1,k_1})\} = \tau_1, \\ min\{\iota(\sigma_{2,1}), \iota(\sigma_{2,2}), \ldots, \iota(\sigma_{2,k_2})\} = \tau_2, \\ \vdots \\ min\{\iota(\sigma_{q,1}), \iota(\sigma_{q,2}), \ldots, \iota(\sigma_{q,k_q})\} = \tau_q, \end{array} \right\}$$

and let o_j be the time equation obtained at time $\tau_j > \tau_q$, where $o_j : min\{\iota(\sigma_{j,1}), \iota(\sigma_{j,2}), \ldots, \iota(\sigma_{j,k_j})\} = \tau_j$, with $q, k_q, j \in \mathbb{N}_{>0}$.

Let $\iota(\sigma_j) \in \{\iota(\sigma_{j,1}), \iota(\sigma_{j,2}), \ldots, \iota(\sigma_{j,k_j})\}$ and decompose σ_j as $\sigma_j = \sigma_j^1 \sigma_j^2 \ldots \sigma_j^r$. Find all $\sigma_{k,l}^i$ in O such that $\sigma_{k,l}^i$ is a subsequence of a σ_j^l and $\forall l, \iota(\sigma_{k,j}^i) \geq \sigma_j^l$. If $\sum_i \iota(\sigma_{k,j}^i) > \tau_j$ then remove $\iota(\sigma_j)$ from o_j.

Proof. Obviously, If the previous conditions are satisfied, $\iota(\sigma_j) > \tau_j$. Hence it is not timed-consistent with the observation. □

4.3 Algorithm for state estimation

In this section, we present an algorithm for state estimation of systems modeled by timed PN's. When a new observation is available, the four steps in Algorithm 1 are performed.

Algorithm 1 Estimate the state of timed PN's

1: Compute the set of basis markings $\mathcal{M}_b(wt_j, \tau_j)$ based on the current observation t_j at τ_j.
2: Compute the time equation o_j.
3: Reduce o_j based on Prop. 4.1.
4: Reduce the set of basis markings $\mathcal{M}_b(wt_j, \tau_j)$ accordingly.

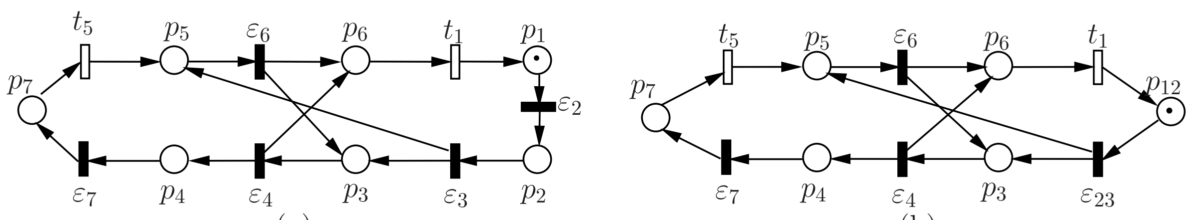

Figure 4.2: Example of the algorithm

Example 4.3. *Let us consider the PN in Fig. 4.2(a). with observable transitions t_1 and t_5, $\theta_1 = \theta_5 = 1$, and the initial marking $\boldsymbol{m}_0 = [p_1, p_2, p_3, p_4, p_5, p_6, p_7]^T = [1, 0, 0, 0, 0, 0, 0]^T$. Applying reduction rule # 1, transitions ε_2 and ε_3 are merged into ε_{23}, and places p_1 and p_2 are merged into p_{12}. Fig. 4.2(b) shows the reduced model. The initial marking is $\boldsymbol{m}_0 = [p_{12}, p_3, p_4, p_5, p_6, p_7]^T = [1, 0, 0, 0, 0, 0]^T$.*

The state estimation algorithm is applied on the reduced PN in Fig. 4.2(b). Let us assume the following observations: t_1 at 5, 9 and t_5 at 10.

