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Highlights

- We analyze price-advertising competition with horizontally differentiated products when firms can use mass advertising, opt-in direct advertising or direct advertising without permission.
- Compared to opt-in advertising, the use of direct advertising without permission results in lower or equal prices and higher or equal profits for firms.
- A fraction of consumers refuse the offer to receive opt-in advertising and we prove that this fraction is, from a social perspective, too large.
- A regulatory policy banning the use of direct advertising without permission in favor of opt-in advertising lowers social welfare and, with high product-differentiation, consumer surplus.

Direct Advertising and Opt-in provisions: policy and market implications

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Abstract

This paper formulates a game of pricing and informative advertising with horizontally-differentiated products in which two firms, first, compete with mass advertising and, later, build a database using their historical sales records and compete by targeting the ads to their potential customers. We study market interaction under two types of direct advertising: opt-in advertising, where firms ask consumers for their consent to send them ads with information about new products, and direct advertising without permission, where sellers use consumer contact information without their explicit consent. We show that, compared to the case where firms only use mass media, the use of direct ads (with or without permission) results in an intertemporal reallocation of market power from the first to the second period and that, compared to opt-in advertising, direct advertising without permission results in lower or equal prices. We also evaluate the impact of a regulatory policy aimed at protecting consumer privacy by banning the use of direct advertising without permission in favor of opt-in advertising. We find that this policy lowers social welfare and, if the degree of product differentiation is sufficiently high (vs. low), it does not affect (vs. lowers) firm profits and lowers (vs. increases) consumer surplus.

Keywords: informative advertising, opt-in advertising, nuisance costs, privacy.

JEL Classification: L13; L59; M37

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1 Introduction

The emergence of new information-transmission technologies has led to fundamental changes in firms' communication strategies. According to the marketing literature, sellers today tend to use *mass* advertising media to capture new clients, but use personal *direct* media when developing a relationship with existing customers. Effective direct advertising begins with a consumer database that includes an organized collection of comprehensive data about existing or prospective customers. On the basis of this information, marketers can target small groups of potential clients and promote their new products through personalized communication, thus providing firms a low-cost, efficient alternative for reaching their markets. This explains why database advertising has become increasingly important and is now extensively used by sellers.[1]

The most important forms of direct advertising include telemarketing and direct mail, which together account for 52% of all sales engendered by direct marketing (Kotler *et al.*, 2013). These information transmission technologies use a telephone call, fax, SMS, postal mail[2] or email to obtain an immediate response from customers and cultivate lasting relationships. Taking into account the old marketing adage of "it is easier to sell something to an existing customer than make a new one", marketers often achieve these goals simply by building a database with the contact details of their *existing* customers and using the database to send them direct advertising with information about new products or services. Against this background, it is interesting to investigate how the strategic use of direct advertising based on historical sales records can affect the functioning of the markets. Further, we are particularly interested in the policy aspects of database advertising. It is clear that the informative role of advertising and the high cost-efficiency of direct advertising make this marketing tool socially desirable. However, a fraction of consumers receiving, for example, telephone calls, may not be interested in the promoted products and consider such advertising intrusive, thus generating "nuisance costs". These costs can have a strong negative impact on social welfare, so the protection of consumers' privacy can justify government intervention. One regulatory solution can be to introduce an opt-in provision in the use of direct advertising, so that firms must ask consumers for their explicit consent to be included in the database and to send them messages, in return for valuable information about new products or services. From a social perspective, the advantage of opt-in advertising is that the audience of the campaign becomes

[1]According to the US Direct Marketing Association (DMA), in 2011, marketers spent $163 billion on direct marketing, which accounted for 52.1% of all ad expenditures in the United States (see DMA's "Power of Direct Marketing Report").

[2]The DMA reports that *offline* marketing channels, mainly direct–mail (non-catalog) and telephone marketing, account for the bulk of advertising dollars in 2016. However, DMA expects digital channels to continue increasing their share of the marketing budget in the future.

highly qualified, thus mitigating the impact of nuisance costs. The aim of this paper is to provide a theoretical framework in which to investigate how the use of direct advertising, with and without opt-in provisions, can affect consumers and firms, to shed light on how public policies towards database advertising (e.g. to allow direct advertising without permission, or to ban this type of advertising with an opt-in provision) can affect the level of social welfare.

We formulate a model of price competition in which two firms sell horizontally-differentiated products. To accommodate direct advertising based on historical sales records into the model, we consider that sellers launch a succession of new products over time. Consumers are unaware of the existence and characteristics of the goods and firms use informative advertising to promote sales. For the sake of simplicity, we reduce the analysis to three stages, $\tau = 0, 1, 2$. In the first, sellers do not have the information necessary to target their ads, so they compete on price and reach consumers by using a TV or radio mass advertising campaign that covers the entire market and provides information about the existence of a new product, price, characteristics, etc. In $\tau = 1$, the informed consumers make their purchasing decisions, i.e. they decide whether or not to purchase a product and from which firm. Finally, in $\tau = 2$, firms launch a new product and compete by choosing their pricing and advertising strategies (mass advertising or direct advertising).

A model of database advertising based on historical sales records only makes sense if the firms' intertemporal demands are correlated. We assume that the different goods produced by the two firms over time are oriented to the same group of potential consumers so, in order to generate repeat business, a firm has a high incentive to foster a relationship with its potential "regular" customers, that is to say, with those consumers who purchased its product in $\tau = 1$. This is done by taking advantage of the first sale to obtain a profile of the clients (name, address, telephone number, email, etc.) and create a database that in $\tau = 2$ can be used to send direct ads with information about new products or services.[3] The distinctive feature of our model is that marketers can implement a direct advertising campaign in two different ways. One, they can send direct messages to all their past clients, without asking for their permission. We will refer to this advertising strategy as "*direct advertising without permission*". The other way is to ask consumers for their consent to include their personal information in the database and send messages only to those clients who have given their explicit consent. We will refer to this advertising strategy as "*opt-in direct advertising*". We assume that only a fraction of first-period buyers are interested in the second-period products and

[3] In-house lists with information about existing customers have been used by firms for a long time. For example, Kotler and Armstrong (1998) report that General Electric has constructed databases containing the purchasing record for electro-domestic goods of each customer. On the basis of this information, the firm has used direct mail to offer them a new electro-domestic or other item that can be used in combination with other products they have recently purchased.

that consumers suffer a "nuisance cost" when they receive direct messages (e.g. telephone calls) informing them about the existence and characteristics of products in which they are not interested. We can interpret this as a "privacy cost", which reflects the opportunity cost of the time spent in receiving and processing the information transmitted by the commercial messages, and we assume that consumers have heterogeneous privacy costs. Clearly, the effectiveness of direct advertising without permission and opt-in advertising will be different. Whereas, in the first case, all first-period clients receive an ad, so all are exposed to the advertising campaign, with opt-in advertising, only a fraction of these consumers (with relatively low privacy cost) will give permission to be included in the database and only they will be reached by the corresponding advertising campaign. This explains that firms have a clear incentive to use direct advertising *without permission*, and the problem is that this type of advertising can generate high privacy costs, because consumers who are not interested in second-period products and have relatively high privacy costs receive undesired direct ads. Thus, consumer protection can justify government intervention aimed at reducing privacy costs by imposing an opt-in provision on the use of direct advertising. The issue is how does this provision affect consumers, firms, and the level of social welfare. To address these questions, the first phase of our work compares the functioning of the market under three competition scenarios: (i) sellers use only mass advertising: this case, which equals the full-information outcome, constitutes a reasonable benchmark against which we can compute the impact of database advertising on the market outcome, (ii) regulation (opt-in provision): in $\tau = 2$, sellers can use only mass advertising or opt-in direct advertising and (iii) no regulation: in $\tau = 2$, sellers can use any type of advertising (mass advertising or direct advertising with or without permission).

When firms employ direct advertising, the key point is that the use, in $\tau = 2$, of an in-house list with contact information about existing clients (with or without their permission) allows sellers to target their advertising campaigns to a distinct set of consumers, which fragments the market into local monopolies due to the information-differentiation that arises. Firms strategically anticipate this effect so, compared to the benchmark, they compete more aggressively for consumers in the initial period, that is, we prove that database advertising results in an intertemporal transfer of market power from $\tau = 0$ to $\tau = 2$. Interestingly, we find that firms could find it optimal to offer their first-period products at zero price. This can be interpreted as an aggressive introductory offer aimed at building a large customer base in $\tau = 1$, which allows sellers to better market their products in $\tau = 2$. Regarding profits, if first-period prices are positive, direct advertising (with or without permission) yields lower firm profits than does mass advertising, whereas, if first-period prices are zero, profits can increase. This means that firms may engage in a typical prisoner's dilemma: both sellers are better off using only mass advertising but, in $\tau = 2$, they have a strong incentive to use their databases, indirectly generating more intense price competition in $\tau = 0$ and, as a result, lower intertemporal aggregate profits. Finally, we find that the pricing strategy and the level of profits can vary depending on whether firms use opt-in advertising or direct advertising without permission. We show that second-period prices coincide with the two types of direct advertising, while opt-in advertising yields higher (or equal) first-period prices and lower (or equal) aggregate firm profits.

The second phase is to evaluate a regulatory policy aimed at protecting consumer privacy by imposing an opt-in provision on the use of direct advertising. It is clear that opt-in advertising reduces consumer privacy costs, thus increasing welfare, but, under this type of advertising, a fraction of consumers refuse the offer to be included in the database and we prove that, compared to the socially-optimal solution, this fraction is too large. This generates a negative externality on firms and a quantity distortion in the product market, with the corresponding welfare loss. The key point, then, is which of these effects is dominant, and we find that a regulatory policy banning the use of direct advertising without permission and generating opt-in advertising results in a welfare loss. To understand this, we note that, given the relatively high cost of a mass advertising campaign, it is socially desirable that firms employ database advertising. However, if the use of direct advertising without permission is prohibited, then, having mass advertising available, firms will find it optimal to use opt-in advertising *only* if it is relatively efficient, i.e. if the campaign has a wide reach, which requires low privacy costs. In this set-up, we show that the gain in social welfare induced by the savings in privacy costs is smaller than the welfare loss generated by the quantity distortion induced by opt-in advertising, so government intervention is socially detrimental. Regarding firms and consumers, we find that, if the degree of product differentiation in the market is relatively large, a regulatory policy banning the use of direct advertising without permission and resulting in opt-in advertising does not affect firms' profits and lowers consumer surplus, whereas, under low differentiation, the regulation lowers profits and increases consumer surplus. In sum, our welfare analysis yields a remarkable result, namely, that a regulatory policy aimed at protecting consumers could end up leaving firm profits unchanged but lowering both consumer surplus and social welfare.

