

Annex I. Theoretical behaviour of electrical conductivity

Calculus of the different behaviours of the theoretical models of different associations had been performed in order to obtain information about the behaviour of our films.

Capacitor is an element that store energy in form of electrostatic field. This storage is quantified by the capacitance (C), measured in Farads. The impedance of a capacitor can be calculated using the following form:

$$X_c = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$$

Series association of a resistance and a capacitor can be calculated adding the impedances of the two components:

$$Z = R - j\frac{1}{\omega C} \quad \text{---} \text{C} \text{---} \text{Rs} \text{---}$$

Figure A.1. Equation and representation of series association (RC).

Parallel association of a resistance and a capacitor can be calculated using the formula for the parallel association and following the next process:

$$Z = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_C}}$$

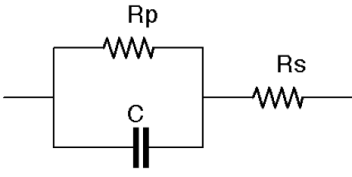
$$Z = \frac{1}{\frac{1}{R} + j\omega C} = \frac{R}{1 + j\omega CR}$$

The last equation can be normalized and reduced in order to separate the real part from the imaginary part of the complex impedance, obtaining the following equation for the parallel association:

$$Z = \frac{R}{1 + \omega^2 C^2 R^2} - j\frac{\omega CR^2}{1 + \omega^2 C^2 R^2} \quad \text{---} \begin{array}{c} \text{Rp} \\ \text{---} \\ \text{C} \end{array} \text{---}$$

Figure A.2. Equation and representation of parallel association (RC)

Finally, a mixture association between the parallel and the series can be calculated, adding a resistance in series to the parallel association.



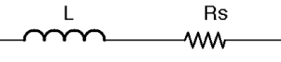
$$Z = R_s + \frac{R_p}{1 + \omega^2 C^2 R_p^2} - j \frac{\omega C R_p^2}{1 + \omega^2 C^2 R_p^2}$$

Figure A.3. Equation and representation of mixed association (RC).

Inductor is an element that store energy in form of magnetic field. This storage is quantified by the inductance (L), measured in Henry. The impedance of a capacitor can be calculated using the following form:

$$Z_L = j\omega L$$

Series association of a resistance and an inductor can be calculated adding the impedances of the two components:



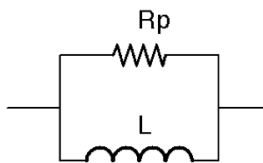
$$Z_L = j\omega L$$

Figure A.4. Equation and representation of series association (RL).

Parallel association of a resistance and an inductor can be calculated using the formula for the parallel association and following the next process:

$$Z = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_L}} = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L}} = \frac{j\omega RL}{R + j\omega L}$$

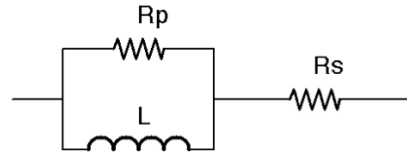
The last equation can be normalized and reduced in order to separate the real part from the imaginary part of the complex impedance; obtaining the following equation for the parallel association:



$$Z = \frac{\omega^2 RL^2}{R^2 + \omega^2 L^2} + j \frac{\omega R^2 L}{R^2 + \omega^2 L^2}$$

Figure A.5. Equation and representation of parallel association (RL).

Finally, a mixture association between the parallel and the series can be calculated, adding a resistance in series to the parallel association.



$$Z = R_s + \frac{\omega^2 R_p L^2}{R_p^2 + \omega^2 L^2} + j \frac{\omega R_p^2 L}{R_p^2 + \omega^2 L^2}$$

Figure A.6. Equation and representation of mixed association (RL).

Nyquist plot of this theoretical behaviour have been represented in order to know about the different relation between the real and the imaginary impedance depending in the type of association. For this purpose, the frequency was varied from 40 Hz to 8 GHz.

