Movies and TV series fragments in mathematics: Epistemic suitability of instructional designs

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Abstract

There are plenty of books, journals and online sites devoted to the relationship between mathematics and cinema, and its educational applications, whose interest is to explore the pertinence of the instruction processes that can be designed around this resource. Instead of watching a full production, mathematics teachers that include movies and TV series in their classroom sessions, usually show short fragments, so the first step should be to consider these fragments alone to identify the mathematical objects and the involved meanings and representations. For this purpose, we use some theoretical notions from the Onto-Semiotic Approach to research in mathematics education, applying them to three excerpts from a movie and to some typical tasks designed based on them. The analysis of the involved mathematical content allows to reflect about the epistemic suitability of the instruction process, in terms of how aligned they are with the institutional meanings. The results show that it is possible to achieve a high suitability level but, most importantly, that this kind of analysis promotes teacher reflection to design teaching and learning processes.

Introduction

Audio-visual fiction can be considered both as an object of research and as a teaching and learning resource. Several authors (Ballesta & Guardiola, 2001; Spanhel, 2011) point out that the use of this resource in the classroom is still far away from being a common practice in many disciplines, for a range of reasons including a lack of instruction, because the existing infrastructure is inadequate, or because it would require a drastic change in the teaching style. However, audio-visual fiction permeates every moment of our lives and it is so rich in representations and symbolic meanings that experts have suggested educational applications be based on it (Quero, 2003).

Videogames could also be considered as audio-visual fiction, but they are beyond the scope of our research and, thus, the term audio-visual media in this paper will denote movies and TV series. Moreover, watching a full movie in a mathematics classroom might not be efficient in terms of time, so the most common approach is to use selected fragments from movies and TV series.

Audio-visual language in educational contexts gathers some traits that make it special, for instance, being able to promote visualization and motivation. There are not a lot of academic descriptions or analyses of experiences which integrate audio-visual media in mathematics teaching and learning processes, and it is even stranger to find research work devoted to the use of movies and TV series.

However, there are a lot of resources, such as online databases and servers (Micolich, 2008), books or even teacher training seminars and courses, that facilitate the use of movies and series as an efficient resource.
When a teacher thinks about designing a classroom activity using some fiction fragments, a reflective process takes place, just as when considering any other resource. In our case, this reflection implies that the fragments are not randomly chosen, but determined by the desired didactical suitability level, understood as grade of adequacy (Godino, 2013).

Now, within this abundance and availability of movies and TV series fragments, we must distinguish two types. On the one hand, there will be fragments with a poor mathematical content, because they contain just a single mathematical object or a few unconnected objects. On the other hand, some fragments will be especially rich in that sense. However, a 'poor' movie excerpt could also lead to a perfectly optimal teaching and learning process, since didactical suitability relies on multiple factors, regarding duration, availability or emotional impact. In this study, we focus on analysing the mathematical content, determining that a high epistemic suitability can be achieved.

Problem, theoretical framework and methodology

Our research is motivated by the following main question: Is it pertinent, from the epistemic perspective, to use selected short fragments taken from movies and TV series as a didactical resource in mathematics education? In other words, herein lies the interest to explore the adequacy of the kind of study processes built upon the use of these fragments, with the emerging mathematical knowledge in mind. However, having in mind how specific this resource is, one previous question arises: are there enough fragments, taken from movies and TV series, with a clear relation to mathematics, to motivate a research line on their mathematics education applications?

The *Mathematical Movie Database* (https://www.qedcat.com/moviemath) is a webpage devoted to the compilation of mathematical references in movies and TV series. On April 2017, the number of entries was beyond 1200. Polster and Ross (2012), creators of that page, compiled a book featuring some films with a special connection with mathematics, such as *Good Will Hunting*, *A Beautiful Mind*, *Stand and Deliver*, *Pi*, *Die Hard with a Vengeance* and *The Mirror Has Two Faces*. Sorando (2004, 2014) maintains also an online site about how mathematics appears in real-world contexts (http://matematicasentumundo.es) with a special section devoted to mathematics and cinema, including 307 references (April 2017). His articles and books also analyse some movies in depth and add extra references to the list, in the same way as other authors (Población, 2006).