• *At time* 0*, the set of basis markings is* $\mathcal{M}_b(\epsilon, 0) = \{\boldsymbol{m}_0\}$ *and the set of time equations is* $O = \emptyset$*.*

• *At time* 6*,* t_1 *is observed* ($w = t_1$)*. The set of minimal explanations is* $\Sigma_{min} = (\boldsymbol{m}_0, t_1) = \{\sigma_1.\sigma_2\}$*, where* $\sigma_1 = \varepsilon_{23}\varepsilon_6$*,* $\sigma_2 = \varepsilon_{23}\varepsilon_4$*, meaning that* σ_1 *or* σ_2 *has been fired in order to enable* t_1*. By firing* $\sigma_1 t_1$ *and* $\sigma_2 t_1$*, the set of basis markings is obtained as* $\mathcal{M}_b(w, 6) = \{\boldsymbol{m}_1, \boldsymbol{m}_2\}$*, where* $\boldsymbol{m}_1 = [1, 2, 0, 0, 0, 0]^T$*,* $\boldsymbol{m}_2 = [1, 0, 1, 1, 0, 0]^T$*, and the sets of minimal firing sequences are* $\Gamma_{min}(\boldsymbol{m}_1, w) = \{\sigma_1 t_1\}$ *and* $\Gamma_{min}(\boldsymbol{m}_2, w) = \{\sigma_2 t_1\}$*. The time equation at time* 6 *is* $min\{\iota(\sigma_1 t_1), \iota(\sigma_2 t_1)\} = 6$*, the only equation that will compose* O*.*

• *At time* 9*,* $w = t_1 t_1$ *and the sets of minimal explanations are* $\Sigma_{min}(\boldsymbol{m}_1, t_1) = \{\sigma_1, \varepsilon_4\}, \Sigma_{min}(\boldsymbol{m}_2, t_1) = \{\sigma_2, \varepsilon_6\}$*.*

By firing $\sigma_1 t_1$ *and* $\varepsilon_4 t_1$ *from* $\boldsymbol{m}_1$*, we obtain* $\boldsymbol{m}_3 = [1, 4, 0, 0, 0, 0]^T$ *and* $\boldsymbol{m}_4 = [2, 1, 1, 0, 0, 0]^T$*, respectively; by firing* $\sigma_2 t_1$ *and* $\varepsilon_6 t_1$ *from* $\boldsymbol{m}_2$*,* $\boldsymbol{m}_4$ *and* $\boldsymbol{m}_5 = [1, 0, 2, 2, 0, 0]^T$ *are obtained. Therefore,* $\mathcal{M}_b(w, 9) = \{\boldsymbol{m}_3, \boldsymbol{m}_4, \boldsymbol{m}_5\}$ *and* $\Gamma_{min}(\boldsymbol{m}_3, w) = \{\sigma_3\}$*,* $\Gamma_{min}(\boldsymbol{m}_4, w) = \{\sigma_4, \sigma_6\}$ *and* $\Gamma_{min}(\boldsymbol{m}_5, w) = \{\sigma_5\}$*, where* $\sigma_3 = \sigma_1 t_1 \sigma_1 t_1\}$*,* $\sigma_4 = \sigma_1 t_1 \varepsilon_4 t_1$*,* $\sigma_6 = \sigma_2 t_1 \varepsilon_6 t_1$*, and* $\sigma_5 = \sigma_2 t_1 \sigma_2 t_1\}$*.*

From previous sets the time equation at time 9 *is obtained as* $min\{\iota(\sigma_3), \iota(\sigma_4), \iota(\sigma_5), iota(\sigma_6)\} = 9$*.*

Observe that $\sigma_3 = \sigma_1(t_1\sigma_1)t_1$ *satisfying Prop. 4.1, and* $\iota(\sigma_3) = \iota(\sigma_1) + \iota(t_1\sigma_1) + \iota(t_1)$*. Form the equations of* O *can be observed immediately that* $\iota(t_1\sigma_1) \geq 6$ *and* $\iota(\sigma_1) = \iota(t_1\sigma_1) - \theta_1 \geq 5$*. Therefore,* $\iota(\sigma_3) \geq 5 + 6 + 1 = 12 > 11$*. Hence,* $\iota(\sigma_3)$ *should be removed. For the same reason,* $\iota(\sigma_5)$ *is also redundant and can be removed. The set of time equations becomes:*

$$O = \left\{ \begin{array}{l} min\{\iota(\sigma_1 t_1), \iota(\sigma_2 t_1)\} = 6, \\ min\{\iota(\sigma_4), \iota(\sigma_6)\} = 9. \end{array} \right\}$$

