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Mass *versus* Direct Advertising and Product Quality

Lola Esteban* José M. Hernández†

Abstract This paper analyzes how the use of mass *vs.* direct advertising can affect the pattern of price and quality competition in a market where two firms compete with vertically differentiated products. We show that, compared to the case where sellers employ only mass advertising, the use of database advertising based on historical sales records improves the competitive position of the low-quality firm, which achieves a larger market share and can obtain higher profits. As a result, the high-quality firm lowers the supply of quality, which decreases the degree of product differentiation in the market and triggers strong price competition, thus decreasing its profits and increasing consumer surplus. Finally, we show that, although database advertising is more cost-efficient than mass advertising, the market distortion in the provision of quality implies that the use of direct advertising can yield a welfare loss.

Keywords database advertising, vertical differentiation

JEL Classification D43; L13; M37

*First author. University of Zaragoza. Departamento de Análisis Económico, Facultad de Economía y Empresa, Gran Vía 2, 50.005 Zaragoza, Spain, E-mail: lesteban@unizar.es.

†Corresponding author. University of Zaragoza. Departamento de Análisis Económico, Facultad de Economía y Empresa, Gran Vía 2, 50.005 Zaragoza, Spain, E-mail: hernande@unizar.es.

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1. INTRODUCTION

The successful launching of new products is the result of an integrated process that involves several aspects of firms' marketing plan, among which the following three are noteworthy: first, the "*product design strategy*" includes the specification of the features of the new goods, including quality levels, second, the "*pricing strategy*" determines a price for the goods and, finally, the "*promotion strategy*" specifies an advertising media plan which helps sellers to create consumer awareness and inform their potential clients about the characteristics and price of their new products. Regarding the advertising media planning strategy, firms have traditionally used the mass communication media to inform potential clients about their new products. However, the huge proliferation of new communication channels has induced a progressive fragmentation of audiences so, nowadays, sellers find it difficult and expensive to efficiently reach their target consumers with the mass media. Faced with this scenario, marketers are increasingly reallocating their advertising budgets towards specialized communication channels (cable TV, local radios or TV stations, specialized magazines, the Internet, etc.), which allows them to reach particular market segments with a higher effectiveness and at a lower cost. However, many sellers find that, in order to reach their potential clients, the specialized advertising media available have a low effectiveness, and mass advertising is too expensive, so they must look for alternatives and direct advertising is often the solution.[1] In particular, and considering the old marketing adage of "it is easier to sell something to an existing customer than make a new one", marketers sometimes build a database with the contact details of their *existing* customers and use it to send them *direct* advertising with information about new products or services. Consequently, it is interesting to analyze how the use of mass *vs.* database advertising can affect market performance and, especially, products' prices and quality levels.

To address these issues, we formulate a model of price competition in which two firms sell vertically differentiated products. Potential buyers are unaware of the existence and characteristics of the goods and firms use informative advertising to promote sales. We analyze price-quality-advertising competition (with mass *vs.* direct advertising based on historical sales records) by assuming that firms launch a succession of new products over time aimed at the same group of potential consumers and play the following three-stage game. In $t = 0$, sellers compete by setting the quality level of their products. In $t = 1$, firms do not have

[1]According to the US Direct Marketing Association (DMA), in 2011, marketers spent $163 billion on direct marketing, which accounted for 52.1% of all ad expenditures in the United States (see DMA's "Power of Direct Marketing Report").

the information necessary to target their ads, so they reach consumers by using a TV or radio mass advertising campaign that covers the entire market and informs potential buyers about the existence, price and characteristics of their first-period products. In $t = 2$, both sellers launch a new product and we study market competition under two advertising scenarios: (i) firms do not have their potential consumers' contact information available, so they again use the mass media to inform them about their new products, and (ii) firms can build a database with the contact information of their first-period buyers and directly target the advertising campaign to these potential consumers, which provides sellers a low-cost, efficient alternative for reaching their markets. In this framework, the goal of this paper is to analyze how the use of these two advertising strategies (mass *vs.* database advertising) can affect products' prices, quality levels, profits, consumer surplus and social welfare.

We begin the analysis by considering the case in which firms use the mass media in $t = 1$ and $t = 2$. This case, which equals the full-information outcome, constitutes a reasonable benchmark against which we can compute the impact of database advertising on the market outcome. Later, we analyze market competition when, in $t = 2$, firms can use database advertising. Regarding prices, the key point is that the use of an in-house list with the contact information about existing clients allows sellers to target their direct advertising campaigns to a distinct set of consumers, which fragments the market into local monopolies due to the informational differentiation that arises. Firms strategically anticipate this effect so, compared to the benchmark case of mass advertising, they compete more aggressively for consumers in $t = 1$, which generates a downwards pressure on the first-period prices. The most interesting effect of this change in the pattern of intertemporal price competition is that the low-quality firm has a stronger incentive to lower the price, so the use of direct advertising allows this firm to increase its market share in both $t = 1$ and $t = 2$. Further, the improved competitive position of the low-quality firm induces the high-quality seller to reduce the supply of quality. As a result, we find that the degree of product differentiation falls, thus further increasing the intensity of price competition in $t = 1$. This explains why, compared to the solution with mass advertising, database advertising yields lower profits to the high-quality firm, whereas the impact on the low-quality firm's profits is ambiguous. We also find that, although, with targeted advertising the market provision of quality falls and buyers pay the monopoly price in $t = 2$, these negative effects on consumers are outweighed by the positive impact of the strong price competition in $t = 1$ so, compared to the benchmark case, consumer surplus increases. Finally, database advertising is more cost-efficient than mass advertising, but we show that the lower supply of quality associated with direct advertising can lead to a welfare loss.