Some authors have already put into practice this resource, suggesting how it can be used to achieve whatever the learning objectives are (Martín & Martín, 2009; Raga, Muedra, & Requena, 2009; Reinhold, 1997; Sorando, 2004, 2014). A result of the interest aroused by this availability of information is the appearance of teacher instruction courses, as it is possible to find fragments with mathematical references for almost any point of the curriculum.

Theory of Didactical Suitability

Oliver (2013, p. 31) noted that 'current accounts of technology provide poor explanations of how technology use leads to - or fails to lead to - learning'. When it comes to the teaching and learning of specific content, such as mathematical objects, specific theoretical frameworks are required to answer the research questions. The aim of the Theory of Didactical Suitability is to synthesize and organize a system of components and quality criteria for mathematical instruction processes (Breda, Pino-Fan, & Font, 2017; Godino, 2013; Godino, Batanero, Font, Contreras, & Wilhelmi, 2016). These criteria are classified into six facets, already depicted in previous works within the Onto-Semiotic Approach to Research in Mathematics Education (OSA) (Godino, Batanero, & Font, 2007) that allow analysis of these processes and to promote teacher reflection: epistemic (mathematical institutional meanings), ecological (socio-professional and curricular context), cognitive (personal meanings), affective (emotional factors), interactional (personal interactions) and mediational (didactical resources).

The first step in the design of an instruction process is to determine what is suitable from the epistemic or institutional point of view (Godino, 2013), because the aim of such process is to develop learning about some specific content. According to OSA's interpretation of these two suitability facets (Breda, Pino-Fan, & Font, 2017; Godino, Arteaga, & Rivas, 2014), we assume that epistemic suitability refers to the institutional contents. It will achieve a high level if the mathematical content is representative of the curricular content and its inclusion in the teaching and learning process is justified. On the other side, cognitive suitability refers to the adequacy of the process to the student's actual knowledge. A high correlation between what the students eventually learn, and what the learning objectives are, is an indicator of it.

As it has been previously mentioned, this work focuses on the epistemic facet, although we infer some conclusions for the cognitive facet. The traditional categorization of mathematics education objects into concepts, procedures and attitudes, which is mostly used in the official curricular guidelines, responds to a dichotomous view of mathematical activity, which reflects neither other important aspects nor interactions among them. Therefore, the OSA framework ontology extends the variety of objects by considering situation-problems, languages (semiotic registers and representations), concepts (as definitions), procedures, propositions (statements about concepts) and arguments (reasoning statements to validate propositions). These objects compose a configuration, that depends on the meaning which emerges from a specific system of mathematical practices (Godino, et al., 2007; Godino, Font, Wilhelmi, & Lurduy, 2011). This categorization, together with the Theory of Didactical Suitability, is the theoretical framework which allows us to answer our research questions. Moreover, Godino (2013) suggests a series of indicators or criteria of didactical suitability for every process facets, which we will refer to.

Procedure and method

This work shows part of the results of the research done by Beltrán-Pellicer (2015) where 18 fragments from movies and TV series were selected to design and implement mathematical teaching and learning processes, covering certain aspects within each curricular block. A didactical suitability analysis was made for each one of these fragments and their related teaching and learning processes. The description of the configurations lies in this kind of analysis, allowing reflection on the suitability of the expected emerging mathematical knowledge. A selection of these fragments was implemented with two student groups of secondary education to collect additional data to develop a new insight into the affective suitability, something which is out of the scope of this paper.

We illustrate the procedure by describing in detail the corresponding configurations of three selected fragments from a movie. Based on these fragments, the authors designed a teaching and learning process to address the emerging mathematical objects (both explicit and implicit) allowing use of them in the classroom. Such objects belong to the algebra curricular block and the process was designed for 13-15 years-old secondary education students.

Epistemic analysis of an instructional design

We begin the procedure by considering the isolated fragments, without the corresponding instructional design. Then, an onto-semiotic analysis to reveal the configuration (emerging mathematical objects and how they interconnect) of each one of the fragments is carried out. Any instructional design based on it will just expand this configuration, maybe including some additional objects, strongly related to the mathematical content of the fragment.