These results have interesting implications for the evaluation of the distinct regulatory approaches towards direct advertising implemented by the European Union and the US. The EU is clearly concerned with the defense of consumer privacy in commercial relationships, which is protected by (i) a common European regulation, (ii) national agencies that supervise and prevent the incorrect use of commercial information, and (iii) a severe penalty mechanism that punishes the incorrect and abusive use of private data. By contrast, the US maintains a permissive use of commercial information and so, for example, there is neither a general regulation nor a central agency of data protection. Our model suggests that the US regulatory approach, close to direct advertising without permission, may well improve market performance. Nevertheless, it is clear that, beyond the gains in efficiency illustrated by our model, the abusive and massive use of private information may be considered by many consumers to be overly intrusive, thus generating substantial losses in social welfare, which can justify the European regulatory approach towards direct advertising, more inclined towards opt-in advertising. While the computation of this "intrusive effect" is beyond the scope of the current paper, our work strongly suggests that, in order to find an "optimal" regulatory policy towards direct advertising, it is important to carefully take into account the potential negative efficiency effects induced by a restrictive approach to the use of private information in commercial relationships.

Our work relates to recent economics literature on informative advertising and targeting, which has focused mainly on how specialized advertising media, such as magazines, newspapers and cable TV, can be used to target potential consumers, as well as on the bearing of these technologies on market outcomes. Some contributions in this area are those of Esteban, Gil and Hernández (2001), Iyer *et al.* (2005), Galeotti and Moraga (2008) and Esteban and Hernández (2011, 2016). However, the analysis of direct advertising in a competitive setting[4] has received little attention and, to the best of our knowledge, only the works of Shaffer and Zhang (1995), Roy (2000) and Esteban and Hernández (2014) have addressed this issue. The first of these analyzes the case in which firms have a fixed-size database containing precise information on consumers which allows them to both locate and classify these customers according to their brand loyalty. Given this information, the authors show that direct advertising allows for price discrimination by way of coupons, which stimulates competition in the market. Roy (2000) places the analysis within a spatial framework in which firms can send their ads directly, given that they know the physical location of all consumers. In this context, the use of direct advertising can lead to market fragmentation and the creation of local monopolies. We note that the results provided by Shaffer and Zhang (1995) depend on the information contained in the database and the origin of this information is exogenous. While Roy's approach provides an answer to the question of how to locate potential consumers, this model[5] applies only to a spatial context in which consumers have homogeneous tastes. Esteban and Hernández (2014) study database advertising based on historical sales records in a context of heterogeneous tastes, and investigate how direct advertising affects both pricing and advertising

[4] Esteban, Gil and Hernández (2004) analyze the use of database advertising in a monopoly setting.

[5] We note that Roy (2000) formulates a two-stage model, where two firms first decide the advertising strategy and then compete in prices (the ads do not mention prices). The timing of this game suggests that advertising has a long-run nature, perhaps to create brand loyalty, and that consumers learn prices later, without cost.

levels when firms face a totally inelastic demand. The present work puts the accent on the analysis of pricing strategies when firms face an elastic demand. Further, our main contribution to the existing literature is to analyze the functioning of the market under opt-in direct advertising, that is, we study database advertising when consumers choose whether or not to be included in a firm's database. On this basis, we provide a theoretical framework in which to evaluate the effects of a regulatory policy towards database advertising.

Finally, our work also connects to the literature on ad-blocking. Johnson (2013) analyzes how the increasing ability of firms to target their ads to particular individuals affects the market outcome when consumers have access to advertising avoidance tools. The paper formulates a model where firms only compete for the attention of consumers, but they do not compete in the product market, and shows that improved targeting leads to higher firm profits, and that consumers may under-utilize advertising-avoidance technologies. Anderson (2010) formulates a two-sided model in which a monopolist provider supplies content to consumers and sells advertising space to firms, and analyzes the content provider's reaction to the use of ad-avoidance technologies (such as remote controls, pop-up adblockers, etc.). The paper shows that the adoption of ad-avoidance technologies may reduce total welfare and content quality. Our model provides a different framework in which to evaluate the effects of consumer ad-blocking on the market outcome, namely, price and direct-advertising competition in the product market, and finds that (i) improved targeting can lead to lower firm profits, (ii) consumers may over-utilize advertising-avoidance tools, and (iii) the possibility of ad-blocking may reduce total welfare.

The remainder of the work is organized as follows. Section 2 sets out the model and describes the functioning of the market under mass advertising and direct advertising. Section 3 discusses the welfare and policy implications of database advertising. Finally, Section 4 contains some concluding remarks. All the proofs are relegated to an Appendix.

2 The model: equilibrium analysis

We consider two firms, $j = 1, 2$, launching one new product in $\tau = 0$ and another in $\tau = 2$, and competing in prices, (p_1^τ, p_2^τ) for a group of consumers. Potential buyers are uninformed of the existence and characteristics of the goods and firms use advertising to promote sales. To model the interaction between uninformed consumers and firms selling differentiated products and competing in prices and advertising, we use a simple version of the familiar model of Tirole (1988). We assume that the firms are located at the extremes of a linear potential market, of unit length, along which there is a mass of uniformly-distributed consumers who demand, at most, one unit of the good per period. We interpret the linear market, $[0,1]$, as the space of a given characteristic and we assume that at each point of the market, $1 \geq x \geq 0$, there is a density N of consumers. Let v^{τ} be the consumers' common valuation of their favorite variety in period τ. For the sake of simplicity, we consider that $v^0 = v^2 = v$. If firm 1 is located at the left end of the market, a consumer located at a distance x from this seller achieves a utility $U = v - tx - p_1$ from buying firm 1's product, where tx represents a linear transportation cost, i.e. the cost in terms of utility of not having the favorite good available.[6] The marginal cost of production is constant and, for the sake of simplicity, we normalize this cost to zero.

Regarding the information structure of the model, we assume that consumers are endowed with preferences over product attributes but, without advertising, they are unaware of the existence of the goods or their characteristics.[7] Advertising provides information about the existence of a new product and its characteristics, including the price, so a consumer can (1) learn the product attributes, (2) evaluate the degree of preference for the good and (3) decide whether to buy it or not. In $\tau = 0$, firms reach consumers by using the mass-advertising media (TV, radio, press, etc.), which cover the entire market at a cost A per consumer. The fundamental feature of our model is that, in $\tau = 0$, a firm anticipates that, in $\tau = 2$, it will again launch a new product, and marketers may have an incentive to develop and maintain a direct relationship with their potential buyers. A model of database advertising based on historical sales records only makes sense if a firm's demands in $\tau = 0$ and $\tau = 2$ are correlated, so we assume that both demands stem from the same mass of consumers located within the interval $[0,1]$. Further, we can expect that the ordering of consumers with respect to the valuation they place on the goods in $\tau = 0$ and $\tau = 2$ do not necessarily have to coincide, so we consider that the location of consumers along the line in $\tau = 2$ is independent of their locations[8] in $\tau = 0$. We also assume that, in the first sale, a firm can capture its clients' contact information (name, address, telephone number, etc.) and build a database which, in $\tau = 2$, can be used to send them a direct ad with information about the existence, price, and characteristics of the new product. Accordingly, we assume that firms have three marketing tools available which can serve to materialize the demand in $\tau = 2$. First, a seller could launch a new *mass advertising*

[6] We are also assuming a similar pattern of product differentiation in $\tau = 0$ and $\tau = 2$, but the model can easily be extended to consider that consumer valuation and/or the pattern of product differentiation changes over time.

[7] We assume that consumers do not search for information about products and firms must periodically launch an advertising campaign to facilitate consideration of its product by consumers. With this assumption, we are in line with the mainstream of the informative advertising literature, which considers that consumers' search cost is high relative to the expected surplus offered by the goods so, in the absence of information, consumers do not purchase any good (see, for example, Grossman and Shapiro, 1984; Stegeman, 1991; Stahl, 1994). Later, in footnote 14, we will specify the conditions under which this assumption holds.

[8] The model can easily be extended to consider that consumers' locations in $\tau = 0$ and $\tau = 2$ are also correlated.

campaign informing all potential consumers of the existence of the new product. The second way to reach consumers in $\tau = 2$ is to ask consumers for their explicit consent to be included in the database, and send a direct message only to those who explicitly agree to receive ads (*opt-in direct advertising*). The last alternative is to use *direct advertising without permission*, that is, the firm send a direct ad to all the clients in the database.

We consider that only a fraction $\lambda < 1$ of first-period buyers are interested in the second-period products and that consumers suffer a "nuisance cost" when they receive direct advertising (e.g. telephone calls) about products in which they are not interested.[9] This loss of utility corresponds to the opportunity cost of the time spent in receiving and processing the information transmitted by the commercial message, and we interpret it as a "privacy cost". We assume that the privacy cost is heterogeneous across consumers. More precisely, we consider that, at each point x on the line $[0, 1]$, the N consumers have privacy costs distributed in the support $[0, \alpha]$. Concretely, at point x, the privacy cost for the N consumers is given by the function $C = \alpha\ y$, where the index $y \in [0, 1]$ measures the degree of disutility of receiving advertising without being interested in the product. Regarding the effectiveness of direct advertising, under opt-in advertising, only a fraction of consumers (with relatively low values of y) located at each point x will be reached by the campaign, whereas, under direct advertising without permission, all the first-period clients receive ads, so all are exposed to the direct advertising campaign.