All these results have interesting implications for policy and business practice. Direct marketing is attractive to many marketers because the use of internal databases makes information management easy and cheap. Further, compared to mass advertising, relationship marketing is traditionally associated with less price-sensitive customers and higher profits. Our work confirms that direct advertising allows both firms to increase their market power, but competition for second-period profits generates a strong first-period price war for market share, which is reinforced by a reduction in the equilibrium degree of product differentiation. As a result, and contrary to the traditional view, we find that direct advertising could yield lower profits to *both* firms. This means that firms can engage in a typical prisoner's dilemma, that is, both sellers could be better off by using only mass advertising but, in order to save advertising costs, they have a strong incentive to use their databases in the second period which, indirectly, generates a price war that can yield lower overall profits. Finally, from a policy perspective, it is clear that database advertising increases cost efficiency. However, due to privacy considerations in the management of personal data, this information technology could impose nuisance costs on consumers, so the policy approach towards direct advertising is currently controversial.[2] Whilst a comprehensive treatment of this issue falls outside the scope of this paper, our work provides a new aspect to this controversy. We show that the use of a database advertising technology generates a distortion in the market provision of quality, with the corresponding welfare loss, which suggests that a welfare evaluation of direct advertising should take these negative quality effects into account.

The analysis of strategic competition with direct advertising has received little attention in the literature and, to the best of our knowledge, only the works of Shaffer and Zhang (1995), Roy (2000) and Esteban and Hernández (2014, 2017a, 2017b) have addressed this issue. These papers study the relationship between only *two variables, price and promotion*, focusing on how direct advertising can affect the pattern of price competition between firms. Shaffer and Zhang (1995) analyze the case in which firms have a fixed-size database containing precise information on consumers, which allows them to both locate and classify these customers according to their brand loyalty. Given this information, these authors show that direct advertising allows firms to price discriminate by

[2]For example, the EU is clearly concerned with the defense of consumer privacy in commercial relationships, whereas the US maintains a permissive use of commercial information. See Esteban and Hernández (2017b) for a discussión about the welfare and policy implications of database advertising.

way of coupons, which stimulates competition in the market. Roy (2000) places the analysis within a spatial framework in which firms can send their ads directly because they are fully informed about the physical location of all consumers. In this context, the use of direct advertising can lead to market fragmentation and the creation of local monopolies. We note that the results provided by Shaffer and Zhang (1995) depend on the information contained in the database, and that the origin of this information is exogenous. Whilst Roy's approach provides an answer to the question of how to locate potential consumers, his model applies only to a spatial context in which consumers have homogeneous tastes. In Esteban and Hernández (2014), we study database advertising based on historical sales records in a context of heterogeneous tastes and analyze the firms' optimal pricing and advertising strategies when sellers face a totally *inelastic* demand and compete with *horizontally* differentiated products. In this setup, we find that, compared to mass advertising, direct advertising yields higher advertising levels, an intertemporal reallocation of market power and a higher level of social welfare. Esteban and Hernández (2017a) and Esteban and Hernández (2017b) leave out the analysis of optimal advertising levels from the model and, considering that firms compete in prices with an *elastic* demand, extend the study of direct advertising based on historical sales records in two directions. The first work studies the conditions under which market interaction yields perfect targeting (i.e. all the informed potential consumers buy a product) or imperfect targeting (some of them do not buy any product) and how perfect vs. imperfect targeting affects the functioning of a market. This analysis reveals that imperfect targeting generates lower intertemporal market prices and quantity distortions so that, compared to mass advertising, the use of direct advertising can be socially detrimental. The second work addresses the policy aspects of database advertising. In particular, the paper studies how a regulatory policy imposing an opt-in provision on the use of database advertising (under which firms must ask consumers for their consent to send them ads with information about new products) affects consumers, firms and social welfare. The analysis suggests that such regulation lowers social welfare. The current paper considers that firms compete with an *elastic* demand and *perfect* targeting and contributes to the existing literature by introducing an *additional variable*, *quality*, into the analysis. This allows us to study (i) how the use of database advertising can affect the pattern of price-quality competition in an oligopolistic market and (ii) how quality-competition influences the effect that direct advertising has on profits, consumer surplus and social welfare. As a result, by considering *endogenous* product quality, we provide a wider view of the basic interactions between pricing and direct advertising and offer some new insights (e.g., direct advertising can lower the degree of product differentiation and social welfare) about how database advertising can affect the functioning of the markets.