This section illustrates this kind of analysis over a sequence of three short fragments (1-2 minutes each one) from the movie *October Sky* (Johnston, 1999). The mathematical content of the narrative is closely related to quadratic equations, as depicted in the situation-problem. We decided to design tasks from the algebraic language perspective and problem-solving strategies, favouring the functional approach (Bednarz, Kieran, & Lee, 1996) for 13/14-year-old students.

Situation-problem

The situation which is outlined in the movie could be classified as a research problem in applied mathematics. The main characters are four high school students who are trying to build a real rocket for a science contest. The three movie fragments tell the part where they are accused of causing a fire, since they were not able to recover one of their rockets, and they prove their innocence by means of a mathematical demonstration.

After the fire disaster, Homer, the student who leads the rocket-designing team, leaves high school to work in the mine and earn some money to help his family. However, Homer begins to study 'advanced' mathematics on his own, thanks to a book that his teacher gave to him. However, he feels that he needs some help, so he asks a friend to assist him with the calculations and solve the problem of finding the lost rocket. All they must do is to find the rocket which presumably caused the disaster. This way, one of them realizes that if they knew the height reached by their rockets at that time and how tilt was the trajectory, they could narrow the search radius. Later, as they don't manage to find it, they take the usual wind direction in that location as an additional variable, finding the rocket in the estimated placement. Then, Homer realizes that he does not want to follow his father's steps and leaves the mine to continue studying at high school, so he decides that he should come back to his classroom and explain to the principal why it was impossible for him and his friends to cause the fire:

> *HOMER: At the time of the fire, the best that we could do was x miles...which is exactly where we found that rocket. See, that rocket fell for about x seconds, which means... that it flew to an altitude of 3000 feet...according to the equation..."S" equals one-half "a-t" squared... where "S" is the altitude, "a" is the gravity constant, or... and "t" is the time it took for that rocket to come back down. Velocity equals acceleration...*
>
> *QUENTIN: Get him, Homer.*
>
> *HOMER: ... times time. Are you following this, Mr. Turner?*

Five tasks were designed around these fragments, composing the situation-problem of interest. Although the teaching methodology is not relevant for our analysis, these tasks were thought to be worked in small groups of 4-5 students. After they discuss the questions within the small group and annotate their conclusions in their notebooks, there is a whole group discussion where every group explains the arguments or how they have reached their solution. At the end of each task, the teacher negotiates the mathematical meanings, in what is called institutionalization.

Task 1 of the instructional process is designed to reinforce student confidence when using algebraic language, as we can assume it is not the first time for them:

> *Task 1. Homer uses the expression $S = 1/2\ at^2$ to support his explanation to prove that their rocket did not started the fire.*
> *a) What do the letters S, a and t stand for in this equation?*
> *b) Are these letters appropriate? Would you had chosen other letters?*

Not only the formula S =1/2 at^2 is explicitly said by Homer during the fragments: he also explains the meaning of each letter (variable), by connecting each one to a physical magnitude. This way, 'S' stands for the height reached by the rocket, 'a' is the gravity constant and 't' is the flying time. Although it is not expected that the students could interpret meaningfully the gravity law classic model, it might be necessary a brief introduction by the teacher. The students might notice that it is not important which letter is used to represent a magnitude, just its meaning. Thus, the second question of this task proposes the students use other letters to represent the variables.

Along the movie, every measurement is expressed in USA customary units and they are also rounded. The teacher should consider that some students might find an obstacle at this point in Task 2:

Task 2. Homer concludes that their rocket fell for 14 seconds. What altitude did it reach?
Note: If using an audio dubbed version (i.e., for Spanish audiences), this task can be extended: Does this altitude agree with what Homer says? And with what it is written at the blackboard?

On the one hand, their calculations will not agree with Homer's, as he is rounding every number. The obstacle that may arise from this fact is epistemic, regarding the mathematical content, and it is related to the student's beliefs about mathematics as a knowledge field. How precise should the result be? Is it enough with just the first digit? On the other hand, audio dubbed versions of this movie convert the USA customary units (3000 feet) into metric system units (900 meters), also rounding, adding another change of semiotic register, as what Homer says and what it is written on the blackboard do not agree.