The timing of the game is as follows. In $\tau = 0$, both firms launch a mass advertising campaign and simultaneously compete in prices, (p_1^0, p_2^0). In $\tau = 1$, consumers make their purchasing decisions and, under opt-in advertising, each seller asks its clients for their permission to use their contact information for commercial purposes. In $\tau = 2$, firms simultaneously choose their advertising strategy, mass or direct advertising,[10] and compete in prices, (p_1^2, p_2^2). Next, we look for subgame-perfect Nash equilibria (SPNE) of the game and, for the sake of simplicity, we focus the analysis on *symmetric pure-strategy equilibria.*

In order to understand how the switch from mass to database advertising can affect the functioning of the market, we begin by studying the benchmark case of mass advertising, i.e. when both firms use only the mass media in $\tau = 0$ and $\tau = 2$. Given that a mass advertising campaign reaches all potential buyers, in this full information situation, the model does not have intertemporal effects, so sellers face the same competitive scenario in $\tau = 0$ and $\tau = 2$, thus yielding the same equilibrium prices in both periods. To obtain the equilibrium, we note that a consumer located at a distance

[9] It is clear that mass advertising can also impose nuisance costs, but our model implicitly assumes that nuisance costs are higher when firms use database advertising and we normalize the nuisance costs of mass advertising to zero.

[10] Notice that, given that mass advertising covers the entire market, it is never optimal to use mass and database advertising simultaneously.

$1 \geq x \geq 0$ from firm 1 achieves a utility $U = v - tx - p_1^\tau$ from buying firm 1's product and a utility $U = v - t(1-x) - p_2^\tau$ from buying firm 2's product. We focus the analysis on the market scenario where, in equilibrium, both firms actively compete for consumers (i.e. the market is covered) and obtain positive profits.[11] In this set-up, consumers located at $\overline{x}$ are indifferent between buying the product from either firm, where $\overline{x}$ is defined by the condition $v - t\overline{x} - p_1^\tau = v - t(1-\overline{x}) - p_2^\tau$. From this equation, we obtain that $\overline{x} = \frac{p_2^\tau - p_1^\tau + t}{2t}$ and the firms' full-information demand functions are $x_1^0 = N\ \overline{x}$, $x_2^0 = N(1-\ \overline{x})$, $x_1^2 = \lambda N\ \overline{x}$ and $x_2^2 = \lambda N(1-\ \overline{x})$. Profit maximization yields that the equilibrium price in $\tau = 0$ and $\tau = 2$ is $p_j^m = t$, where the superscript m denotes the mass advertising solution and, in equilibrium, the market is covered, $x_1^0 + x_2^0 = N$, $x_1^2 + x_2^2 = \lambda N$. Finally, sellers obtain a profit $\Pi_j^{m0} = N\left(\frac{t}{2} - A\right)$, in $\tau = 0$, and $\Pi_j^{m2} = N\left(\frac{\lambda t}{2} - A\right)$, in $\tau = 2$, and, in order to guarantee the existence of a pure-strategy equilibrium with mass advertising, throughout the paper we will assume that $A < \frac{\lambda t}{2}$, so firms obtain an overall (intertemporal) profit $\Pi_j^{mT} = N\left(\frac{t}{2} - A\right) + \delta N\left(\frac{\lambda t}{2} - A\right) > 0$, where δ is the discount rate from $\tau = 2$ to $\tau = 0$.

Next, we assume that, in $\tau = 2$, firms have database advertising available. Considering that the main goal of this paper is to evaluate distinct public policies towards direct advertising, we analyze the functioning of the market under two regulatory scenarios: (i) firms can freely use direct advertising without asking for permission, i.e. no-regulation, and (ii) the government introduces an opt-in provision, i.e. direct advertising without permission is banned. Our first step is to characterize the market outcome when *regulation is active*, that is, when the use of direct advertising without permission is banned and both sellers can choose only between opt-in advertising and mass advertising. We look for an SPNE in which both firms find it optimal to use opt-in advertising, so we begin by solving the game in $\tau = 2$. Let us assume that firms set first-period prices (p_1^0, p_2^0) and that, in $\tau = 1$, both firms ask consumers for their consent to send them direct ads in the future. In line with seller expectations, we assume that, in $\tau = 1$, consumers can anticipate that, in $\tau = 2$, firms will again launch a new product with a "similar nature", that is, with a similar consumer valuation (v) and a similar pattern of product differentiation (t). However, in $\tau = 1$, the particular characteristics of the firms' future products are unknown, so we consider that (i) first-period buyers expect to be interested in a future product only with a probability $1 > \lambda > 0$, and (ii) they cannot anticipate a greater preference for any of the two future products, that is, all of them consider that their future (expected) location on the linear potential market is $x^e = \frac{1}{2}$. In this framework, let us assume that, at each point x on the line $[0, 1]$, a fraction $1 > \overline{y} > 0$ of consumers accepts being included in the database. The size of, for example, firm 1's database will be $N\ \frac{p_2^0 - p_1^0 + t}{2t}\ \overline{y}$, and we

[11] This implies that $A < \frac{t}{2}$ and $Min\left[\frac{v - p_1^\tau}{t}; \frac{p_2^\tau - p_1^\tau + t}{2t}\right] = \frac{p_2^\tau - p_1^\tau + t}{2t}$ which, in the symmetric equilibrium, implies $v \geq \frac{3t}{2}$.

consider that the cost of a direct advertising campaign depends on the size of the market segment covered with ads so that, for example, firm 1 incurs a cost $\beta \left[N \; \frac{p_2^0 - p_1^0 + t}{2t} \; \overline{y}\right]$, where β denotes the mailing (or telephone call) cost per ad, and we assume that reaching the market with targeting is cheaper than with mass advertising, i.e. $N\beta < NA$.

Given that a seller's database contains contact information *only* about its first-period clients, the use of direct advertising based on previous sales records allows firms to target their direct ads to a distinct set of consumers. As a result, in $\tau = 2$, all consumers become captives of one of the sellers, that is, direct advertising fragments the market into local monopolies. Accordingly, firm 1, maximizes profits taking into account that the maximum demand that it can serve is $N \; \overline{x} \; \overline{y} \; \lambda$ so, in $\tau = 2$, this seller faces a demand function

$$x_1^2 = N\left(\frac{p_2^0 - p_1^0 + t}{2t}\right) \; \overline{y} \; \lambda \; Min\left[\frac{v - p_1^2}{t}; 1\right],$$

sets a price $\widehat{p}_1^2 = \arg\max\left[p_1^2 x_1^2 - \beta N \; \overline{x} \; \overline{y}\right]$ and obtains a profit $\Pi_1^2 = \widehat{p}_1^2 x_1^2 - \beta N \; \overline{x} \; \overline{y}$. For the sake of simplicity, we assume that the firm pricing strategies do not generate quantity distortions, that is, consumers' valuations of the products are sufficiently high (or transportation costs sufficiently low) so, under monopoly pricing, the market is covered, which implies $v \geq 2t$ (see the Appendix).

In $\tau = 1$, firms build their databases and, under opt-in advertising, the key point is how many consumers authorize the use of their contact information for commercial purposes. If, in $\tau = 2$, firms use opt-in advertising, consumers can obtain a (potential) positive benefit from consumption only if they agree to receive direct advertising. However, if, ex-post, consumers are not interested in the products, when they receive direct ads (e.g. telephone calls) they suffer a privacy cost. This implies that consumers will agree to be included in the database only if the expected benefit surpasses the expected cost, that is, if $\lambda(v - p_1^e - tx^e) \geq (1-\lambda)\alpha y$, where p_1^e represents the expected price of the second-period product and $x^e = \frac{1}{2}$ is the expected location on the line in $\tau = 2$. From this condition, we can identify the consumer ($\overline{y}$) who is indifferent between accepting and rejecting the inclusion in the database, $\lambda(v - p_1^e - \frac{t}{2}) = (1-\lambda)\alpha\overline{y}$, so the fraction $\overline{y}$ of the N consumers located at point x agree to be in the database, with

$$\overline{y} = \frac{\lambda(v - p_1^e - \frac{t}{2})}{\alpha(1-\lambda)}.$$

Finally, in $\tau = 0$, firms maximize the discounted value of their total expected profit, considering a common discount factor δ. Therefore, firm 1 faces the following problem:

$$Max_{p_1^0} \; \Pi_1^T = p_1^0 N\left(\frac{p_2^0 - p_1^0 + t}{2t}\right) - NA + \delta\left[\widehat{p}_1^2 x_1^2 - \beta N \; \overline{x} \; \overline{y}\right].$$

From the above discussion, it follows that database advertising could fragment the market into local monopolies. However, if, for example, in $\tau = 2$, firm 2 targets the advertising and charges the monopoly price, it is clear that firm 1 could react by extending the reach of its advertising campaign (by using the mass media) and undercutting the price, in order to poach some consumers from the rival's database. The key issue then is whether, in equilibrium, the market can be fragmented, that is, if both firms can find it optimal to use opt-in advertising, rather than mass advertising, and to charge the monopoly price. The following proposition addresses this point.

Proposition 1 *(active regulation). If $\frac{4t}{\overline{y}} \geq v \geq 2t$ and $A \geq \lambda(v-t)(1-\frac{3\overline{y}}{4})+\frac{\beta\overline{y}}{2}$, there exists a symmetric pure-strategy SPNE in which both firms employ opt-in advertising and*

(i) in $\tau = 0$, the equilibrium prices are $p_j^0 = Max\left[t-\delta\left[(v-t)-\frac{\beta}{\lambda}\right]\lambda\,\overline{y}\ ;\ 0\right] < p_j^m$,

(ii) in $\tau = 1$, the fraction of consumers who agree to be in the database is $\overline{y} = \frac{\lambda t}{2\alpha(1-\lambda)}$,

(iii) in $\tau = 2$, the equilibrium prices are $p_j^2 = v - t > p_j^m$ and the aggregate intertemporal profits are:

$$\Pi_j^T = \begin{cases} \frac{t\,N}{2} - AN < \Pi_j^{mT} & if \quad p_j^0 > 0 \\ -AN + \delta\left(v-t-\frac{\beta}{\lambda}\right)\frac{N\lambda\overline{y}}{2} \lesseqgtr \Pi_j^{mT} & if \quad p_j^0 = 0 \end{cases}$$

Given $v \geq 2t$, when $v \leq \frac{4t}{\overline{y}}$ and $A \geq \lambda(v-t)(1-\frac{3\overline{y}}{4})+\frac{\beta\overline{y}}{2}$, the deviation strategy of mass advertising in $\tau = 2$ is not profitable because the deviation price is not sufficiently high and is too expensive to poach consumers from the rival, so the game has an equilibrium in which, in $\tau = 2$, the market fragments into local monopolies.[12] The comparison between the market outcome when firms use only mass advertising, and when they use opt-in direct advertising, yields some interesting results. The use of opt-in advertising implies, on the one hand, that firms achieve a monopolistic position in their local market and, on the other, that advertising cost-efficiency increases, which help marketers to achieve high profits in $\tau = 2$. Given that the level of second-period profits depends on the number of consumers included in the database, firms have a strategic incentive to advertise low first-period prices in order to increase their customer base. In line with this intuition, Proposition 1 shows that, compared to the benchmark case (where firms use only mass advertising), opt-in advertising results in a *reallocation* of market power from the first to the second period. Another remarkable result is that, if $\delta\lambda\frac{[\lambda(v-t)-\beta]}{2\alpha(1-\lambda)} > 1$, firms find it optimal to sell their first-period goods at zero price,[13] which yields negative profits in $\tau = 1$. This can be interpreted as an aggressive

[12] It is possible to prove that, for sufficiently low values of A ($A \leq \frac{t\lambda}{4}+\frac{\beta}{2}$), the game has an equilibrium in which both firms find it optimal to use only mass advertising. Further, we note that, if (i) $A > \frac{t\lambda}{4}+\frac{\beta}{2}$ and (ii) $v > \frac{4t}{\overline{y}}$ or $A < \lambda(v-t)(1-\frac{3\overline{y}}{4})+\frac{\beta\overline{y}}{2}$, the game does not have a symmetric pure-strategy equilibrium, so the equilibrium must be in mixed strategies where firms combine, within a planning period, the use of mass and direct advertising. The analysis of optimal pricing when firms combine the use of mass and direct advertising is addressed in Esteban and Hernández (2014).

