30605 - Mathematics II

Información del Plan Docente

Academic Year	2016/17
Academic center	109 - Facultad de Economía y Empresa
Degree	432 - Joint Law - Business Administration and Management Programme
ECTS	6.0
Course	1
Period	Second semester
Subject Type	Basic Education
Module	---

1.Basic info

1.1.Recommendations to take this course

The students should have a good command of all the contents of the subject Mathematics I, taught during the first semester of the first year. They must, in any case, know the meaning and implications of the differentiability of a function and be skilled in the calculus of partial derivatives. The students also have to know how to determine the sign of a quadratic form. They must also be able to present and support an argument with a logical sequence and to connect various mathematical aspects previously learnt.

1.2.Activities and key dates for the course

1. Presentation of the subject in the first session of the semester, in accordance with the timetable established by the Faculty.
2. Continual attendance at, and productive use of, theoretical and practical classes.
3. Attendance at practical classes P6, which may include computer practice if the global schedule allows it.
4. Midterm exams, scheduled in accordance with the academic calendar.
5. Final exam, on the day established by the Faculty.

2.Initiation

2.1.Learning outcomes that define the subject

The student, in order to pass the course, will have to show her/his competence in the following skills:

1. To have gained good skills in using mathematical language, both in comprehension and writing.
2. To be able to identify the fundamental elements of an optimisation problem: variables, objective function and constraints.
3. To be able to formulate static optimisation problems: unconstrained, and with equality and/or inequality constraints.
4. To know how to solve an optimisation problem by the graphical method, when that is possible.
5. To be able to evaluate whether or not a mathematical programme meets the conditions that allow it to be solved by the techniques learnt.
6. To be able to distinguish between critical points and extrema (optima).
7. To be able to discriminate between local and global optima.
8. To be able to distinguish between necessary conditions and sufficient conditions for local optimality.
9. To be able to calculate the critical points by solving the system of equations obtained by applying the first-order conditions for local optimality, both for unconstrained cases and for problems with equality constraints.
10. To know how to classify the obtained critical points by using the second-order conditions, both for unconstrained optimisation programmes and for problems with equality constraints.
11. To be able to apply the conditions which guarantee that an optimum is global.

12. To be able to interpret economically the Lagrange multipliers obtained in an optimisation problem with equality constraints.
13. To be able to evaluate whether a mathematical programme is linear. If it is, they must know how to solve it by the graphical method (when that is possible) and by the simplex algorithm.
14. When varying a parameter of a linear optimisation programme, the students must be able to analyse how the solution changes, without solving the new problem.
15. To be able to use some computer programmes to find the solution to an optimisation problem and to be able to interpret the results obtained.
16. To be able to identify a dynamic process in an economic scenario and be able to represent this process (when possible) by an ordinary differential equation.
17. To understand the concept of the solution of an ordinary differential equation and to be able to distinguish between general solution and particular solution.
18. To be able to discriminate between a first-order differential equation and a linear differential equation of order n.
19. To be able to identify whether a first-order differential equation is with separable variables, homogeneous, exact, or of linear type, and to know how to solve the equation by the appropriate method.
20. For a linear differential equation with constant coefficients, they must be able to write the complementary (homogeneous) equation and obtain its general solution.
21. To be able to find a particular solution of a linear differential equation with constant coefficients.
22. To have the know-how to calculate the general solution of a linear differential equation with constant coefficients.
23. To be able to work out the solution of a linear differential equation of order n with constant coefficients, given n initial conditions.

2.2.Introduction

Brief presentation of the course

Mathematics II is a basic-training subject with a value of 6 ECTS credits and it is taught during the second semester of the first year. It is based on and complements Mathematics I, a subject of the first semester of the first year.

The subject Mathematics II consists of two different parts: Mathematical Programming and Dynamical Analysis, which, respectively, apply to two different points of view of economic reality. After learning the first part, the students will be able to formulate and solve a wide variety of classical optimisation problems: both linear and non-linear, whether unconstrained or with equality and/or inequality constraints. In the case of optimisation programmes where both the objective function and the constraints are linear, the solving technique used is the simplex algorithm. This topic may be used to connect the traditionally-taught solving methods with the use of computer software, which simplifies the calculations and introduces students to professional practice.

The second part, Dynamical Analysis, is concerned with solving differential equations and with the analysis of the solutions. Its inclusion in the syllabus is necessary because many of the processes that Economic Analysis deals with are non-static. Some examples of these dynamic processes are: optimal economic growth, optimal management of renewable and non-renewable resources, optimal long-term investment, etc.

3.Context and competences

3.1.Goals

The expected results of the course respond to the following general aims

The general objectives of the mathematical subjects in this Degree are included in the following two main goals: (1) Mathematical education, (2) Training to apply Mathematics to the challenges that the students will encounter in their careers.

The subject Mathematics II supposes a step forward in these objectives which Mathematics I also dealt with. Education in Mathematics is significant not only because of the transmission of new concepts, but also because the students gain a rigorous and accurate perspective, as well as the capacity for abstraction and the scientific method that characterise Mathematics. Regarding the second goal, this subject introduces students to modelling, using the mathematical analysis

approach through two different ways: classical optimisation, and dynamical analysis.