The remainder of the work is organized as follows. Section 2 sets out the model and describes the optimal firms' pricing and product design strategies under mass *vs.* direct advertising. Section 3 discusses the market and welfare implications of the use of direct advertising. Finally, Section 4 contains some concluding remarks. All the proofs are relegated to an Appendix.

2. THE MODEL: EQUILIBRIUM STRATEGIES

We consider two firms, $j = 1,2$, competing with vertically-differentiated products. In order to accommodate database advertising based on historical sales records into the model, we assume that, over a period of time, each firm launches a line of new products, with quality $s_j \geq 0$, aimed at the same group of potential consumers. In particular, we consider a game where each firm launches one new good in $t = 1$ and, within the same product line (i.e. with the same quality level), another good (with different characteristics) in $t = 2$, and compete in prices, (p_1^t, p_2^t), $t = 1,2$, for a group of potential consumers. The market is comprised of a unitary mass of potential buyers who demand, at most, one unit of a product per period. A consumer's utility in period t is $U = v^t + \theta s_j - p_j^t$ when he buys a good of quality s_j at a price p_j^t, and 0 if he does not buy. The parameter $v^t > 0$ represents consumers' common valuation of the product, independent of the level of quality and, for simplicity, we assume that $v^1 = v^2 = v$. Consumers are heterogeneous in their valuation of quality and, following Tirole (1988), we assume that the parameter θ of taste for quality is uniformly distributed across the population of consumers in the interval $[a,b]$, with $b - a = 1$.

Regarding the information structure of the model, we consider that consumers are endowed with preferences over product attributes but, without advertising, they are unaware of the existence of the goods or their characteristics.[3] Advertising provides information about the existence of a new product and its characteristics, including price and quality, so a consumer can (1) learn the product attributes, (2) evaluate the degree of preference for the good and (3) decide whether to buy it or not. Initially, sellers do not have the information necessary to target their ads so, in $t = 1$, they reach consumers by using a TV or

[3] We assume that consumers' search cost is high relative to the expected surplus offered by the goods so, in the absence of information, consumers do not purchase any good (see, for example, Grossman and Shapiro, 1984; Stegeman, 1991; Stahl, 1994).

radio mass advertising campaign which spreads the ads to the entire population of potential buyers at a cost $C_A = A$. The fundamental feature of our model is that the goods produced by the two firms in $t = 1$ and $t = 2$ are orientated to the same group of potential consumers, so a firm has a high incentive to foster a direct relationship with the consumers who purchased its product in $t = 1$. However, this is not always possible so, regarding the advertising technology available in $t = 2$, we will analyze two scenarios. The first assumes that sellers have direct access to the contact information (name, address, telephone number, etc.) of their first-period buyers, e.g. because the products are delivered to their homes, so they can compile this information in a database which, later, is used to send them direct ads informing them about the characteristics and price of the new products. The second scenario considers that firms have no access (or difficult access) to the contact information of first-period buyers, e.g. because the purchases are made in a traditional shop and, for privacy considerations, a large fraction of consumers refuse to give sellers their contact information,[4] so they reach consumers with a new mass advertising campaign. We consider that the cost of a direct advertising campaign is proportional to the size of the market segment covered with ads so, for example, if a firm informs one third of a market of size 1, the cost of the campaign is $C_A = \beta\frac{1}{3}$, where β denotes, for example, the mailing cost per ad. Finally, according to the empirical evidence, which suggests that mass advertising is nowadays very expensive, we also assume that reaching consumers with database advertising is cheaper than with mass advertising, i.e. $\beta < A$.

The timing of the game is as follows. In $t = 0$, both firms compete in the design of their line of products by simultaneously choosing their supply of quality, (s_1, s_2). The value of s_j determines the quality level of the two goods produced by firm j in $t = 1$ and $t = 2$, and we assume that the cost of developing a line of products with quality s_j is $C_j = \alpha s_j^2$. In $t = 1$, firms launch a mass advertising campaign informing potential buyers about the existence, price and characteristics (including quality) of their first-period products, and compete in prices, (p_1^1, p_2^1) for the unit mass of potential buyers. Again, following Tirole (1988), we assume that the unit cost of production for both qualities[5] is c and we nor-

[4]Esteban and Hernández (2017b) analyzes when consumers allow fims the use of their contact information.

[5]The model and the results can be extended to the case in which producing higher quality is more expensive. However, this extension does not provide any new interesting insight to our work so, for analytical simplicity, we consider symmetric production costs. This means that if, for example, $s_2 > s_1$, firm 2 makes a one-time fixed-cost investment, which allows it to produce the high-quality product at the same cost as the low-quality product.

malize this marginal cost to zero. In $t = 2$, firms decide their advertising strategy, mass or database advertising (if available), to inform potential buyers about the existence, price and characteristics of their second-period products, and compete in prices, (p_1^2, p_2^2), for the same unit mass of potential buyers. We look for subgame-perfect Nash equilibria (SPNE) of the game and, for the sake of simplicity, we focus the analysis on *pure-strategy equilibria*.