Once the students have gained insight into what the expression for rocket altitude means, Task 3 introduces some related procedures, like polynomial evaluation:

Task 3. Imagine that just before the scientific contest, Homer and his friends launch some rockets, modifying the fuel quantity and other design parameters. They fill a table to collect the data for each one of the launches (altitude and flying time), but the day before the best project award, they lose the sheet of paper and they do not remember the data. Help them.

Launch number	*Altitude (feet)*	*Flying time (s)*
1		*8*
2		*9*
3		*11*
4	*3622,5*	

The last row requires some algebraic manipulation, and it might fall outside the zone of proximal development for most students. However, this task can be solved by working in heterogeneous groups and with little guidance by the teacher, who may remind them of equivalent equations.

The method used by the students to fill the last row of the table in Task 3 is not relevant. Some groups might have used a linear model to guess the blank value. This is an attitude which should be encouraged by the teacher, to promote discussion and negotiate personal meanings. Task 4 prepares the path to institutionalize the standard procedure to find the algebraic expression for t, related to incomplete second grade equations:

Task 4. Could you write down a formula to calculate the flying time when given the altitude reached by a rocket?

Task 5 relates second grade expressions to first grade equations. A fragment from *The Simpsons* (Season 17, *Girls Just Want to Have Sums*) can be projected to promote discussion, as it shows a teacher asking the students for the solution of a simple second grade equation:

> *TEACHER: Everyone take out your math books, come on. [At the blackboard: YxY=25] Now, how many different numbers can "Y" be?*
>
> *LISA: That's easy, just one, the number five.*
>
> *TEACHER: Wrong.*
>
> *MARTIN: There are two possible solutions; five and negative five.*
>
> *LISA: Oh, my god, I was wrong! And by being corrected, I learned! And no one cared about my feelings!*
>
> *Task 5. What is the difference between this formula and first grade equations?*

The solution of each one of these tasks requires operative and discursive mathematical practices from which mathematical objects and meanings emerge. These entities define a configuration, where audio-visual language plays an important role. The three selected fragments allow the design of mathematical tasks, which can be viewed as sub-configurations that are contextualized thanks to the narrative thread. It is interesting to note that this is a classic context in applied mathematics: ballistic trajectories. As described, the suggested tasks for these fragments do not add any other mathematical object, and they just provide new insight into the tasks performed by the film's characters.

Languages

The selected fragments show a wide range of semiotic register and representations. Apart from showing the rocket launches in a very realistic way, the following linguistic registers appear:

- Verbal register: the main characters talk about the steps followed to solve the situation. In other words, there is a narrative support to the problem-solving procedure. Besides, one of them explains (speaking, using the blackboard) some of the equations and calculations.
- Graphical register: when Homer explains how he solved the problem and why it was impossible for them to provoke the fire at that time, he uses the classroom blackboard to draw the quadratic function representing the height over a Cartesian plane.
- Symbolic register: Homer complements his explanation at the blackboard by writing some of the equations. The equation $S = 1/2\ at^2$ can be observed.
- Numeric register: Homer writes down the height reached by the rocket at the blackboard, using the imperial metric system (3000 ft.) and the flight time (14 s).

Concepts (definitions-rules)

In the OSA framework, the term concept is used to denote definitions as rules which are introduced explicitly. Thus, in that sense there are not any concept here. However, we could say that some mathematical abstractions can be traced here: polynomials, quadratic equations and algebraic expressions evaluation. The definitions do not appear in the movie, so if they were not institutionalized in previous lessons yet, the teacher can find a time through the tasks to introduce the standard notation and vocabulary and to explain the referred concepts by such notation.

Procedures

Although the complete procedures are not introduced along the fragments, it is clear from the different linguistic objects, that once the main character has found the algebraic expression for the rocket trajectory, he proceeds to evaluate $S = 1/2\ at^2$ for $t=14$.