The set of basis markings is reduced to $\mathcal{M}_b(w, 9) = \{\boldsymbol{m}_4\}$*.*

• *At time* 10*,* t_5 *is observed* ($w = t_1 t_1 t_5$)*. The set of minimal explanations is* $\Sigma_{min} = (\boldsymbol{m}_4, t_5) = \{\varepsilon_7\}$*. Firing* $\varepsilon_7 t_5$*, the set of basis markings is obtained as* $\mathcal{M}_b(w, 10) = \{\boldsymbol{m}_6\}$*, where* $\boldsymbol{m}_6 = [2, 1, 0, 1, 0, 0]^T$*, and the set of minimal firing sequences as* $\Gamma_{min}(\boldsymbol{m}_6, w) = \{\sigma_7, \sigma_8\}$*, where* $\sigma_7 = \sigma_4 \varepsilon_7 t_5$ *and* $\sigma_8 = \sigma_6 \varepsilon_7 t_5$*. The time equation obtained at this time moment is* $min\{\iota(\sigma_7), \iota(\sigma_8)\} = 10$*. Hence, the set of time equations is*

$$O = \left\{ \begin{array}{l} min\{\iota(\sigma_1 t_1), \iota(\sigma_2 t_1)\} = 6, \\ min\{\iota(\sigma_4), \iota(\sigma_6)\} = 9, \\ min\{\iota(\sigma_7), \iota(\sigma_8)\} = 10. \end{array} \right\}$$

Being an online procedure, seems that the set of time equations is growing indefinitely. However, dealing only with time deterministic Petri nets, this is not true and there exists a moment from which any other time equation does not provide new information and the set of time equations is not updated anymore.

In the following, we discuss the time in a structurally live (SL) and structurally bounded (SB) choice-free net with a minimal T-semiflow $\boldsymbol{x}$. We assume the upper bound of time duration of every transition is u, and then the upper bound of a firing vector σ is $u(\boldsymbol{\sigma}) = u \cdot \sum_{i=1}^{|T|} \boldsymbol{\sigma}[i]$. Let $\boldsymbol{m}_h$ be *home state*, i.e., it can be reached from every reachable marking [5]. Based on [13], $\boldsymbol{m}_h$ will be reached by a firing sequence σ_h, with $\boldsymbol{\sigma}_h \leq \boldsymbol{x}$.

Proposition 4.2. *In a SL&SB choice-free net with minimal T-semiflow $\boldsymbol{x}$, if the initial marking is live, it is not necessary to update the set of time equations after the time instant $2 \cdot u(\boldsymbol{x})$.*

Proof. Because the net is SL&SB and the initial marking is live, then there exists a circle in the reachability graph and a home state $\boldsymbol{m}_h$. From $\boldsymbol{m}_0$, after firing σ_h, the home state is reached and the system behavior starts to repeat. Therefore, from this moment, it is not necessary to update the set of time equations. □

4.4 Extension to nets with choices

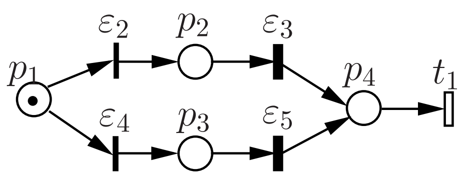

Figure 4.3: Example of PN's with choice

Let us consider the PN in Fig. 4.3 with ε_2 and ε_4 immediate transitions, i.e., $\theta_2 = \theta_4 = 0$, $\theta_1 = 1$, and $\boldsymbol{m}_0 = [1, 0, 0, 0]^T$. Assume t_1 is observed at time 4. Obviously, $\varepsilon_2\varepsilon_3$ or $\varepsilon_4\varepsilon_5$ has been fired to enable t_1, but we don't know exactly which one. Since t_1 has been observed at 4, we can say that $\iota(\varepsilon_2\varepsilon_3 t_1)$ or $\iota(\varepsilon_4\varepsilon_5 t_1)$ is 4, but we cannot say nothing about the time duration of the other. Hence, we cannot say that the minimum of $\iota(\varepsilon_2\varepsilon_3 t_1)$ and $\iota(\varepsilon_4\varepsilon_5 t_1)$ is 4.