We begin by studying the benchmark case of mass advertising, i.e. when both firms use only the mass media in $t = 1$ and $t = 2$. In this case, the model does not have intertemporal effects, so sellers face the same competitive scenario in $t = 1$ and $t = 2$, thus yielding the same market outcome in both periods. To obtain the equilibrium, we note that a mass advertising campaign reaches all potential buyers, so consumers have full information about the firms' products. In period $t = 1,2$, a consumer with a taste parameter θ achieves a utility $U = v + \theta s_1 - p_1^t$ from buying firm 1's product and a utility $U = v + \theta s_2 - p_2^t$ from buying firm's 2 product so, considering $s_2 > s_1$, the consumer with a taste parameter $\overline{\theta}^t$, with $\overline{\theta}^t = \frac{p_2^t - p_1^t}{\Delta s}$, $\Delta s = s_2 - s_1 > 0$, is indifferent between buying the two products. Accordingly, firms 1 and 2 face a demand[6] $x_1^t = \left(\overline{\theta}^t - a\right)$ and $x_2^t = \left(b - \overline{\theta}^t\right)$, respectively, and they compete by choosing the optimal prices, (p_1^m, p_2^m), where the superscript m means the solution under mass advertising. Finally, in $t = 0$, firms choose (s_1^m, s_2^m) to maximize total intertemporal profits. We assume that sellers consider a common discount rate, δ, and, for simplicity, we assume that $\delta = 1$. Proposition 1 describes an equilibrium of the game.[7]

Proposition 1. *Let us assume that firms use mass advertising in $t = 1$ and $t = 2$. Then, if $v \geq \frac{(2+a)^2(1-a)}{27\alpha}$, and $A < \frac{\left(2-a-a^2\right)^2}{81\alpha}$ the game has the following SPNE:*

(i) in $t = 0$, firms set $s_1^m = 0$ and $s_2^m = \frac{(2+a)^2}{9\alpha}$,

(ii) in $t = 1,2$, firms charge a price $p_1^m = \frac{(2+a)^2(1-a)}{27\alpha} < p_2^m = \frac{(2+a)^3}{27\alpha}$, serve a demand $x_1^m = \frac{1-a}{3} < x_2^m = \frac{2+a}{3}$, and obtain an overall profit $\Pi_1^m = \frac{2\left(2-a-a^2\right)^2}{81\alpha} - 2A < \Pi_2^m = \frac{(2+a)^4}{81\alpha} - 2A$.

[6]We focus the analysis on the case in which, in equilibrium, the market is covered and firms obtain positive profits. This implies (see proposition 1) that (i) $x_1^t > 0$, i.e. $a < 1$, (ii) $v - p_1^m \geq 0$, i.e. $v \geq \frac{(2+a)^2(1-a)}{27\alpha}$, and (iii) firm 1 achieves positive profits, i.e. $A < \frac{\left(2-a-a^2\right)^2}{81\alpha}$.

[7]Regarding proposition 1 (and proposition 2), we note that the game has two "similar" pure-strategy Nash equilibria, and the second one can be obtained by simply reversing the firms' indices. If one of the firms entered first into the market (preemption game with secuential choice of quality) that firm would choose the high quality (see Tirole, 1988).

From Proposition 1 it follows that, under mass advertising, the market outcome yields the standard results of competition in vertically-differentiated markets, that is, (i) compared to the low-quality firm, the high-quality firm enjoys a dominant position in the market with a higher market share, price and profit, and (ii) firm 1 supplies the minimum quality level, i.e. maximum quality differentiation.

Next, we analyze the optimal quality-pricing strategies when firms have database advertising available. We note, first, that, in this scenario, the game does not have an equilibrium where firms use only mass advertising. This occurs because, if both firms employed mass advertising in $t = 2$, the use of the database would allow a firm to reach the same consumers at a lower cost, i.e. there is a profitable deviation.[8] Therefore, in a pure-strategy SPNE, firms will use only database advertising.[9] We begin by solving the game in $t = 2$. For a given values of (s_1, s_2) and first-period prices (p_1^1, p_2^1), in $t = 1$, both firms serve a demand $x_1^1 = \left(\frac{p_2^1 - p_1^1}{\Delta s} - a\right)$ and $x_2^1 = \left(b - \frac{p_2^1 - p_1^1}{\Delta s}\right)$. Firms compile the contact information of these clients in a database and, in $t = 2$, each firm uses this information to send them a direct ad with information about the price and characteristics of its new product. Given that a seller's database contains information *only* about its first-period clients, the use of direct advertising based on previous sales records allows firms to target their direct ads to a distinct set of consumers. As a result, in $t = 2$, all consumers become captives of one of the sellers, that is, direct advertising fragments the market into local monopolies. Accordingly, and assuming that each consumer is identified in $t = 1$ and $t = 2$ by the same point on the unit interval,[10] firms 1 and 2 face a demand function $x_1^2 = Min\left[\frac{p_2^1 - p_1^1}{\Delta s} - a; \frac{p_2^1 - p_1^1}{\Delta s} - \frac{p_1^2 - v}{s_1}\right]$, $x_2^2 = Min\left[b - \frac{p_2^1 - p_1^1}{\Delta s}; b - \frac{p_2^2 - v}{s_2}\right]$, set a price $\widehat{p}_j^2 = \arg\max\left[p_j^2 x_j^2 - \beta x_j^1\right]$ and obtain a profit $\Pi_j^2 = \widehat{p}_j^2 x_j^2 - \beta x_j^1$. For the sake of

[8]It is also straightforward to check that the game does not have a pure-strategy equilibrium in which one firm uses mass advertising and the other database advertising. Under these advertising strategies, there is informational differentiation in one local market and firms would find it optimal to play a *mixed* pricing strategy, which combines high prices (when the firm that uses mass advertising chooses to monopolize its local market) with low prices (when the same firm chooses to compete for the rival's market share).