3.2.Context and meaning of the subject in the degree

As Mathematics is a tool and a support for other subjects that are essential in the education of the students (Microeconomics, Macroeconomics, Econometrics, etc.), Mathematics II continues the line of work of Mathematics I by bringing Mathematics closer to problems in economic scenarios, which will undoubtedly facilitate a deeper comprehension of and, as a consequence, better skills in applying Mathematics.

After passing the mathematical subjects in the Degree, the students will have worked towards attaining one of the most important goals of mathematical theory: to formulate models that explain the real world. Prospective graduates will be able to use the language of science and to understand the role played by Mathematics in the development of their thinking skills, given that the students' logical reasoning, accuracy, rigour, capacity for abstraction and skills in interpreting results will be improved. This is why the subjects of Mathematics are indispensable tools which allow the designing of appropriate models that are used for researching, describing, understanding and thinking about the realities of companies

3.3.Competences

After completing the course, the student will be competent in the following skills:

Problem solving

Analysis and synthesis

Decision-making.

Applying knowledge to practice

3.4.Importance of learning outcomes

They permit the comprehension of theoretical concepts and models that are part of the contents of other related subjects studied in the Degree. Mathematics is most important in this goal because it facilitates the analysis and discussion of the models and concepts studied. In this regard, it is worth mentioning that Optimisation techniques allow the laying of the foundations of the two basic paradigms of Microeconomics, namely, the theory of consumer choice and the production theory. The concepts of convex set and concave/convex function, whose economic interpretations are, respectively, the diversity in consumption and the law of diminishing marginal returns, have important applications. Linear Programming is very useful in production planning problems and it allows the solving of some simple exercises of comparative statics. Different techniques are required for the analysis of dynamic processes in continuous time, which is essential, for example, in models of economic growth. The theory of differential equations provides the necessary tools to deal with some key concepts such as trajectory over time, evolution of the system, stability, etc.

4.Evaluation

The student will prove that he/she has achieved the expected learning results by means of the following assessment tasks:

The evaluation will be global in both the first and second sittings. It will consist of a final exam to be taken on the dates determined by the Faculty. The global exam will be written and will assess the proposed learning outcomes through questions that are theoretical, practical, or of a mixed theoretic-practical character and that will be based on the topics taught. It will be worth 10 points.

In addition, in the first sitting, it will be possible to take a voluntary intermediate test. This test will assess the student's knowledge about the topics of the subject that will be indicated by the teacher in advance. The date of the test will be announced in advance in class and/or in the virtual teaching platform.

The students who obtain a mark of at least 40% of the maximum in this test will be able to eliminate the corresponding topics from the global exam at the first sitting. In this case, the mark corresponding to the eliminated topics will be added to the mark of the global exam in a weighted fashion. This weight will range between 35% to 50% of the global value (10 points). In order to pass the subject, students have to obtain at least 5 points out of 10.

It has to be taken into account that the evaluation process closes at the end of the academic year, so it is not possible to claim academic merits from one academic year in a later one.

Evaluation Criteria

Students will be assessed on whether they have acquired the learning outcomes mentioned above. In particular, they will be assessed on the following aspects:

1. Correct mathematical writing.

2. Logical reasoning in the posing and solving of the problems.

3. Reference to the theoretical results used, when relevant.

4. The choice of the most appropriate method for the solving of problems.

5. Clarity in the application of mathematical concepts and procedures.

6. Computations carried out with care.

7. The correct expression of the results obtained when solving problems.

Students taking their exams at their fifth or sixth opportunity will be marked following the rules established under the Governing Council Agreement on 22 December 2010, which sets out the assessment regulations in the University of Zaragoza.

5.Activities and resources

5.1.General methodological presentation

The learning process that has been designed for this course is based on the following activities:

The objective of this subject is that the students should develop the analytical skills, rigour and intuition needed for using mathematical concepts and results and that they should be able to apply these abilities to the analysis of problems of an economic nature. Therefore, the teaching should aim to provide students with a solid mathematical knowledge and to train them in a way of reasoning that will allow them thereafter to successfully solve a wide variety of questions in an economic scenario.

5.2.Learning activities

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The programme offered to the students to help them achieve the learning results includes the following activities :

Theoretical lessons which will be based on lectures to present the concepts and results corresponding to the contents. At the same time, some exercises will be solved with the participation of the students to help them comprehend the theoretical concepts presented. These classes are face-to-face and will be given to the full group.

Time allotted: 1.2 ECTS credits (30 hours).

Practical lessons, in which the students will apply the theoretical results in order to solve, with the teacher's help, more complete exercises, and problems of an economic nature. Problem sheets will be available for the students and the teacher will announce in advance the problems that will be solved in each practical lesson so that the students can prepare them beforehand. These classes are face-to-face and will be given separately to each subgroup.

Time allotted: 1.2 ECTS credits (30 hours each subgroup).

Seminars (practical classes P6), which may consist of a number of different activities designed to support the learning process, including: follow-up of some simple projects that had been assigned to small teams of students and the presentation of these projects; answering questions that students may have regarding some of the contents taught; solving problems of an economic nature by using some of the mathematical tools taught during the classes, etc. These seminars may also be devoted to the teaching of more advanced topics, intended for the students interested in learning some further mathematical tools that would allow them to deal with more general problems. In this way, the students are shown thatboth Mathematics and Economics are vibrant sciences with many facets to be studied.

Time allotted: 3 hours each subgroup

Out of class work: 3.6 ECTS credits.

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Unit 2: Unconstrained programming

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