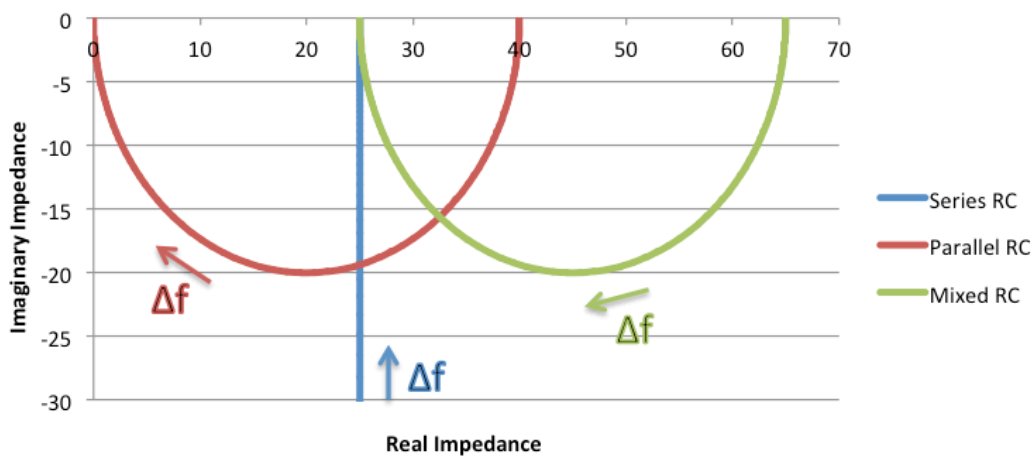


Figure A.7. Nyquist plot of Resistance-Capacitance associations (RC).

Series association corresponds to a straight vertical line because the real part only depends on the fixed resistance, and the imaginary part depends on the frequency. For the parallel association, a semicircle shape was obtained and finally, for the mixed association, the semicircle is obtained but in this case the series resistance shifts it. These shapes are the same for both the RC and the RL associations; but the sign of the imaginary impedance is changed. This fact is due Capacitor and Inductor do the same effect but each in a direction.

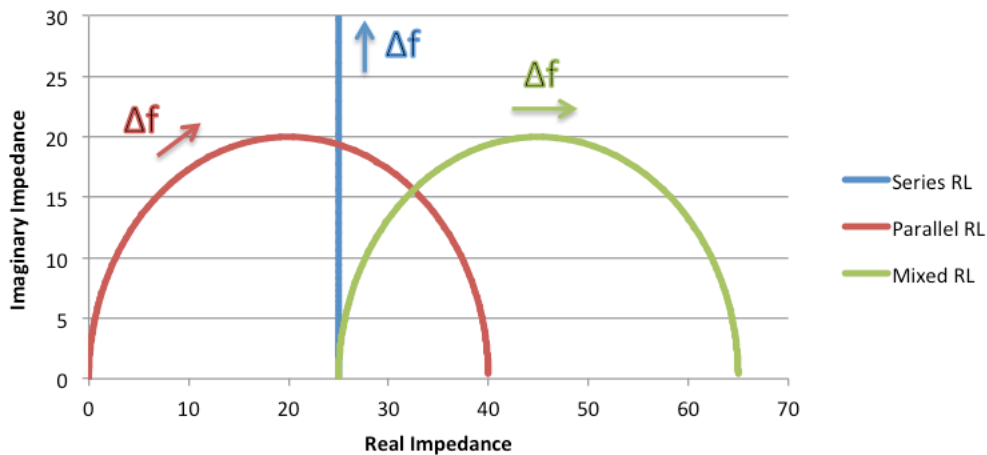


Figure A.8. Nyquist plot of Resistance-Inductance association (RL).

References

In addition, the direction of the frequency changes in the two different circuits (RC and RL). For the circuit RC the real impedance decreases when the frequency is increased. However, in the circuit RL, the real impedance increases when the frequency is increased.