[9]Notice that, given that mass advertising covers the entire market, it is never optimal to use mass *and* direct advertising simultaneously.

[10]This assumption means that the inclination of previous customers to continue to buy from a specific firm is high, so firms compete under high intertemporal consumer loyalty. Esteban and Hernández (2017a) extends this analysis by studying price-advertising competition when the location of consumers along the line in $t = 2$ is independent of their locations in $t = 1$, so firms compete under low consumer loyalty.

simplicity, we assume that the firms' pricing strategies do not generate quantity distortions, that is, consumers' valuation of the products is sufficiently high so that, under monopoly pricing, it holds that[11] $x_1^2 = \frac{p_2^1 - p_1^1}{\Delta s} - a$ and $x_2^2 = b - \frac{p_2^1 - p_1^1}{\Delta s}$. In $t = 1$, each firm chooses the price p_j^1 to maximize the discounted value of the total expected profit $Max_{p_j^1}\ \Pi_j^{T1} = p_j^1 x_j^1 - A + \left[\widehat{p}_j^2 x_j^2 - \beta x_j^1\right]$. Finally, taking into account the optimal first and second-period prices, $(\widehat{p}_1^1, \widehat{p}_2^1)$, $(\widehat{p}_1^2, \widehat{p}_2^2)$, in $t = 0$, each firm sets the quality level that maximizes total intertemporal profits, Max_{s_j} $\Pi_j^T = \widehat{p}_j^1 x_j^1 - A + \left[\widehat{p}_j^2 x_j^2 - \beta x_j^1\right] - \alpha s_j^2$.

From the above discussion, it follows that database advertising can fragment the market into local monopolies. However, if, for example, in $t = 2$, firm 2 targets the advertising and charges the monopoly price, it is clear that firm 1 could react by extending the reach of its advertising campaign (using the mass media) and undercutting the price, in order to poach some consumers from the rival's database. The key issue then is whether, in equilibrium, the market can be fragmented, that is, if both firms can find it optimal to target the ads and charge the monopoly price. The following proposition addresses this issue.

Proposition 2. *If* $\frac{4(3-a-2a^2)^2}{625\alpha} > A \geq \overline{A}$ *(see the Appendix),* $\frac{2(1-a)(3+2a)^2}{125\alpha} + \beta \geq v \geq \frac{(1-a)(3+2a)^2}{125\alpha}$*, and* $a \leq \frac{4}{9}$*, the game has the following SPNE:*

(i) in $t = 0$*, firms set the quality levels* $s_1 = 0$*,* $s_2 = \frac{(3+2a)^2}{25\alpha}$*,*

(ii) in $t = 1$*, firms charge positive prices* $p_1^1 = \frac{2(1-a)(3+2a)^2}{125\alpha} - v + \beta < p_2^1 = \frac{(3+2a)^2(4+a)}{125\alpha} - v + \beta$ *and serve a demand* $x_1^1 = \frac{2(1-a)}{5} < x_2^1 = \frac{3+2a}{5}$*,*

(iii) in $t = 2$*, firms use direct advertising, charge the monopoly prices,* $p_1^2 = v < p_2^2 = v + \frac{(3+2a)^2(2+3a)}{125\alpha}$*, and the overall intertemporal profit is* $\Pi_1^T = \frac{4(3-a-2a^2)^2}{625\alpha} - A < \Pi_2^T = \frac{(3+2a)^4}{625\alpha} - A$.

Proposition 2 describes an equilibrium of the game where, in $t = 0$, firm 1 chooses the minimum level of quality. Further, the proposition confirms that database advertising arises in a market scenario where the use of mass advertising is expensive ($A \geq \overline{A}$), which is consistent with the current advertising landscape where (i) the progressive fragmentation of audiences makes it very expensive to reach the targeted consumers by using the mass-communication media

[11] We again focus the analysis on the case in which, in equilibrium, the market is covered and firms obtain positive profits. These assumptions imply that (i) $x_1^t > 0$, i.e. $a < 1$, (ii) $v \geq \frac{(1-a)(3+2a)^2}{125\alpha}$, and (iii) firm 1 achieves positive profits, i.e. $A < \frac{4(3-a-2a^2)^2}{625\alpha}$ (see the Appendix and proposition 2).

