

27004 - Numbers and Sets

Información del Plan Docente

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| Academic Year | 2016/17 |
| Academic center | 100 - Facultad de Ciencias |
| Degree | 453 - Degree in Mathematics |
| ECTS | 6.0 |
| Course | 1 |
| Period | First semester |
| Subject Type | Basic Education |
| Module | --- |

1.Basic info

1.1.Recommendations to take this course

1.2.Activities and key dates for the course

2.Initiation

2.1.Learning outcomes that define the subject

2.2.Introduction

3.Context and competences

3.1.Goals

3.2.Context and meaning of the subject in the degree

3.3.Competences

3.4.Importance of learning outcomes

4.Evaluation

5.Activities and resources

5.1.General methodological presentation

The general teaching methodology designed for this class is based in the following:

1. Lectures
2. Participatory sessions addressing issues and problems
3. Office hours
4. Students' individual work

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5.2.Learning activities

Support for training through documents and links on the page of the subject in college ADD, moodle.unizar.es (restricted to students registered with the PIN and password provided by the University Access)

For the on-line course "Information Management" is expected:

training session of 50 minutes, to explain to students the objectives and mechanics of operation of the virtual course in Moodle

participatory sessions addressing issues, and troubleshooting methods

5.3.Program

Part I: ELEMENTARY NUMBER THEORY

Chapter 1: INTRODUCTION- Polygonal Numbers. Pythagorean Triangles. Fibonacci, Lucas and Pell Numbers. Continued Fractions and Pell's Equation. Exercises.

Chapter 2: ARITHMETIC OF INTEGERS AND THE THEORY OF CONGRUENCES - DIOPHANTINE EQUATIONS.

1. The Division Algorithm . The Greatest Common Divisor, the Euclidean Algorithm and the Bezout Identity.

2. Congruences. The ring $\mathbb{Z}/n\mathbb{Z}$. On Sums of two and three squares . Difference of Two Squares. The group $U(\mathbb{Z}/n\mathbb{Z})$.

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3. Arithmetical Properties of Fibonacci and Pell Numbers
4. The Diophantine Equation $aX + bY = c$ and the Equation $aX \equiv b \pmod{n}$.
5. The Chinese Remainder Theorem.
6. Fermat's Little Theorem- Wilson's Theorem and the equation $X^2 \equiv -1 \pmod{p}$, p prime.
7. The Fundamental Theorem of Arithmetic- Existence of infinitely many primes of the form $4n + 1$ - Existence of infinitely many primes of the form $4n + 3$
- The Functions τ , σ , and ϕ ;
8. Perfect Numbers and Mersenne Primes
9. The Diophantine Equation $X^2 + Y^2 = Z^2$
10. The Diophantine Equation $X^2 - dY^2 = (\pm)1$ where $d-1$ is a perfect square.

PART I':

Chapter 1': SETS AND MAPS

1. Operations: Union, Intersection, Difference, Cartesian product- The Power Set of a Set.
2. Maps. injective, surjective and bijective maps, composition of maps- left inverse, right inverse and the two-sided inverse.
3. Symmetric Group. Action of a Group on a Set.
4. Infinite Sets: On Cardinal Numbers, case of the sets N , Z , Q , R , R^2 , a set and its power set.
5. Relations. Relations of Equivalence and Partitions-The Quotient Set. Cardinal numbers again. Order Relations. The well Ordering Principle.
6. Mathematical Induction.

Chapter 2': COMPLEX NUMBERS

1. Complex Numbers-Argand's Diagram. Modulus and Argument of a Complex Number. Complex Conjugate. Exponential Representation. The field of Complex Numbers. Products and powers.
2. Polynomials over the field of complex numbers. Roots.
3. Roots of complex numbers. Geometric representation. The Group of the n -th roots of unity.
4. The Primitive n -th roots of unity. Cyclotomic Polynomials

5.4. Planning and scheduling

See the academic calendar of the Universidad de Zaragoza and the class schedules published on the School of Sciences (Facultad de Ciencias) webpage.

5.5. Bibliography and recommended resources

- Burton, D.M. "Elementary Number Theory". Allyn and Bacon, 1998
- Hungerford, T.W. "Algebra" Springer-Verlag 1974
- Nicolaidis, A. "Complex Numbers" (Success in Pure Mathematics), Private Academic & Scientific Studies Ltd; 2007
- Stein, W: Elementary Number Theory, Springer-Verlag 2008, <http://wstein.org/ent/>
- Stewart, I.; Tall, D. "The Foundations of Mathematics". Oxford Univ. Press, 1977
- Shockley, J.E. "Introduction to Number Theory". Holt, Rinehart and Winston, 1967

<http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Pythag/pythag.html#twofractgenall>

<http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibrefs.html>

<http://primes.utm.edu/mersenne/index.html#c>

<http://www.mersenne.org/primes/>

<http://www.numbertheory.org/ntw/N4.html#fibonacci>

<https://calendar.google.com/calendar/embed?src=4fmd2ed0p25vqvk7skquh4hbmo@group.calendar.google.com&ctz=Europe/M>

<https://calendar.google.com/calendar/embed?src=4fmd2ed0p25vqvk7skquh4hbmo@group.calendar.google.com&ctz=Europe/M>