Therefore, to apply the algorithm to general nets, there exist two possibilities: (1) reduce the net using the reduction rules, to obtain a choice-free one (2) enumerate all possible combinations of firing sequences. This approach is similar with the one of state estimation of untimed PN's.

Chapter 5

State estimation of distributed systems

Let us consider distributed system, for which each site is a timed Petri net system monitored by an agent. Every agent knows the structure and the initial marking of its site. The model of each site is a state machine, while the model of global system is a *Deterministically Synchronized Sequential Processes (DSSP)* system [3].

Definition 5.1. *[3] A PN system,* $\mathcal{S} = \langle P_1 \cup \cdots \cup P_K \cup B, T_1 \cup \cdots \cup T_K, \boldsymbol{Pre}, \boldsymbol{Post}, \boldsymbol{m}_0 \rangle$*, is a DSSP, if:*

1. $P_i \cap P_j = \emptyset$, $T_i \cap T_j = \emptyset$, $P_i \cap B = \emptyset$, $\forall i, j \in \{1, \ldots, K\}$, $i \neq j$;
2. $\langle \mathcal{SM}_i, \boldsymbol{m}_{0i} \rangle = \langle P_i, T_i, \boldsymbol{Pre}_i, \boldsymbol{Post}_i, \boldsymbol{m}_{0i} \rangle$, $\forall i \in \{1, \ldots, K\}$ *is a strongly connected and 1-bounded state machine (where* $\boldsymbol{Pre}_i$, $\boldsymbol{Post}_i$ *and* $\boldsymbol{m}_{0i}$ *are the restrictions of* $\boldsymbol{Pre}$, $\boldsymbol{Post}$ *and* $\boldsymbol{m}_0$ *to* P_i *and* T_i*);*
3. *The set* B *of buffers is such that* $\forall b \in B$:
   (a) $|^\bullet b| \geq 1$ *and* $|b^\bullet| \geq 1$,
   (b) $\exists i \in \{1, \ldots, K\}$ *such that* $b^\bullet \subset T_i$,
   (c) $\forall p \in P_1 \cup \cdots \cup P_K : t, t' \in p^\bullet \Rightarrow \boldsymbol{Pre}[b, t] = \boldsymbol{Pre}[b, t']$.

Transitions belonging to the set $TI = {}^\bullet B \cup B^\bullet$ *are called* interface transitions. *The remaining ones* ($T_1 \cup \cdots \cup T_K \backslash TI$) *are called* internal transitions.

In this chapter, immediate transitions, whose time delays are 0, are introduced to solve conflicts, i.e., if $|p^\bullet| > 1$, then $\forall t \in p^\bullet$ has its time delay $\theta_t = 0$. In order to let representation of models to be compact, immediate transitions are not shown in models. In Fig. 5.1(a), immediate transitions are t_1 and t_2, while they are not shown in Fig. 5.1(b),

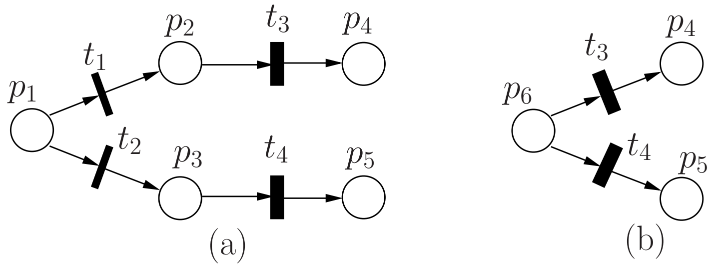

Figure 5.1: Immediate transitions

which is the compact representation, and the marking of p_6 is $\boldsymbol{m}[p_6] = \boldsymbol{m}[p_1] + \boldsymbol{m}[p_2] + \boldsymbol{m}[p_3]$.