Propositions
Propositions are statements about concepts. While polynomials, quadratic equations or algebraic expressions are not explicitly defined in the movie fragments, there are implicit propositions, some of them with a physical perspective. For instance: 'Projectiles (in this case, the rockets) travel with a parabolic trajectory.' 'The required time to reach the maximum height equals the required time to descend.'

Arguments
Arguments are statements that validate propositions and explain why some procedures might work, like deductive or inductive ones. The selected fragments offer a narrative thread whose purpose is to elaborate a valid argument to show that the students were not guilty of causing the fire. Homer, the main character, builds this argument on mathematical calculations. This way, he finds the expression for a projectile trajectory and evaluate it to estimate the height reached by their rockets. Besides, the argument is demonstrated empirically, because they manage to find the lost rocket.
However, the mathematical arguments needed to support the narrative outlined by the fragments are not shown in the movie. They appear just as results which are articulated in the higher-level argument. Therefore, they must be elaborated by the students when tackling the tasks.

Discussion: epistemic suitability

Godino (2013, p. 119) suggests indicators for each configuration component. This system of indicators serves as a guide to support reflective teaching practice. Therefore, the main objective here is not to assess the teaching and learning process with a number. On the contrary, it is enough to handle three basic achievement levels in order to promote teacher reflection: low, medium and high. Regarding the situation-problem to be considered, it will affect positively the epistemic suitability when it includes a representative and articulated sample of contextualization, exercising, application and problem-generation situations.

When it comes to languages, the movie fragments themselves provide an assorted set of registers and representations. The mathematical object configurations from the previous section shows that the verbal, graphic (parabolic drawing) and symbolic registers are embroidered within the narrative. It is interesting to notice how they contribute to the story, because they allow it to advance by showing translations from one register to another. The epitome comes when Homer interrelates all the registers in his final argument, suggesting that he has been able to solve the problem by learning how these registers contribute to the meaning of the interwoven mathematical objects. Moreover, Homer's verbal explanations are appropriate for the students, as they are of a similar age to the characters. The designed activities bring together the already introduced mathematical objects and define mathematical expression and interpretation situations. Therefore, according to Godino (2013, p. 119) all these linguistic aspects add value to the epistemic suitability of the study process.

Concepts, procedures and propositions do not appear clearly in the fragments, mainly because the movie was produced without educational purposes. However, procedures and propositions are suggested and partially shown, since the main character solves a problem and operates with the formulas. It is obvious that there are neither explicit nor implicit concepts (understood as definitions), and the teacher should plan some time to institutionalize them. As some of the proposed activities ask the students to elaborate their own propositions, in the same way as Homer does, the instructional design achieves a high epistemic suitability here. Moreover, Homer's explanations facilitate student's argumentation, as they serve as example.

All these mathematical objects (problems, concepts, propositions, etc.) are not isolated, as they interrelate thanks to the movie narrative. According to Godino (2013, p. 119), this fact is an indicator for high epistemic suitability, because it allows recognition and articulates the different meanings of the mathematical objects which emerge from the practices. In this regard, the processes carried out by the students in the mathematical object interplay are essential. There is a decontextualization process when the physical problem (finding the rocket) is translated to each one of the linguistic registers which support mathematical argumentation. Moreover, after the calculations, the number which is obtained as the result must be interpreted, so there is also a contextualization process.

The configuration of mathematical objects within the movie fragments is extended by the designed instructional sequence. This design is not the only one possible, but a suggestion inspired by a literature review with a minimal introduction of new mathematical meanings. The contextualization and decontextualization processes demonstrate how important the situation-problems within OSA framework are, as they provide a place from which the rest of the mathematical objects emerge. This conception of mathematics education is aligned with other theoretical frameworks, such as the Theory of Didactic Situations (Brousseau, 1997) and Realistic Mathematics Education (Van den Heuvel-Panhuizen, & Drijvers, 2014), based on Freudenthal's didactical phenomenology (1983; 1991). Furthermore, problem-solving is not only a matter of interest in mathematics education, but also an essential way of doing mathematics, and it plays a central role in curriculum guidelines (NCTM, 2000).