[13] Notice that, if we relax the assumption of zero production cost by introducing a positive marginal cost, $c > 0$,

introductory offer, where a firm initially ($\tau = 1$) sells the good for free in order to foster sales and gain a large customer base which, later ($\tau = 2$), can be used to obtain high profits. We also observe that the equilibrium fraction of consumers who agree to be included in the database, $\overline{y}$, is positively related to both t and λ. Given that, in $\tau = 2$, firms charge the monopoly price and that, in $\tau = 1$, potential consumers do not know the second-period products' characteristics, all of them can anticipate only that their expected location on the line will be $x^e = \frac{1}{2}$ so, if they are interested in the product, the expected surplus from consumption is $\frac{t}{2}$, which explains why, as t increases, more consumers will be willing to be in the database.[14] Regarding λ, when this parameter increases, consumers anticipate a higher benefit and a lower cost of being in the database, so the equilibrium value of $\overline{y}$ increases. Proposition 1 also indicates that, unlike the benchmark case of mass advertising, when $p_j^0 > 0$, the optimal first-period prices depend on the advertising costs. This is so because, under mass advertising, a change in the first-period price has no effect on the cost of the advertising campaign, whereas, under direct advertising, given, for example, firm 2's price, a decrease in firm 1's first-period price yields an increase in the size of the database and, therefore, a higher advertising cost in $\tau = 2$. This implies that firms internalize the cost of advertising when setting first-period prices and, as a result, when $p_j^0 > 0$, this price is positively related to the advertising cost parameter. We also note that first-period prices are positively related to α and negatively related to λ. This occurs because an increase in α (*vs.* λ) reduces (*vs.* increases) $\overline{y}$ and, therefore, the efficiency of opt-in advertising, thus softening (*vs.* stimulating) price competition in $\tau = 0$. We finally observe that a higher level of product differentiation has two effects on first-period prices. A higher t directly softens first-period price competition but, indirectly, increases $\overline{y}$, i.e. the efficiency of opt-in advertising, which stimulates price competition in $\tau = 0$. Some simple calculations[15] show that the first effect dominates, so prices are positively related to the degree of product differentiation in $\tau = 0$ and negatively related in $\tau = 2$.

Regarding profits, compared to the benchmark case, where firms use only mass advertising, opt-in direct advertising generates two effects that work in opposite directions. The greater cost

the model will simply give more weight to the positive-prices market outcome.

[14]As mentioned in footnote 7, a key aspect of our model is that, in $t = 2$, consumers do not purchase a product unless they receive an ad. This occurs because we are implicitly assuming that consumers incur a search cost (e.g. a common transportation cost, s) to visit the store. Although potential buyers may be aware of product existence, they ignore the characteristics of the products so, in the absence of ad information, we consider that the expected surplus of visiting the store is negative, $\lambda(v - p_1^e - tx^e) < s$. By contrast, if consumers receive information about the new product, with a probability λ, they are interested in purchasing the new good and we assume that the expected surplus of visiting the store is positive, $(v - p_1^e - tx^e) > s$. This means that, in equilibrium, it holds that $\frac{t}{2} > s > \frac{\lambda t}{2}$.

[15]When $p_j^0 > 0$, we have that $\frac{\partial p_j^0}{\partial t} = 1 - \delta\lambda\frac{\lambda(v-2t)-\beta}{2\alpha(1-\lambda)}$, and the condition $p_j^0 > 0$, which implies $\delta\lambda\frac{\lambda(v-t)-\beta}{2\alpha(1-\lambda)} < 1$, guarantees that $\frac{\partial p_j^0}{\partial t} > 0$.

efficiency and monopoly power that firms enjoy in $\tau = 2$ increases profits, but targeting yields lower market prices in $\tau = 0$, which results in lower profits. According to proposition 1, the final effect on profits depends on market conditions. In particular, we find (see the Appendix) that database advertising yields higher profits in markets where the consumers' valuation of their favorite variety, v, and the savings in advertising costs, $A - \beta$, are high. A notable result is that, when $p_j^0 > 0$, then the switch from mass to targeted advertising yields lower profits whereas, when $p_j^0 = 0$, profits can increase. This occurs because firms have a strong incentive to lower prices in $\tau = 0$ in order to take advantage of database advertising in $\tau = 2$. The fact that, in equilibrium, it holds that $p_j^0 > 0$, implies that the parameter t is relatively large. In this scenario, firms fully internalize the intertemporal effect generated by targeting, which results in a strong decrease in first-period profits that leads to lower intertemporal profits. This means that firms engage in a typical prisoner's dilemma, that is, both sellers are better off using only mass advertising but, in order to save advertising costs, have a strong incentive to use their databases in $\tau = 2$ which, indirectly, generates more intense price competition in $\tau = 0$ and, finally, lower overall profits. By contrast, if t is small, firms cannot fully internalize this intertemporal effect, i.e. $p_j^0 = 0$ works as a lower bound that limits the decrease in first-period prices, and, as a result, targeting can yield higher total intertemporal profits.

Having described the market outcome under regulation (direct advertising is banned without permission), an interesting question is whether the use of opt-in advertising can lead to an equilibrium in the absence of government intervention.

Proposition 2 *(No regulation) If firms can freely use direct advertising without permission, the game does not have a symmetric pure-strategy SPNE in which firms use opt-in advertising.*

Market interaction cannot generate an equilibrium in which both firms use opt-in advertising because if, for example, firm 2 used opt-in direct advertising, the use of direct advertising without permission would allow firm 1 to simply increase the reach of its advertising campaign and, therefore, the level of profits, so there is a profitable deviation. This means that firms have a clear incentive to use direct advertising without permission, rather than opt-in advertising. Next, we describe the market equilibrium under *no regulation*, that is, when direct advertising without permission is permitted. If both sellers use direct advertising, then, given first-period prices (p_1^0, p_2^0), the size of firm 1's and firm 2's databases are, $N\frac{p_2^0 - p_1^0 + t}{2t}$ and $N(1 - \frac{p_2^0 - p_1^0 + t}{2t})$, respectively. As in the case of opt-in advertising, with direct advertising without permission, firms can target their direct ads to a distinct set of consumers, thus fragmenting the market into local monopolies, so profit maximization is identical to the opt-in advertising case, with the only difference being that, in $\tau = 2$, firm 1 and firm 2 face the demand functions:

$$x_1^2 = \; N \; \left(\frac{p_2^0 - p_1^0 + t}{2t}\right) \; \lambda \; Min \left[\frac{v - p_1^2}{t}; 1\right], \; x_2^2 = N \left[1 - \left(\frac{p_2^0 - p_1^0 + t}{2t}\right)\right] \; \lambda \; Min \left[\frac{v - p_2^2}{t}; 1\right],$$

respectively. The following proposition describes the equilibrium.

Proposition 3 *(No regulation) If $4t \geq v \geq 2t$ and $A \geq \frac{\lambda(v-t)}{4} + \frac{\beta}{2}$, there exists a symmetric pure-strategy SPNE in which, in $\tau = 2$, both firms employ direct advertising without permission and the equilibrium prices and profits are $p_j^2 = v - t > p_j^m$, $p_j^0 = Max \left[t - \delta \left[(v-t) - \frac{\beta}{\lambda}\right] \lambda \; ; \; 0\right] < p_j^m$,*

$$\Pi_j^T = \begin{cases} \frac{t\,N}{2} - AN < \Pi_j^{mT} & if \quad p_j^0 > 0 \\ -AN + \delta\left(v - t - \frac{\beta}{\lambda}\right)\frac{N\lambda}{2} \lesseqgtr \Pi_j^{mT} & if \quad p_j^0 = 0 \end{cases}$$

Comparing propositions 1 and 3, we conclude that, with both opt-in advertising and direct advertising without permission, first-period prices (vs. second-period prices) are positively (vs. negatively) related to the degree of product differentiation, t. Further, compared to opt-in advertising, the effect of direct advertising without permission on firm profits and pricing strategies,[16] and the comparative static properties of the equilibrium, are similar, so we can conclude that, compared to the benchmark case of mass advertising, opt-in advertising and direct advertising without permission have, in qualitative terms, a similar effect on the pattern of price competition and the functioning of the market. However, it is important to note that first-period prices with opt-in advertising are higher than or equal to direct advertising without permission, whereas aggregate profits with opt-in advertising are lower than or equal to direct advertising without permission. This occurs because, with opt-in advertising, selling in the first period is less valuable for firms, as it does not lead to as many subsequent sales, so sellers have a lower incentive to decrease first-period prices to increase market share. Further, opt-in advertising is less efficient than direct advertising without permission in reaching the set of potential clients. This explains why, compared to direct advertising without permission, opt-in advertising yields lower profits.

The next section discusses how direct advertising without permission and opt-in advertising can affect market performance, and how a regulatory policy of database advertising can affect social welfare.