and (ii) direct advertising is extensively used. In this context, it is too costly to poach consumers from the rival's database by launching a new mass advertising campaign in $t = 2$, so neither firm has an incentive to deviate from monopoly pricing.[12] Accordingly, the use of database advertising implies, on the one hand, that the firms achieve a monopolistic position in their local markets and, on the other, that advertising cost efficiency increases, which helps marketers to achieve high profits in $t = 2$. Given that the level of second-period profits depends on the number of consumers in the database, firms have a strategic incentive to lower first-period prices in order to increase their customer base. This incentive is positively related to the profit achieved in $t = 2$ which, in turn, is positively related to v, which explains that an equilibrium where firms set positive prices in $t = 1$ exists only if v is sufficiently low. Finally, the prospect of enjoying a monopolistic position in $t = 2$ might give the low-quality firm an incentive to increase the quality of its line of products, so the equilibrium with $s_1 = 0$ exists if a is sufficiently low. This means that proposition 2 does not provide a full characterization of all the pure-strategy equilibria of the game. If a is sufficiently large, in equilibrium, firm 1 may find it optimal to provide a positive level of quality, $s_1 > 0$. In this case, the optimization problems are analytically intractable, so it is not possible to provide an explicit solution of the game. However, extensive numerical simulations of the model indicate that, for a small set of parameter values, there exists a pure-strategy equilibrium in which $s_1 > 0$, and the interesting point is that the functioning of the market is similar to that described in proposition 2, that is, it holds that $s_2 > s_1 \approx 0$ and both firms charge the monopoly price in $t = 2$. On this basis, we consider that the equilibrium described in proposition 2 is a good reference point in order to illustrate how the use of direct advertising can affect market performance. The following section carries out this analysis.

3. MARKET AND WELFARE IMPLICATIONS OF MASS *VS.* DIRECT ADVERTISING

In this section, we wonder, first, how mass *vs.* direct advertising affects the functioning of the market and, second, how these advertising strategies affect firms, consumers and the level of social welfare. Regarding the comparative static properties of the model, we note that the parameter $a \in [0, 1)$, which measures the degree of heterogeneity in the taste for quality, determines both the

[12] If $A < \overline{A}$, the equilibrium must be in *mixed strategies* with firms combining, within a planning period, the use of mass *vs.* direct advertising with competitive *vs.* monopoly pricing. The analysis of optimal pricing when firms combine the use of mass and direct advertising falls outside the scope of the current work and is addressed in Esteban and Hernández (2014).

consumers' willingness to pay for quality and the competitive pressure in the market, so it is particularly interesting to study how changes in this parameter affect the pattern of competition between firms. It is easy to check that $\frac{\partial s_2^m}{\partial a} > 0$, $\frac{\partial p_1^m}{\partial a} < 0$, $\frac{\partial x_1^m}{\partial a} < 0$, $\frac{\partial \Pi_1^m}{\partial a} < 0$, $\frac{\partial p_2^m}{\partial a} > 0$, $\frac{\partial x_2^m}{\partial a} > 0$,$\frac{\partial \Pi_2^m}{\partial a} > 0$ so, with mass advertising, a higher level of competition allows the high-quality firm to raise prices and to increase its market share and profits, whereas the effect of a higher a on the low-quality firm is the opposite.

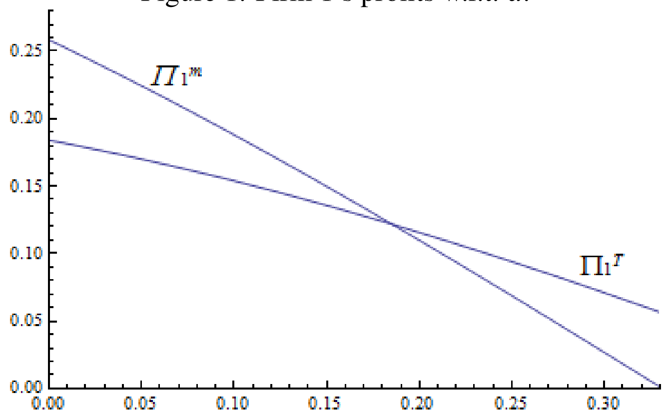
Turning to the market equilibrium with direct advertising, it is straightforward to prove that $\frac{\partial s_2}{\partial a} > 0$, $\frac{\partial p_1^1}{\partial a} = \frac{2(3+2a)(1-6a)}{125\alpha} \lesseqgtr 0$, $\frac{\partial p_1^2}{\partial a} = 0$, $\frac{\partial x_1^t}{\partial a} < 0$, $\frac{\partial \Pi_1^T}{\partial a} < 0$, $\frac{\partial p_2^1}{\partial a} > 0$, $\frac{\partial p_2^2}{\partial a} > 0$, $\frac{\partial x_2^t}{\partial a} > 0$, $\frac{\partial \Pi_2^T}{\partial a} > 0$. Therefore, we find that, in qualitative terms, the use of direct advertising only changes the way in which an increase in a affects firm 1's prices. In $t = 2$, firm 1 charges the monopoly price and, given the equilibrium level of quality, $s_1 = 0$, this price does not depend on the parameter a. Interestingly, we find that p_1^1 is non-monotonic with the parameter a and reaches a maximum when $a = \frac{1}{6}$. This occurs because a higher competitive pressure, i.e. an increase in a, generates two effects: first, it directly decreases the degree of heterogeneity in consumers' taste for quality, thus increasing price competition in $t = 1$ and, second, a higher a increases the quality differential (Δs), i.e. the level of product differentiation, thus softening price competition. When $a < \frac{1}{6}$, under direct advertising, the latter effect is dominant so, in equilibrium, it holds that $\frac{\partial p_1^1}{\partial a} > 0$. Obviously, the fact that a higher a is associated with higher prices comes, in part, from the fact that this parameter reflects the consumers' willingness to pay for quality. However, at the same time, our model suggests that, under database advertising based on historical sales records, a higher competitive pressure can yield a general increase in market prices.

