27017 - Galois Theory

Información del Plan Docente

Academic Year	2017/18
Faculty / School	100 - Facultad de Ciencias
Degree	453 - Degree in Mathematics
ECTS	6.0
Year	3
Semester	First semester
Subject Type	Compulsory
Module	---

1.General information

1.1.Introduction

The problema of finding explicit expressions for the solutions of polynomial equations in terms of its coefficients in a way similar to the one used in basic algebra with the second degree equation attracted much effort from some of the best mathematicians during several centuries. After its solution for equations of degrees 3 and 4, the impossibility of a general solution started to be considered as a possibility. The solution of this problema gave rise in the hands of Evariste Galois to the theory that bears his name.

The solution of that problem was achieved by Galois through the study of some permutations of the roots of the polynomial, thus inaugurating the theory of groups, and the important idea of studying mathematical objects through its group of symmetries.

The aim of the curse is introducing the student to both the theory of (finite) groups, and the Galois' theory of fields.

1.2.Recommendations to take this course

This course assumes an interactive approach in its structure and in its presentation, which requires engaged participation from all members of the class. The student's presence is essential to the liveliness of this course and concomitantly to their individual success in it. Therefore, regular attendance is expected.
Students should work on the exercises and problems sheets regularly, should study on a continuous basis and should make use of the office hours (their schedule will be communicated at the beginning of the course).

1.3.Context and importance of this course in the degree

This course presents a key tool in any mathematical area: the Theory of Groups, which is the tool to measure and take advantage of the symmetries that may appear in any system. It is therefore a basic course.

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1.4.Activities and key dates

The exams will be given following the official dates established by the School of Sciences at the beginning of the academic year.
Students should follow regularly both the theorical classes and the classes devoted to exercises and problems.

2.Learning goals

2.1.Learning goals

- Become familiar with group concepts.
- Be able to use Group Theory to take advantage of symmetry.
- Learn about classical problems, like the unsolvability of the quintic by radical.

2.2.Importance of learning goals

Numbers and symmetry constitute two of the basic mathematical concepts. Both are blended in this course.

3.Aims of the course and competences

3.1.Aims of the course

Introduce the students to the basic aspects of Group Theory, which deals with symmetry, as well as Galois Theory, which uses Group Theory to study field extensions and algebraic equations.

3.2.Competences

Being succesful in this course should mean that the student is competent to

- Reason in an abstract way.
- Recognize the symmetries of a given situation and is able to use Group Theory to study it.
- Know about some of the classical mathematical problems, like the unsolvability of the quintic by radicals.
- Be able to write and communicate abstract concepts of Mathematics.
- Be able to learn by oneself, and to look for information through different media.

4.Assessment (1st and 2nd call)

4.1.Assessment tasks (description of tasks, marking system and assessment criteria)

20% of the final grade will be obtained by means of a continuous evaluation throughout the course. This will include solving exercises sheets and share the information with the classmates.
There will be a final exam which will amount for the remaining 80% of the final grade.
The student has the right to base his/her final graded on just a global exam.

5.Methodology, learning tasks, syllabus and resources

5.1.Methodological overview

The course will contain theoretical classes that will include examples and proposed exercises. The interaction between students and the teacher will be encouraged, so as to increase the students' capacity for abstract reasoning, and to improve their level of mathematical expression. In addition, some problems to be solved either individually or in small groups will be proposed to the students, notwithstanding the fact that exams will be individual. Students will be attended

by the teacher at the office hours.

5.2.Learning tasks

During the lectures, the teacher will provide explanations on the subjects covered by the notes that will be available at the ADD. In addition, the solutions of the proposed problems, previously considered by the students either in their individual or collective work outside the class hours, will be discussed.

5.3.Syllabus

- Groups: basic notions

- Groups of permutations

- Actions of groups

- Structure of finite groups

- Field extensions. Algebraic extensions

- Splitting extensions. Extensions of homomorphisms

- Normal extensions. The Galois group

- The Galois Theorem

- Solving equations by radicals

5.4.Course planning and calendar

Four weekly lecture hours. Theoretical and practical parts will not be separated in advance. Some problems will be proposed to be solved in groups by the students which will have the opportunity of asking doubts during the office hours of attention to students. Those exercises will be similar to the ones that will be asked in the exams.

Office hours: they will be announced at the beginning of the course. After request, the students have also the option of meeting the teacher at times different from the ones scheduled to that purpose.
Please contact the teacher through e-mail for any further request or question.

5.5.Bibliography and recommended resources

Galois Theory textbooks

1.- J. Gaal, Classical Galois Theory. Chelsea Publishing Company, New York,

1971.

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2.- P. M. J. McCarthy, Algebraic extensions of fields, Dover Publ. Inc., New

York, 1991

3.- G. Navarro, Un curso de álgebra. Publicaciones de la Universidad de Valencia, 2002

4.- P. Ribemboim, L'Arithmétique des corps, Hermann, Paris, 1972.

5.- J. Rotman, Galois Theory, Springer Verlag, New York, 1990.

6.- I. Stewart, Galois theory, Chapman and Hall, London, 1973.

General algebra textbooks

1.- N. Bourbaki, Elements of Mathematics, Algebra II. Springer Verlag, New

York, 1990.

1.- P. M. Cohn, Algebra, vols. 1 and 2, J. Wiley & sons, Chichester, 1989.

2.- T. W. Hungerford, Algebra Springer-Verlag, New York, 1974.

1.- N. Jacobson, Basic algebra, vols. I and II, Freeman and Co., San Francisco,

1985.

Additional reading

1.- E. Artin, Galois theory, University of Notre Dame Press, London, 1985.

2.- E. Galois, Oeuvres Mathématiques. Publiées en 1846 dans le Journal de

Liouville, Editions Jacques Gabay, Paris, 1989.