[16] The impact of direct advertising on prices and profits in our model contrasts with Esteban and Hernández (2014), where firms face a totally inelastic demand and (i) first-period prices are always positive and, (ii) compared to the benchmark case of mass advertising, firms always achieve higher profits.

3 Welfare and policy implications of direct advertising

Our previous analysis suggests that database advertising generates (i) high advertising-cost efficiency, (ii) an intertemporal transfer of market power from $\tau = 0$ to $\tau = 2$, and (iii) privacy costs for consumers who receive ads and are not interested in second-period products. This section addresses the welfare and policy aspects of direct advertising. In particular, we evaluate the impact of the two types of direct advertising (without permission and opt-in) on social welfare, so that we can shed light on how distinct public policies about direct advertising, e.g. to allow or to restrict the use of direct advertising without permission, can affect firms and consumers.

We begin by noting that total welfare equals the value of the good for all buyers, minus the advertising, transportation and privacy costs. Regardless of the type of direct advertising used by firms, in $\tau = 0$, both sellers use mass advertising and the market is covered, so the level of social welfare coincides. However, in $\tau = 2$, if firms use direct advertising without permission, the level of welfare is:[17]

$$W^A = N\left[\lambda \int_0^1 (v - tx)\ dx - \ (1-\lambda)\ \alpha \int_0^1 y\ dy - \beta\right] = N\ \lambda\left(v - \frac{t}{2}\right) - \frac{N\ \alpha\ (1-\lambda)}{2} - N\beta$$

whereas, under opt-in advertising, it is:

$$\begin{aligned} W^B \quad &= \quad N\left[\overline{y}\ \lambda \int_0^1 (v - tx)\ dx - \ (1-\lambda)\ \alpha \int_0^{\overline{y}} y\ dy - \beta\ \overline{y}\right] = \\ & N\ \overline{y}\ \lambda\left(v - \frac{t}{2}\right) - N\ \alpha\ (1-\lambda)\frac{\overline{y}^2}{2} - N\beta\ \overline{y}. \end{aligned}$$

In order to understand how opt-in advertising *vs.* direct advertising without permission can affect social welfare, we first compare the private level of $\overline{y}$ with the socially-optimal level, $\overline{y}^*$, that is, with the value of $\overline{y}$ that maximizes social welfare.

Proposition 4 *If firms use opt-in advertising, the private level of $\overline{y}$ is below the socially optimal value, i.e. $\overline{y} < \overline{y}^*$.*

From a social perspective, the differences between direct advertising without permission and opt-in advertising affect several aspects. First, under opt-in advertising, consumers decide whether or not to be included in a firm's database and, given $\overline{y} < 1$, a fraction of them refuse the offer, thus reducing the firms' potential demand in $\tau = 2$. Proposition 4 indicates that, from a social perspective, consumers have too little incentive to accept their inclusion in a database, which

[17] We assume that consumers' valuation of the products is sufficiently high, so that the level of welfare under both DA and PBA is positive.

generates a negative externality on firms in the form of a quantity distortion in the product market, $\overline{y} < \overline{y}^*$, with the corresponding welfare loss. Second, compared to direct advertising without permission, under opt-in advertising a lower number of consumers receive direct ads, which reduces both the cost of the advertising campaign and the privacy costs.

According to propositions 1, 2 and 3, in the absence of a regulation, firms have a clear incentive to use direct advertising without permission rather than opt-in advertising, and the problem is that consumers who are not interested in second-period products and have a relatively high privacy cost receive undesired direct ads, which has a negative effect on welfare. Therefore, the protection of consumer privacy can justify government intervention aimed at restricting the use of direct advertising without permission, in favor of opt-in advertising. However, the use of opt-in advertising generates quantity distortions, so there is a policy dilemma with no easy answer. The following proposition addresses this problem,

Proposition 5 *A regulatory policy banning the use of direct advertising without permission and generating opt-in advertising results in a welfare loss.*

In order to evaluate a regulatory policy for database advertising, it is necessary to consider three different goals. First, government intervention must favor advertising cost efficiency, that is, firms should use database advertising rather than mass advertising. Second, the advertising strategy used by firms should favor socially-positive exchanges between firms and consumers, i.e. quantity distortions should be minimized. Finally, it is also important to reduce consumer privacy costs. If the government prohibits the use of direct advertising without permission, then firms will employ either the mass media or opt-in direct advertising and proposition 5 indicates that, in the latter case, the regulatory policy yields a welfare loss. This occurs because, if mass advertising is available, firms will find it optimal to use opt-in advertising *only* if it is relatively efficient, that is, if $\overline{y}$ is sufficiently large, which requires a small α. In this scenario, proposition 5 indicates that the regulatory policy will fail because the gain in social welfare induced by the savings in privacy and advertising costs is lower than the welfare loss generated by the quantity distortion associated with opt-in advertising. Put another way, given the quantity distortion induced by opt-in advertising, the only possibility that, compared to direct advertising without permission, opt-in advertising increases welfare is that the savings in privacy costs are very high, i.e. α is large. However, when α reaches the corresponding critical level, opt-in advertising has a low effectiveness ($\overline{y}$ is low) and firms always find it optimal to deviate by using mass advertising so, once again, the regulatory policy will fail, this time by generating an increase in advertising costs (firms use mass advertising rather than database advertising).

Next, we evaluate how a ban on direct advertising without permission affects firms and consumers.

Proposition 6 *If the degree of product differentiation is relatively large (so that $p_j^0 > 0$ with direct advertising without permission), a regulatory policy banning the use of direct advertising without permission and generating opt-in advertising does not affect firms' profits whereas, under low differentiation (so that $p_j^0 = 0$ with opt-in advertising), the regulation lowers profits.*

If, in equilibrium, it holds that $p_j^0 > 0$ with direct advertising without permission (which implies $p_j^0 > 0$ with opt-in advertising), then profits with the two types of direct advertising coincide, so a ban on direct advertising without permission, generating opt-in advertising, does not affect firms. However, if $p_j^0 = 0$ with opt-in advertising (which implies $p_j^0 = 0$ with direct advertising without permission), firms achieve higher profits with direct advertising without permission than with opt-in advertising, so we conclude that the regulatory policy yields lower firm profits. Concerning consumers, we note that, regardless of the type of direct advertising used by the firms, in $\tau = 0$, they achieve a surplus:

$$CS^0 = 2N \left[\int_0^{\frac{1}{2}} \left(v - p_j^0 - tx\right) \, dx \right].$$

In $\tau = 2$, consumers who receive direct ads achieve a different surplus, depending on whether they are interested (buyers) or not (non-buyers) in second-period products. Non-buyers only suffer a privacy cost and, therefore, their (negative) surplus under opt-in advertising and direct advertising without permission, respectively, is:

$$CS^{NB} = -N \, (1-\lambda) \, \alpha \int_0^{\overline{y}} y \; dy, \;\; CS^{NB} = -N \, (1-\lambda) \, \alpha \int_0^{1} y \; dy.$$

The (positive) surplus of buyers under opt-in advertising and direct advertising without permission, respectively, are:

$$CS^B = N \, \overline{y} \, \lambda \int_0^1 \left(v - p_j^2 - tx\right) \, dx, \;\; CS^B = N \, \lambda \int_0^1 \left(v - p_j^2 - tx\right) \, dx,$$

and total consumer surplus is $CS = CS^0 + \delta(CS^{NB} + CS^B)$. Assuming that $\delta = 1$, the following proposition illustrates how the prohibition of direct advertising without permission can affect consumers.

Proposition 7 *If the degree of product differentiation is relatively large (so that $p_j^0 > 0$ with direct advertising without permission), a regulatory policy banning the use of direct advertising without permission, and generating opt-in advertising, results in a decrease of total consumer surplus,* *whereas, under low differentiation (so that $p_j^0 = 0$ with opt-in advertising), the regulation increases total consumer surplus.*

From the consumer perspective, the switch from direct advertising without permission to opt-in advertising generates three effects. First, they receive less information about new products in $\tau = 2$, which reduces the surplus achieved from the consumption of second-period products. Second, compared to direct advertising without permission, the use of opt-in advertising generates higher first-period prices, which also reduces consumer surplus. Finally, the use of opt-in advertising reduces privacy costs, which benefits consumers. According to proposition 7, the result of this trade-off depends crucially on the extent of product differentiation. If the level of product differentiation is sufficiently high, so that $p_j^0 > 0$, under both direct advertising without permission and opt-in advertising, firms fully internalize the intertemporal effects of database advertising on first-period prices, that is, database advertising yields strong price decreases in $\tau = 0$, and the reduction in prices is greater with direct advertising without permission than with opt-in advertising. This explains why the transition from direct advertising without permission to opt-in advertising produces a loss in consumer surplus. By contrast, when the level of product differentiation is sufficiently low, so that $p_j^0 = 0$, the intertemporal effects of database advertising on first-period prices are weaker, and first-period prices with opt-in advertising and direct advertising without permission coincide. In this case, the savings in privacy costs induced by opt-in advertising dominates the model, and the regulatory policy benefits consumers.

Considering Propositions 5 , 6 and 7 together, we can conclude that, when the level of product differentiation is sufficiently high, so that $p_j^0 > 0$, a regulatory policy banning the use of direct advertising without permission, and generating opt-in advertising, leaves profits unchanged, whereas consumer surplus and, therefore, social welfare, decreases. This means that direct advertising without permission, not only increases welfare, but also (weakly Pareto) dominates opt-in advertising, so a regulatory policy aimed at protecting consumers ends up lowering both consumer surplus and social welfare. By contrast, when $p_j^0 = 0$, government intervention benefits consumers but, in this case, firms obtain lower profits and this effect dominates, so social welfare decreases. Accordingly, our model suggests that the permissive use of consumers' private information for commercial purposes, that is, the use of direct advertising without permission, improves market performance.

4 Conclusions

This paper studies, first, how two different types of direct advertising, without permission and opt-in, affect the functioning of a horizontally-differentiated market and, second, the welfare and policy implications of database advertising. We formulate a three-stage price competition game in which two firms, initially, use mass advertising to reach consumers, then, build an in-house list with the contact information of their past clients and, finally, depending on regulatory provisions, can choose between using mass advertising, direct advertising without permission, or opt-in advertising to provide consumers with information about a new product. We find that, compared to the benchmark case of mass advertising, the use of direct advertising (with or without permission) can fragment the market into local monopolies. This leads sellers to engage in a race to gain a customer base by competing more aggressively in first-period prices. As a result, database advertising yields an intertemporal transfer of market power from the first to the second period. A notable result is that firms can find it optimal to follow an aggressive introductory offer (zero first-period prices) aimed at building a large customer base which, later, allows them to obtain high profits. Further, we show that, if first-period prices are positive, then database advertising generates lower firm profits than mass advertising, whereas profits can increase if first-period prices are zero. Finally, we find that, compared to direct advertising without permission, the use of opt-in advertising can lead to higher first-period prices and lower profits.