In each site, the set of transitions T_i is partitioned into two sets: T_{io} and T_{iu}, where T_{io} is the set of *observable* transitions, whose firing can be detected by an external observer, and T_{iu} is the set of *unobservable* transitions. The firing sequence σ^o is an observable firing sequence, if $t \in \sigma^o$, then $t \in T_{io}$; σ^u is an unobservable firing sequence, if $t \in \sigma^u$, then $t \in T_{iu}$. An observation function $\lambda : T_i^* \to T_{io}^*$, where T_{io}^* is the Kleene closure of T_{io}, extracts a sequence of observable transitions $\lambda(\sigma)$ from σ. Let $\sigma = \sigma_1^u \sigma_1^o \sigma_2^u \sigma_2^o \cdots \sigma_n^u$, then $\lambda(\sigma) = \sigma_1^o \sigma_2^o \cdots \sigma_{n-1}^o$. Observable transitions are represented as white rectangles, while unobservable ones as black rectangles.

We make the following assumptions:

(A1) the initial marking and the net structure are known;

(A2) the unobservable induced subnet is acyclic;

(A3) the time durations of transitions are unknown;

(A4) an agent only observes the firing of transition in its site;

(A5) an agent only knows the structure and initial marking of its site.

When the firing of an observable transition t_j is observed, the marking of $\mathcal{SM}_i$ is $\boldsymbol{m}_i$, that, for $k = 1, \ldots, |P_i|$,

$$\boldsymbol{m}_i[k] = \begin{cases} 1, & p_k \in t_j{}^\bullet, \\ 0, & \textit{otherwise}. \end{cases}$$

Definition 5.2. *Two agents A_i and A_j are neighbor agents, if $\exists b \in B, {}^\bullet b \cap T_i \neq \emptyset, b^\bullet \cap T_j \neq \emptyset$, where T_i and T_j are sets of transitions of sites monitored by A_i and A_j, respectively.*

The proposed state estimation algorithm estimates the firing of transitions. An agent observes its site and computes possible firing sequences using the algorithm. The local estimation is improved based on the communication between agents. Using the communicaton information, the marking of buffers are interchanged between agents.

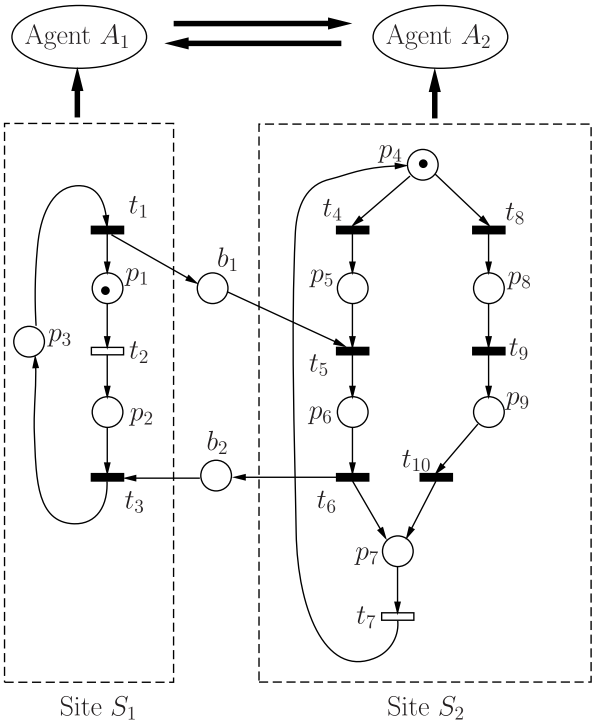

Figure 5.2: Global model for Example 5.1

Example 5.1. *The model in Fig. 5.2 represents a DSSP system. There are two sites and two agents monitoring each sites. The sites S_1 and S_2 are connected with buffers b_1 and b_2. The only observable transition in S_1 is t_2, while in S_2 is t_7. The observation is given in Tab. 5.1. We assume that an agent sends information to its neighbor agents immediately after it computes the estimation. Let us discuss the state estimation of A_2 according to its observation and to the information received from A_1.*