Next, we compare proposition 1 and 2 in order to illustrate how the use of mass *vs.* direct advertising can affect firms.

Proposition 3. *Compared to mass advertising, database advertising yields a lower level of quality in the market, $s_2 < s_2^m$, a higher second-period price and a lower first-period price for firm 1, $p_1^2 > p_1^m$, $p_1^1 < p_1^m$, a higher (vs. lower) market share for firm 1 (vs. firm 2), $x_1^t > x_1^m$, $x_2^t < x_2^m$, and, finally, lower profits for firm 2, $\Pi_2^T < \Pi_2^m$.*

Proposition 3 indicates that, compared to the benchmark case (where firms can use only mass advertising), database advertising results in a *reallocation* of firm 1's market power from the first to the second period. The most interesting result is that the low-quality firm has a strong incentive to lower the first-period price so, in equilibrium, firm 1 (vs. firm 2) achieves a higher (vs. lower) market

Figure 1: Firm 1's profits w.r.t. a.



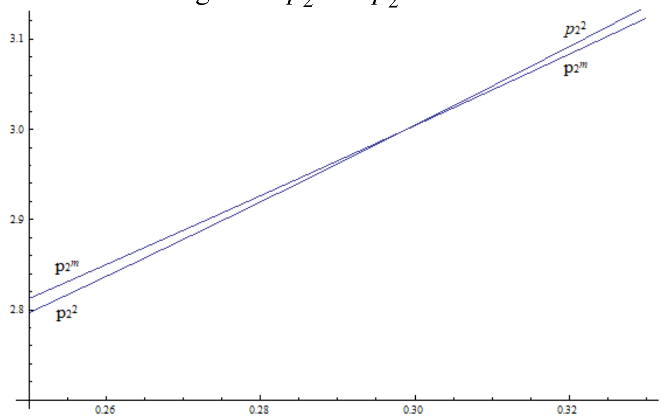
share. The improved competitive position of firm 1 lowers firm 2's incentive to invest in quality, $s_2 < s_2^m$, so we find that the use of database advertising yields a lower level of differentiation (and quality differential, Δs) in the market, which further stimulates price competition in $t = 1$. All this explains why, although, in $t = 2$, the firms enjoy a monopolistic position in their local markets, the use of targeted advertising yields a lower overall profit for firm 2.[13] By contrast, firm 1 benefits from its higher market share, so the impact of targeting on the low-quality firm's profit is, a priori, ambiguous.

Figure 1 displays, for a given market scenario ($v = 1, A = 0.2, \beta = 0.05, \alpha = 0.15$), firm 1's profits under both mass and database advertising and indicates that, when the competitive pressure is sufficiently high, i.e. a is large, database advertising is an effective tool to soften price competition, thus increasing the firm's profits. However, if a is relatively low (i.e. weak competitive pressure), the use of direct advertising yields lower profits for both firms. This means that firms can engage in a typical prisoner's dilemma, that is, both sellers are better off by using only mass advertising but, in order to save advertising costs, they have a strong incentive to use their databases in $t = 2$ which, indirectly, generates more intense price competition in $t = 1$ and, finally, lower overall profits.

Regarding firm 2's prices, we note that, although the use of direct advertising allows the firm to enjoy a monopolistic position in $t = 2$, compared to the mass advertising outcome, firm 2 has a lower market share and provides a lower

[13] We note that the numerical simulation of the model for the case in which $s_2 > s_1 > 0$ yields the same results described in proposition 3.

Figure 2: p_2^2 and p_2^m w.r.t. a.



level of quality, which reduces both the demand and the consumers' willingness to pay for the high-quality product. This trade-off generates a remarkable result. Figure 2 shows that, if a is sufficiently low, the monopoly price is *lower* than the competitive price of the mass advertising outcome, i.e. $p_2^2 < p_2^m$. Finally, extensive numerical simulations of the model indicate that $p_2^1 < p_2^m$, so we find that, compared to the benchmark case of mass advertising, the use of direct advertising can lead the high-quality firm to set *lower intertemporal prices*. This result challenges the traditional marketing view, which relates relationship marketing with higher prices.