Our analysis also indicates that marketers have a clear incentive to use direct advertising without permission rather than opt-in advertising, which has important welfare implications. Under direct advertising without permission, some consumers with relatively high privacy costs are not interested in second-period products and they may receive a great deal of undesired direct mail, which has a strong negative effect on welfare. Therefore, the protection of consumer privacy may justify government regulation aimed at restricting the use of direct advertising without permission in favor of opt-in advertising. However, under opt-in advertising, a fraction of consumers refuse the offer to be included in the database and we show that, compared to the socially optimal solution, this fraction is too large, which generates a strong quantity distortion in the product market, with the corresponding welfare loss. Regarding this policy dilemma, we find that a regulatory policy banning the use of direct advertising without permission and generating opt-in advertising results in a welfare loss. Regarding firms and consumers, if the degree of product differentiation in the market is relatively large, the regulatory policy does not affect firms' profits and generates a lower consumer surplus, whereas, under low product-differentiation, the regulation lowers profits and increases consumer surplus.

In summary, a regulatory ban on direct advertising without permission aimed at protecting consumer privacy can generate a welfare loss, for two reasons. First, the prohibition may lead firms to use mass advertising, which increases advertising costs, and second, if firms choose to use opt-in advertising, then the loss in welfare associated with the quantity distortion generated by such advertising exceeds the saving in privacy costs. Accordingly, we find that the regulatory policy fails. Nevertheless, it is obvious that, beyond the gains in efficiency illustrated by our model, the abusive and massive commercial use of consumer private information may be considered overly intrusive, thus generating substantial losses in social welfare, which can justify restrictions on the use of direct advertising without permission. However, our work suggests that government intervention can generate significant distortions in the functioning of markets so, in order to find an "optimal" regulatory policy, it is important to carefully consider the potential negative impact on efficiency induced by a restrictive approach towards the commercial use of private consumer information.

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5 Appendix

Proof. Proposition 1. We look for an SPNE of the game in which, in $\tau = 2$, both firms use only opt-in direct advertising. Solving the game backwards, if both firms use opt-in advertising, then, given the first-period prices (p_1^0, p_2^0) and $\overline{y}$, in $\tau = 2$, for example, seller 1 faces the following problem:

$$Max_{p_1^2}\ \Pi_1^2 = p_1^2 N \left(\frac{p_2^0 - p_1^0 + t}{2t}\right)\ \overline{y}\ \lambda\ Min\left[\frac{v - p_1^2}{t}; 1\right] - \beta N \left(\frac{p_2^0 - p_1^0 + t}{2t}\right)\overline{y}.$$

Suppose that $Min\left[\frac{v-p_1^2}{t}; 1\right] = \frac{v-p_1^2}{t}$, which implies $p_1^2 > v - t$. Then, the solution of the maximization problem is $p_1^2 = \frac{v}{2}$, so the condition $p_1^2 > v - t$ implies $v < 2t$, which contradicts our assumption that $v \geq 2t$. Therefore, we conclude that, in equilibrium, it holds that $Min\left[\frac{v-p_1^2}{t}; 1\right] = 1$, i.e. $p_1^2 \leq v - t$. Under this condition, we have that $\frac{d\,\Pi_1}{dp_1^2} = N\lambda\overline{x}\ \overline{y} > 0$, which implies that the firm charges the maximum possible price, $p_1^2 = v - t$, and achieves a profit:

$$\Pi_1^2 = (v - t)N\left(\frac{p_2^0 - p_1^0 + t}{2t}\right)\ \overline{y}\ \lambda\ - \beta N\left(\frac{p_2^0 - p_1^0 + t}{2t}\right)\overline{y}.$$

Going backwards, in $\tau = 1$, consumers decide whether or not to be included in a database. According to our previous analysis, the indifferent consumer is $\overline{y} = \frac{\lambda(v - p_1^e - \frac{t}{2})}{\alpha(1-\lambda)}$ and, under rational expectations, $p_1^e = p_1^2 = v - t$, the fraction of consumers who agree to be in a database is $\overline{y} = \frac{\lambda t}{2\alpha(1-\lambda)}$.

In $\tau = 0$, firm 1 maximizes the discounted value of the total expected profit:

$$Max_{p_1^0}\ \Pi_1^T = p_1^0 N\left(\frac{p_2^0 - p_1^0 + t}{2t}\right) - NA + \delta\left[(v - t)N\left(\frac{p_2^0 - p_1^0 + t}{2t}\right)\ \overline{y}\ \lambda\ - \beta N\left(\frac{p_2^0 - p_1^0 + t}{2t}\right)\overline{y}\right]$$

The first-order condition (FOC) of this problem yields:[18]

$$\frac{d\ \Pi_1^T}{dp_1^0} = N\left(\frac{p_2^0 - p_1^0 + t}{2t}\right) - N\frac{p_1^0}{2t} + \delta\left[\left(-\frac{1}{2t}\right)(v-t)N\ \overline{y}\ \lambda + \frac{1}{2t}\beta N\overline{y}\right] = 0,$$

and, using the symmetry condition, $p_2^0 = p_1^0$, we obtain $p_j^0 = t - \delta\left[\lambda(v-t) - \beta\right]\overline{y} \geq 0$. The condition $\Pi_j^{m2} > 0$ implies $A < \frac{\lambda t}{2}$, and, therefore, our assumption that $\beta < A$ yields $\beta < \frac{\lambda t}{2}$. Considering also that $v \geq 2t$, we have that $\lambda(v-t) - \beta \geq \lambda t - \beta > 0$, so $p_j^0 = t - \delta\left[\lambda(v-t) - \beta\right]\overline{y} < p_j^m = t$, and that $p_j^2 = v - t \geq p_j^m$. Finally, considering the optimal pricing strategies, it is easy to check that, if $p_j^0 > 0$, the equilibrium level of profits is $\Pi_1^T = \frac{tN}{2} - NA < \Pi_j^{mT}$, whereas, if $p_j^0 = 0$, then $\Pi_1^T = -NA + \delta\left(v - t - \frac{\beta}{\lambda}\right)\frac{N\lambda\overline{y}}{2} \lesseqgtr \Pi_j^{mT}$, depending on the parameter values.

In order to confirm that these pricing-advertising strategies are an equilibrium, we must check that firms have no profitable deviations. In this regard, we note that, in $\tau = 2$, there are two possible deviations:

(A) Firms can deviate by launching a mass advertising campaign and competing for the segment of fully-informed consumers, i.e. those consumers informed by firm 2. Let us assume that firm 2 uses opt-in advertising and sets $p_2^2 = v - t$. Taking into account that the size of firm 2's database is $x_2^2 = (1-\overline{x})N\overline{y}$, if firm 1 deviates by using mass advertising and charging a deviation price p_1^d, then it captures a demand $x_1^d = N\lambda\overline{y}(1-\overline{x})\ Min\left[\frac{v-t-p_1^d+t}{2t};1\right] + N\lambda\left[1-\overline{y}(1-\overline{x})\right]\ Min\left[\frac{v-p_1^d}{t};1\right]$. In order to obtain the optimal deviation price, let us begin by considering that $\frac{v-p_1^d}{t} > 1 > \frac{v-p_1^d}{2t}$. In this scenario, the firm faces the following optimization problem:

$$Max_{p_1^d}\ \Pi_1^d = p_1^d N\lambda\left[\overline{y}\ (1-\overline{x})\left(\frac{v-p_1^d}{2t}\right) + (1-\overline{y}(1-\overline{x}))\right] - NA, \tag{1}$$

which, considering $\overline{x} = \frac{1}{2}$, yields the optimal deviation price, $p_1^d = \frac{\overline{y}(v-2t)+4t}{2\overline{y}}$, and profit, $\Pi_1^d = \frac{N\lambda[v\ \overline{y}+2t(2-\overline{y})]^2}{16t\ \overline{y}} - NA$. This is the maximum deviation profit when $\frac{v-p_1^d}{2d} < 1$, i.e. when $v < 2t + \frac{4t}{\overline{y}}$, and when $\frac{v-p_1^d}{t} \geq 1$, i.e. $v \geq \frac{4t}{\overline{y}}$. Firm 1 will not deviate if $\Pi_1^d \leq (v-t-\frac{\beta}{\lambda})\frac{N\lambda\overline{y}}{2}$, which implies $\frac{A}{\lambda} \geq \frac{[v\ \overline{y}+2t(2-\overline{y})]^2}{16t\ \overline{y}} - (v-t-\frac{\beta}{\lambda})\frac{\overline{y}}{2}$. Next, we show that $\frac{[v\ \overline{y}+2t(2-\overline{y})]^2}{16t\ \overline{y}} - (v-t-\frac{\beta}{\lambda})\frac{\overline{y}}{2} > \frac{t}{2}$. Some algebraic operations yield that this expression holds if $\left[v^2\overline{y}^2 + v\overline{y}t(8-12\overline{y})\right] + t\left[16t + 12t\overline{y}(\overline{y}-2)\right] > 0$ and, considering that $\overline{y} \leq 1$ and $v \geq \frac{4t}{\overline{y}}$, it follows that $\left[v^2\overline{y}^2 + v\overline{y}t(8-12\overline{y})\right] \geq 0$ and $\left[16t + 12t\overline{y}(\overline{y}-2)\right] > 0$, thus proving that $\frac{[v\ \overline{y}+2t(2-\overline{y})]^2}{16t\ \overline{y}} - (v-t-\frac{\beta}{\lambda})\frac{\overline{y}}{2} > \frac{t}{2}$. Therefore, the condition $\Pi_1^d \leq (v-t-\frac{\beta}{\lambda})\frac{N\lambda\overline{y}}{2}$ implies $\frac{A}{\lambda} > \frac{t}{2}$, thus yielding a contradiction. This means that, if $2t + \frac{4t}{\overline{y}} > v \geq \frac{4t}{\overline{y}}$ firm 1 always finds it optimal to deviate, so the pricing-advertising game does not have an equilibrium with opt-in advertising.