Table 5.1: The observation in Example 5.1

t	t_2	t_7	t_2, t_7
τ	3	6	11

- *At time 3, agent A_1 observes the firing of t_2. At this moment, the information regarding b_1 and b_2 are: "from time 0 to 3, no token has been produced in b_1" and "from time 0 to 3, no token has been consumed from b_2", respectively. These information are sent to A_2.*
- *At time 6, t_7 is observed by A_2. From the initial marking, the possible firing sequences are $\sigma_1 = t_4t_5t_6t_7$ and $\sigma_2 = t_8t_9t_{10}t_7$. From time 0 to 6, σ_1 consumes 1 token from b_1 and*

produces 1 token in b_2, while σ_2 does not consume or produce any token to buffers. The information from A_1 says that no token is produced in b_1 from time 0 to 3, but there is no information of b_1 from time 3 to 6. Therefore, σ_1 may be consistent with the observation and with the information received from A_1. Obviously, σ_2 is consistent with the received information.

- *At time 11, A_1 observes t_2 and A_2 observes t_7. The agent A_1 computes the information of b_1 and b_2, which are "from time 0 to 11, one token has been produced in b_1" and "from time 0 to 11, one token has been consumed from b_2", respectively. The possible firing sequences of S_2 are $\sigma_5 = \sigma_1\sigma_1$, $\sigma_6 = \sigma_1\sigma_2$, $\sigma_7 = \sigma_2\sigma_1$ and $\sigma_8 = \sigma_2\sigma_2$. With the information from A_1, A_2 concludes that: 1) only one token has been produced in b_1 from time 0 to 11, and then the firing sequences which consume more than one tokens from b_1 are not possible; 2) one token has been consumed from b_2 from time 0 to 11, and then the firing sequences which do not produce more than one tokens in b_2 should be eliminated. Because σ_5 consumes two tokens from b_1 and σ_8 does not produce any token in b_2, so they are eliminated from possible firing sequences. The possible firing sequences at time 11 in S_2 are σ_6 and σ_7.*

Therefore, the agents should use the information of buffers to eliminate inconsistent firing sequences. Inconsistent can come from the following situations: (1) if the information says i tokens are produced in a buffer, then firing sequences which consume more than i tokens are inconsistent; (2) if in the information, i tokens are consumed from a buffer, then firing sequences which produce less than i tokens are inconsistent sequences.

When a system starts to evolve, each agent performs local estimation and computes information of buffers. Because the information includes time information, which is "from time τ_1 to τ_2, i tokens are produced to (or consumed from) a buffer", so the problem that it should be considered whether there exists a global clock in the system or not. In the affirmative case, the time instants are interchanged between agents, as the one in Example 5.1. Otherwise, in the situation that each site has a local clock, the time instants are not included in information, while when an agent receives an information, it computes time instants for the information as following: (1) the information is "from last commmunication until this moment, i tokens are produced into (or consumed from) a buffer", (2) assume last and present communication are at τ_1 and τ_2, (3) the information in the receiver agent is "from τ_1 (last communication) until τ_2 (current communication), i tokens are produced into (or consumed from) a buffer". All these consideration will be considered when the state estimation will be developed for distributed systems.

Chapter 6

Conclusions and Future Work

In this work, we provide an online algorithm for state estimation of timed choice-free PNs, and we give some ideas on how the procedure is adapted to a paticular class of distributed systems. First, an algorithm to compute the set of consistent markings is given, and then the time information are grouped into a set of time equations that is used to reduce the set of consistent markings. Some reduction rules are presented that can be used also to reduce the state space of the timed systems merging the indistinguishable transitions. Second, we discuss the general case, i.e., nets with choices, and we show that the procedure is similar with the standard one of untimed Petri nets. Finally, the adaption of the approach to distributed systems is illustrated with an example. Communication is introduced into state estimation by agents in distributed systems. As a future work, we plan to continue working on state estimaton of distributed systems and to implement the algorithms in MATLAB.

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