The different design facets are closely interrelated in the Theory of Didactical Suitability, making no sense to consider one of them alone. For example, the epistemic analysis allows one to carry out an a priori cognitive analysis and predict some of the obstacles which may arise when implementing the instructional process. This is not only feasible, but a required step to promote significant reflection afterwards. The cognitive suitability expresses how the expected or implemented meanings are in the student's Zone of Proximal Development (ZPD) (Vygotsky, 1980) and the personal meanings proximity to the expected meanings.

As the instructional process is addressed to 13/14-year-old students, we can presume that they may have learnt some notions about equations and algebraic expressions. It could be possible that previously they had spent a lot of time working on the idea of equivalent equations, by means of algebraic scales or similar techniques. They may also have been introduced to graphical representations and function interpretation, so they may know that a first-degree equation can be represented by a straight line. Therefore, the teacher should detect any difficulty with this concepts and procedures, adapting the activities.

We have shown that the movie fragments include a wide range of linguistic registers, and this is an indicator of epistemic suitability. Semiotic functions between these registers convert one representation into another, and they can be a source where some obstacles arise. The term obstacle is not used here as a synonym for difficulty. Instead, obstacles are opportunities for the students to put into play their personal knowledge and confront it with the situation, in such a way that new knowledge will be acquired. One of these obstacles takes place when Homer expresses the calculated numbers for the altitude, verbally and writes them on the blackboard, as he is rounding the result, saying '3000 feet' and writing '3000'. When the students replicate Homer's calculations with the formula, they may notice the rounding procedure that has been carried out by the movie character.

Conclusions

The research done by Beltrán-Pellicer (2015) carried on this kind of analysis over a representative sample of 18 movie fragments, covering aspects from the different secondary education main topics, was analysed with the same methodology introduced in this article. The resulting configurations from these fragments reveal a huge variety of mathematical objects articulation, which leads to epistemic suitability evaluations for the fragments alone from low to high. However, it is obvious that the suitability depends not only on the fragments themselves, but also on the instructional designs around them.

Apart from this observation, according to the Theory of Didactical Suitability, the design should cover every one of the six components. Therefore, sometimes a low epistemic suitability might be justified if it is balanced by high levels on the other aspects. For example, if a fragment presents a low mathematical objects representation, but takes just five minutes of class time (including the activity), it may achieve a high resource suitability, as it consumes very little time and thus it uses efficiently the available resources. Besides, high cognitive and ecological suitability are easy to achieve if the mathematical meanings are aligned with the student's meanings and curriculum.

Then, although the use of any movie fragment in the classroom could be justified in the previous terms, by showing in this paper a full analysis of an instructional design based on this resource, we have revealed that it is feasible to find fragments with a rich network of mathematical objects, facilitating a natural design of related tasks with high epistemic suitability. The epistemic and cognitive analysis is a necessary condition for the introduction of any resource. However, the inherent class complexity, full of personal experiences will affect the instruction process (Medina, 2010), and other aspects as well, such as resource availability and ecological and interactional issues. Beltrán-Pellicer and Asti (2014) describe a classroom experience where these kind of videos -indeed, a passive media- are used to promote group discussions, something which is shared by other researchers (Lee & Sharma, 2008). That experience involved the use of a blended learning environment, a teaching methodology that has shown its benefits (Hill, Sharma & Xu, 2017) where short videos find easily a place.

A common fact, related to the mathematical content involved in these fragments, is that they do not include concepts (in the sense of definitions or rules) in an explicit way (Beltrán-Pellicer, 2015; Beltrán-Pellicer & Asti, 2014). The reason is that we are analysing fiction movies or TV series which have not been produced with an educational objective in mind. Thus, the teacher (or the instructional process designer) should consider the configurations and extend them, planning institutionalization moments. A regular exposure to instructional designs around movie fragments, plenty of discussions about them, and problem-solving activities, might result in media literacy improvement, promoting critical thinking attitudes (Feuerstein, 1999). Today, Internet and ICTs provide a medium for the distribution of audio-visual content, which includes popular TV series and films aimed at young audiences. The work that we have presented is oriented towards the 'good' use of a resource that belongs to the home and leisure scope, retaining an educational relevance and value (Selwyn, Potter, & Cranmer, 2009).

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