If $v \geq 2t + \frac{4t}{\overline{y}}$, the solution of (1) yields $Min\left[\frac{v-p_1^d}{2t};1\right] = 1$, i.e. $p_1^d \leq v - 2t$. In this case, we have that $\frac{d\ \Pi_1}{dp_1^d} = N\lambda > 0$, so the firm charges the maximum possible price, $p_1^d = v - 2t$, and the

[18] It is straightforward to check that the second-order conditions are always satisfied.

maximum deviation profit is $\Pi_1^d = (v-2t)N\lambda - NA$. We now prove, by contradiction, that this deviation is always optimal, that is, $(v-2t)N\lambda - NA > (v-t-\frac{\beta}{\lambda})\frac{N\lambda\overline{y}}{2}$. Let us assume that $(v-2t)N\lambda - NA \leq (v-t-\frac{\beta}{\lambda})\frac{N\lambda\overline{y}}{2}$, that is, $\frac{A}{\lambda} \geq v-2t-v\frac{\overline{y}}{2}+t\frac{\overline{y}}{2}+\beta\frac{\overline{y}}{2\lambda}$. Taking into account that $\frac{A}{\lambda} < \frac{t}{2}$, this implies that $\frac{t}{2} > \frac{A}{\lambda} \geq v-2t-v\frac{\overline{y}}{2}+t\frac{\overline{y}}{2}+\beta\frac{\overline{y}}{2\lambda}$, which yields $v < \frac{2t}{(1-\frac{\overline{y}}{2})}+\frac{t}{2}\frac{1-\overline{y}}{(1-\frac{\overline{y}}{2})}-\frac{\beta\overline{y}}{2\lambda(1-\frac{\overline{y}}{2})}$. Finally, considering that $v \geq 2t+\frac{4t}{\overline{y}}$ and $\overline{y} \leq 1$, we have that $2t+\frac{4t}{\overline{y}} \leq \frac{2t}{(1-\frac{\overline{y}}{2})}+\frac{t}{2}\frac{1-\overline{y}}{(1-\frac{\overline{y}}{2})}-\frac{\beta\overline{y}}{2\lambda(1-\frac{\overline{y}}{2})}$, i.e. $\frac{4}{\overline{y}} < \frac{5}{2}+\frac{\overline{y}}{2}-\frac{\beta\overline{y}}{2\lambda t}$, thus yielding a contradiction. From this it follows that, if $v \geq 2t+\frac{4t}{\overline{y}}$, the pricing-advertising game does not have an equilibrium with opt-in advertising.

Finally, if $v < \frac{4t}{\overline{y}}$, then $Min\left[\frac{v-p_1^d}{t};1\right] = \frac{v-p_1^d}{t}$ and the firm faces the following optimization problem:

$$Max_{p_1^d}\ \Pi_1^d = p_1^d N\lambda\left[\overline{y}\ (1-\overline{x})\left(\frac{v-p_1^d}{2t}\right)+[1-\overline{y}(1-\overline{x})]\left(\frac{v-p_1^d}{t}\right)\right]-NA,$$

which, considering $\overline{x} = \frac{1}{2}$, yields the deviation price, $p_1^d = \frac{v}{2}$. Given that $\frac{v-p_1^d}{t} = \frac{v}{2t} \geq 1$, the optimization problem generates a kinked solution, $p_1^d = v-t$, and the corresponding profit is $\Pi_1^d = \frac{N\lambda(v-t)(4-\overline{y})}{4} - NA$. Firm 1 will not deviate if $\Pi_1^d \leq (v-t-\frac{\beta}{\lambda})\frac{N\lambda\overline{y}}{2}$, which implies $A \geq \frac{\lambda(v-t)(4-3\overline{y})}{4}+\frac{\beta\overline{y}}{2}$ so, if $\frac{4t}{\overline{y}} > v \geq 2t$ and $A \geq \frac{\lambda(v-t)(4-3\overline{y})}{4}+\frac{\beta\overline{y}}{2}$, the deviation is not profitable.

(B) Firms can also deviate by launching a mass advertising campaign and charging the monopoly price, $p_1^d = v-t$, to the segment of uninformed consumers, i.e. those consumers who are not informed by firm 2. Let us assume that firm 2 uses opt-in advertising and sets $p_2^2 = v-t$. Under this deviation strategy, the indifferent (fully-informed) consumer is $\widehat{x} = \frac{p_2^2-p_1^d+t}{2t} = \frac{1}{2}$, and the firm achieves a deviation profit:

$$\Pi_1^d = (v-t)\ \left[N\lambda - N\lambda(1-\overline{x})(1-\widehat{x})\overline{y}\right] - NA$$

The firm will not deviate if $\Pi_1^d \leq (v-t-\frac{\beta}{\lambda})\frac{N\lambda\overline{y}}{2}$, which yields $A \geq \frac{\lambda(v-t)(4-3\overline{y})}{4}+\frac{\beta\overline{y}}{2}$, i.e. the same condition as in part (A).

In summary, if $\frac{4t}{\overline{y}} > v \geq 2t$ and $A \geq \frac{\lambda(v-t)(4-3\overline{y})}{4}+\frac{\beta\overline{y}}{2}$, firms do not have profitable deviation so, if direct advertising without permission is banned, the game has a pure-strategy equilibrium where marketers find it optimal to use opt-in direct advertising. ■

Proof. Proposition 2. Let us assume that firm 2 uses opt-in direct advertising and sets $p_2^2 = v-t$. If firm 1 also uses opt-in advertising, it obtains a profit $\Pi_1^2 = (v-t)\frac{N\overline{y}\ \lambda}{2} - \frac{\beta N\overline{y}}{2}$. If the firm deviates by using direct advertising without permission, the optimal deviation price is $p_1^d = v-t$ and the resulting profit is $\Pi_1^d = \left(v-t-\frac{\beta}{\lambda}\right)\frac{N\lambda}{2} > (v-t-\frac{\beta}{\lambda})\frac{N\overline{y}\ \lambda}{2}$, so there is a profitable deviation. ■

Proof. Proposition 3. The proofs of propositions 1 and 3 are similar, the only difference being that, with direct advertising without permission, the reach of the database is $N\left(\frac{p_2^0-p_1^0+t}{2t}\right)$

and the cost of the advertising campaign is $\beta N\left(\frac{p_2^0-p_1^0+t}{2t}\right)$. Accordingly, when both firms use direct advertising without permission, in $\tau=2$, they will charge the monopoly price, $p_j^2=v-t$, and firm 1 obtains a profit:

$$\Pi_1^2=(v-t)N\left(\frac{p_2^0-p_1^0+t}{2t}\right)\lambda\ -\beta N\left(\frac{p_2^0-p_1^0+t}{2t}\right).$$

In $\tau=0$, firm 1 maximizes the discounted value of the total expected profit:

$$Max_{p_1^0}\ \Pi_1^T=p_1^0N\left(\frac{p_2^0-p_1^0+t}{2t}\right)-NA+\delta\left[(v-t)N\left(\frac{p_2^0-p_1^0+t}{2t}\right)\lambda\ -\beta N\left(\frac{p_2^0-p_1^0+t}{2t}\right)\right]$$

The first-order condition (FOC) of this problem yields:

$$\frac{d\ \Pi_1^T}{dp_1^0}=N\left(\frac{p_2^0-p_1^0+t}{2t}\right)-N\frac{p_1^0}{2t}+\delta\left[\left(-\frac{1}{2t}\right)(v-t)N\ \lambda+\frac{1}{2t}\beta N\right]=0,$$

and, using the symmetry condition, $p_2^0=p_1^0$, we obtain $p_j^0=t-\delta\left[\lambda(v-t)-\beta\right]\geq 0$. Considering the optimal pricing strategies, it is easy to check that, if $p_j^0>0$, the equilibrium level of profits is $\Pi_1^T=\frac{t\,N}{2}-NA<\Pi_j^{mT}$, whereas, if $p_j^0=0$, then $\Pi_1^T=-AN+\delta\left(v-t-\frac{\beta}{\lambda}\right)\frac{N\lambda}{2}\lesseqgtr\Pi_j^{mT}$, depending on the parameter values.

Next, we check that firms have no profitable deviations. In $\tau=2$, there are three possible deviations:

(A) Firms can deviate by launching a mass advertising campaign and competing for the segment of fully informed consumers. Let us assume that firm 2 uses direct advertising without permission and sets $p_2^2=v-t$. Taking into account that the size of firm 2's database is $x_2^2=(1-\overline{x})N$, if firm 1 deviates by using mass advertising and charging a deviation price p_1^d, then it captures a demand $x_1^d=N\lambda(1-\overline{x})\ Min\left[\frac{v-t-p_1^d+t}{2t};1\right]+N\lambda\left[1-(1-\overline{x})\right]\ Min\left[\frac{v-p_1^d}{t};1\right]$. Following the same reasoning as in the proof of proposition 1, if $\frac{v-p_1^d}{t}>1>\frac{v-p_1^d}{2t}$, the optimal deviation price is $p_1^d=\frac{v+2t}{2}$ and the corresponding profit is $\Pi_1^d=\frac{N\lambda[v+2t]^2}{16t}-NA$. This is the maximum deviation profit when $\frac{v-p_1^d}{2d}<1$, i.e. when $v<6t$, and when $\frac{v-p_1^d}{t}\geq 1$, i.e. $v\geq 4t$. Firm 1 will not deviate if $\Pi_1^d\leq\left(v-t-\frac{\beta}{\lambda}\right)\frac{N\lambda}{2}$, which implies $\frac{A}{\lambda}\geq\frac{[v+2t]^2}{16t}-\left(v-t-\frac{\beta}{\lambda}\right)\frac{1}{2}$. Next, we show that $\frac{[v+2t]^2}{16t}-\left(v-t-\frac{\beta}{\lambda}\right)\frac{1}{2}>\frac{t}{2}$. Some algebraic operations yield that this expression holds if $(v-2t)^2>0$ so, as in the proof of proposition 1, we obtain a contradiction. This means that, if $6t>v\geq 4t$, firm 1 always finds it optimal to deviate, so the pricing-advertising game does not have an equilibrium with direct advertising without permission.