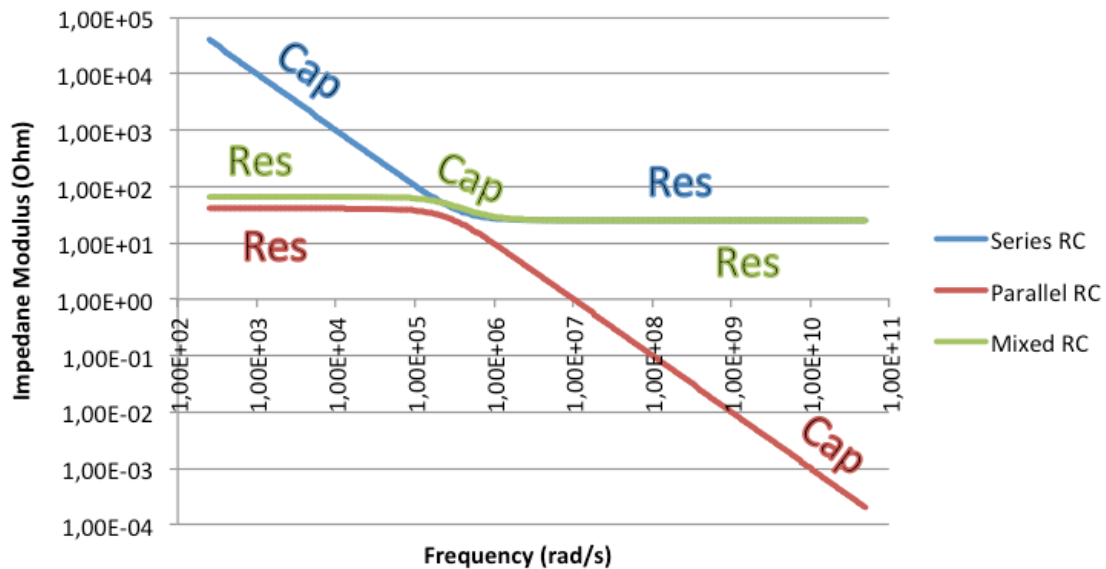


Figure A.9. Bode plot of Resistance-Capacitance association (RC).

The bode plot represent the Impedance modulus versus the frequency, with this plot we obtain information about the behaviour of the material at different frequencies. In the case of RC circuit, the horizontal lines correspond to resistive behaviour, and negative inclination lines correspond to capacitive behaviour. For series association, capacitive behaviour at a low frequencies and resistance behaviour at high frequencies. For the case of parallel, the behaviour is backwards. And finally for the mixed association, mostly resistive behaviour predominates but there are a zone where capacitive behaviour.

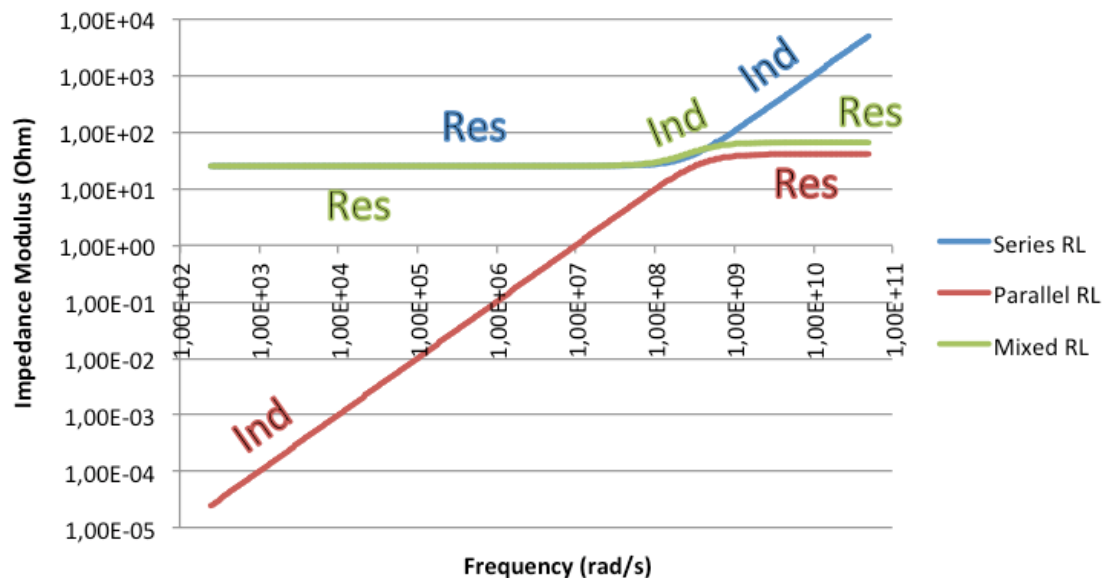


Figure A.10. Bode plot of Resistance-Inductance association (RL).

In the case of RL circuit, the horizontal lines continue to represent the resistive behaviour, but in this plot, positive inclination lines appear, that represent the inductive behaviour. In addition, the behaviours are inversed from the RC circuit. In the mixed association, resistive behaviour continues to predominate except in a zone, where inductive behaviour appears.