If $v\geq 6t$, it holds that $Min\left[\frac{v-p_1^d}{2t};1\right]=1$, i.e. $p_1^d\leq v-2t$. In this case, we have that $\frac{d\ \Pi_1^d}{dp_1^d}>0$, so the firm charges the maximum possible price, $p_1^d=v-2t$, and the maximum deviation profit is $\Pi_1^d=(v-2t)N\lambda-NA$. We now prove, by contradiction, that this deviation is always optimal, that is, $(v-2t)N\lambda-NA>\left(v-t-\frac{\beta}{\lambda}\right)\frac{N\lambda}{2}$. Let us assume that $(v-2t)N\lambda-NA\leq\left(v-t-\frac{\beta}{\lambda}\right)\frac{N\lambda}{2}$, that is, $\frac{A}{\lambda} \geq v - 2t - \frac{v}{2} + \frac{t}{2} + \frac{\beta}{2\lambda}$. Taking into account that $\frac{A}{\lambda} < \frac{t}{2}$, this implies that $\frac{t}{2} > A \geq v - 2t - \frac{v}{2} + \frac{t}{2} + \frac{\beta}{2\lambda}$, which yields $v < 4t - \frac{\beta}{\lambda}$, thus yielding a contradiction. From this it follows that, if $v \geq 6t$, the pricing-advertising game does not have an equilibrium with direct advertising without permission.

Finally, if $v < 4t$, then $Min\left[\frac{v-p_1^d}{t};1\right] = \frac{v-p_1^d}{t}$ and, as in the proof of proposition 1, the optimal deviation price is $p_1^d = v - t$ and the corresponding profit is $\Pi_1^d = \frac{3N\lambda(v-t)}{4} - NA$. Firm 1 will not deviate if $\Pi_1^d \leq \left(v - t - \frac{\beta}{\lambda}\right)\frac{N\lambda}{2}$, which implies $A \geq \frac{\lambda(v-t)}{4} + \frac{\beta}{2}$ so, if $4t > v \geq 2t$ and $A \geq \frac{\lambda(v-t)}{4} + \frac{\beta}{2}$, the deviation is not profitable.

(B) Firms can deviate by launching a mass advertising campaign and charging the monopoly price, $p_1^d = v - t$, to the segment of uninformed consumers. Let us assume that firm 2 uses direct advertising without permission and sets $p_2^2 = v - t$. Under this deviation strategy, the indifferent (fully- informed) consumer is $\widehat{x} = \frac{p_2^2 - p_1^d + t}{2t} = \frac{1}{2}$ and the firm achieves a deviation profit:

$$\Pi_1^d = (v-t)\ \left[N\lambda - N\lambda(1-\overline{x})(1-\widehat{x})\right] - NA$$

The firm will not deviate if $\Pi_1^d \leq \left(v - t - \frac{\beta}{\lambda}\right)\frac{N\lambda}{2}$, which yields $A \geq \frac{\lambda(v-t)}{4} + \frac{\beta}{2}$, i.e. the same condition as in part (A).

(C) Firms can deviate in $\tau = 1$ by using opt-in direct advertising instead of direct advertising without permission. Let us assume that firm 2 uses direct advertising without permission and sets $p_2^2 = v - t$. If firm 1 uses opt-in advertising, the optimal deviation price is $p_1^d = v - t$ and the firm achieves a deviation profit:

$$\Pi_1 = (v-t)\ N\lambda\left(\frac{p_2^0 - p_1^0 + t}{2t}\right)\overline{y} - \beta N\left(\frac{p_2^0 - p_1^0 + t}{2t}\right)\overline{y} = (v-t)\ \frac{N\lambda\overline{y}}{2} - \frac{\beta N\overline{y}}{2}.$$

This deviation is never optimal because $\Pi_1 = (v-t)\ \frac{N\lambda\overline{y}}{2} - \frac{\beta N\overline{y}}{2} < \Pi_1^2 = \left(v - t - \frac{\beta}{\lambda}\right)\frac{N\lambda}{2}$.

In summary, if $4t > v \geq 2t$ and $A \geq \frac{\lambda(v-t)}{4} + \frac{\beta}{2}$, firms do not have profitable deviation, so the game has a pure-strategy equilibrium where marketers find it optimal to use direct advertising without permission. We finally note that, as in proposition 1, when $p_j^0 = t - \delta\left[\lambda(v-t) - \beta\right] > 0$, it holds that $\left[\lambda(v-t) - \beta\right] > 0$, so $p_j^0 < p_j^m = t$. ■

Proof. Proposition 4. We obtain the socially optimal value of $\overline{y}$ by maximizing social welfare:

$$Max_{\ \overline{y}}\ W^B = N\ \overline{y}\ \lambda\left(v - \frac{t}{2}\right) - N\ \alpha\ (1-\lambda)\frac{\overline{y}^2}{2} - N\beta\ \overline{y}.$$

The FOC of this problem yields $\overline{y}^* = \frac{\lambda\left[v - \frac{t}{2} - \frac{\beta}{\lambda}\right]}{\alpha\ (1-\lambda)}$. The condition $\overline{y}^* > \overline{y} = \frac{\lambda t}{2\alpha\ (1-\lambda)}$, implies $\lambda(v - t) - \beta > 0$, which, according to the proof of proposition 1, holds. ■

Proof. Proposition 5. Taking into account that $\overline{y} = \frac{\lambda t}{2\alpha\ (1-\lambda)}$, total welfare under opt-in advertising is $W^B = \frac{N\lambda t[\lambda(4v-3t)-4\beta]}{8\alpha\ (1-\lambda)}$ whereas, under direct advertising without permission, total welfare is $W^A = N\ \lambda\left(v - \frac{t}{2}\right) - \frac{N\ \alpha\ (1-\lambda)}{2} - N\beta$. Some calculations yield that:

$$W^B - W^A = \frac{N\left[\lambda t - 2\alpha(1-\lambda)\right]}{8\alpha(1-\lambda)}\left[\lambda(4v-3t) - 4\beta - 2\alpha(1-\lambda)\right].$$

The first step of the proof is to show that, if $\overline{y} > \frac{1}{3}$, i.e. $\alpha < \frac{3\lambda t}{2(1-\lambda)}$, then $W^B < W^A$. Given $\overline{y} < 1$, we have that $[\lambda t - 2\alpha(1-\lambda)] < 0$, so $W^B < W^A \iff [\lambda(4v-3t) - 4\beta - 2\alpha(1-\lambda)] > 0$, that is, $\alpha < \frac{\lambda(4v-3t)-4\beta}{2(1-\lambda)}$. We now prove that $\alpha < \frac{3\lambda t}{2(1-\lambda)}$ implies $\alpha < \frac{\lambda(4v-3t)-4\beta}{2(1-\lambda)}$, that is, $\frac{3\lambda t}{2(1-\lambda)} \leq \frac{\lambda(4v-3t)-4\beta}{2(1-\lambda)}$, i.e. $\lambda(4v-6t) - 4\beta \geq 0$. To this end, we simply note that $v \geq 2t$, so $\lambda(4v-6t) - 4\beta \geq \lambda 2t - 4\beta > 0$, given that $A < \frac{\lambda t}{2}$, and our assumption that $\beta < A$ implies $\beta < \frac{\lambda t}{2}$.

Next, given that, under the regulation, firms cannot use direct advertising without permission, if they find it optimal to use opt-in advertising it is because there is no profitable deviation, which, according to proposition 1, means that $A \geq \lambda(v-t)\left(1 - \frac{3\overline{y}}{4}\right) + \frac{\beta\overline{y}}{2}$. Therefore, considering that $A < \frac{\lambda t}{2}$, we obtain

$$\frac{\lambda t}{2} > A \geq \lambda(v-t)\left(1 - \frac{3\overline{y}}{4}\right) + \frac{\beta\overline{y}}{2},$$

and, given $v \geq 2t$, this yields

$$\frac{\lambda t}{2} > \lambda t\left(1 - \frac{3\overline{y}}{4}\right) + \frac{\beta\overline{y}}{2},$$

that is $\overline{y} > \frac{2\lambda t}{3\lambda t - 2\beta} > \frac{2}{3}$. In summary, if, under the regulatory ban of direct advertising without permission, firms find it optimal to use opt-in advertising, we know that $\overline{y} > \frac{2}{3}$ and this condition directly implies $W^B < W^A$. ■

Proof. Proposition 6. We first note that the condition $p_j^0 > 0$ with direct advertising without permission implies that $p_j^0 > 0$ also with opt-in advertising, whereas the condition $p_j^0 = 0$ with opt-in advertising implies that $p_j^0 = 0$ also with direct advertising without permission. Having clarified this, if $p_j^0 > 0$ with direct advertising without permission, propositions 1 and 3 indicate that the levels of profit with the two types of direct advertising coincide and, therefore, a ban on direct advertising without permission generating opt-in advertising does not affect profits. By contrast, if $p_j^0 = 0$ with opt-in advertising, then profits with opt-in advertising are, $\Pi_j^T = -NA + \delta\left(v - t - \frac{\beta}{\lambda}\right)\frac{N\lambda\overline{y}}{2}$ whereas, under direct advertising without permission, they are $\Pi_j^T = -NA + \delta\left(v - t - \frac{\beta}{\lambda}\right)\frac{N\lambda}{2}$. Given that $\overline{y} < 1$, we have that a ban on direct advertising without permission, which yields opt-in advertising, results in lower firm profits. ■

Proof. Proposition 7. Taking into account that $\overline{y} = \frac{\lambda t}{2\alpha\ (1-\lambda)}$, and considering $\delta = 1$, when $p_j^0 = 0$ with opt-in advertising, total consumer surplus under opt-in advertising is $CS^B = N(v - \frac{t}{4}) + N\frac{\lambda^2 t^2}{8\alpha(1-\lambda)}$ and, under direct advertising without permission, is $CS^A = N(v - \frac{t}{4}) + N\frac{\lambda t}{2} - N\frac{(1-\lambda)\alpha}{2}$

and some calculations yield:

$$CS^B - CS^A = \frac{N\left[\lambda t - 2\alpha(1-\lambda)\right]^2}{8\alpha(1-\lambda)} > 0.$$

If $p_j^0 > 0$ with direct advertising without permission, proposition 6 indicates that profits with direct advertising without permission and opt-in advertising coincide, whereas proposition 5 implies that $W^B < W^A$. Given that total welfare is the sum of profits and consumer surplus, we can conclude that $CS^B < CS^A$. ■