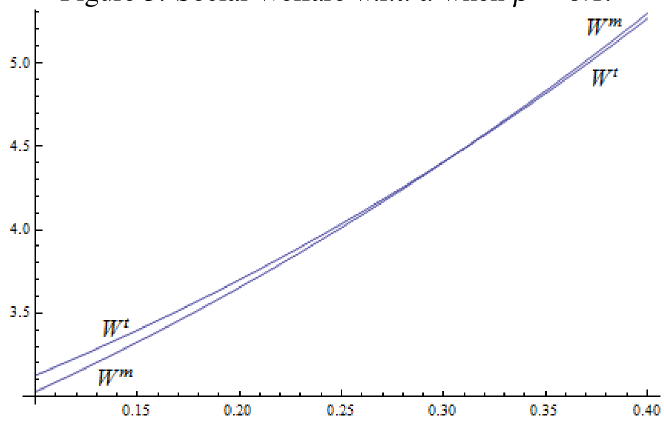
The next proposition addresses how database advertising affects consumers.

Proposition 4. *Compared to mass advertising, database advertising yields a higher level of consumer surplus.*

From the consumers' perspective, the use of direct advertising implies that (i) they pay lower first-period prices, (ii) a higher fraction of the market purchase the low-quality product, and (iii) the buyers of the high-quality product consume a lower level of quality. Proposition 4 indicates that the first effect dominates so, although direct advertising reduces the consumption of quality, the use of this information technology benefits consumers.

Our last goal is to evaluate mass *vs.* database advertising from a social welfare perspective. Taking into account that total welfare equals the value of the

Figure 3: Social Welfare w.r.t. a when $\beta = 0.1$.



goods for all buyers minus quality and advertising costs,[14]

$$W = 2\int_a^\theta (v + x\, s_1)\,dx + 2\int_\theta^b (v + x\, s_2)\,dx - \alpha s_1^2 - \alpha s_2^2 - C_A, \qquad (1)$$

we observe that, compared to the case in which firms use only mass advertising, database advertising has two effects on social welfare. On the one hand, welfare decreases because (i) firm 2 supplies lower quality and (ii) a higher proportion of consumers buy the low-quality product. On the other, database advertising reduces advertising costs, which increases welfare.

If the savings in advertising costs are not very large (for the previous market scenario, if $\beta > 0.071$), Figure 3 confirms, that either of these effects can be dominant, so the final effect on welfare depends on the market conditions. In particular, given that $\frac{\partial s_2}{\partial a} > 0$, when a is relatively large, the level of quality in the market is high, so the welfare loss associated with the lower consumption of the high-quality product is substantial. In this scenario, we find that social welfare with database advertising is lower than with mass advertising. Therefore, our model generates a striking result, namely, that the use of a more cost-efficient advertising technology might not be socially desirable.

It is instructive to compare these results with our previous work. In Esteban and Hernández (2014, 2017a), when firms use direct advertising with (i) horizontally differentiated products, (ii) perfect targeting and (iii) given product designs,

[14]Notice that, under mass adverising, $C_A = 4A$ whereas, under database advertising, $C_A = 2A + \beta$.

compared to the benchmark case of mass advertising, the use of this advertising technology increases social welfare, but it can decrease consumers' surplus. The present paper indicates that the use of direct advertising with *vertically* differentiated products, perfect targeting and *endogenous quality* competition yields the opposite results, that is, direct advertising always benefits consumers but it can yield a welfare loss.

4. CONCLUSIONS

This paper formulates a model of price competition with vertically differentiated products and find that, compared to the case in which firms use only mass advertising, the use of database advertising allows both firms to enjoy a monopolistic position in their local markets. This triggers a first-period price war for market share which favours the low-quality firm. The improved competitive position of the low-quality seller lowers the high-quality firm's incentive to invest in quality, so targeted advertising yields a lower level of product differentiation. All this has interesting implications about the pattern of price competition in the market where we find, for example, that, with direct advertising, the first-period price of the low-quality firm is non-monotonic in the level of competitive pressure or that, contrary to the traditional view of the existing literature, compared to the case of mass advertising, with targeting, the high-quality firm can charge lower intertemporal prices. This explains why, although direct advertising allows firms to increase their ability to exercise market power, this information technology yields a lower overall profit for the high-quality firm. By contrast, the low-quality seller benefits from its higher market share, so the impact of targeting on its profit is ambiguous. Finally, we show that the use of a database increases consumer surplus and that, although this technology is more cost-efficient than mass advertising, direct advertising distorts the market provision of quality and, as a result, can yield a welfare loss. In sum, our analysis shows that (i) the use of different advertising strategies, mass advertising *vs.* database advertising based on historical sales records, has a substantial impact on the firms' pricing and product design strategies, and (ii) the analysis of targeted advertising with *endogenous product quality* substantially changes the current view about how the use of direct advertising can affect the functioning of an oligopolistic market.

REFERENCES

[1] Esteban, L. and J.M. Hernández (2014), "Endogenous Direct Advertising and Price Competition," *Journal of Economics*, 112:3, 225-251.

[2] Esteban, L. and J.M. Hernández (2017a), "Perfect vs. Imperfect Direct Advertising, and Market Performance," *Journal of Economics* 122:1, 1-27.

[3] Esteban, L. and J.M. Hernández (2017b), "Direct advertising and opt-in provisions: Policy and market implications," *Information Economics and Policy* 39,15–25.

[4] Grossman G. and C. Shapiro (1984), "Informative Advertising with Differentiated Products," *Review of Economic Studies* 51, 63-82.

[5] Roy, S. (2000), "Strategic Segmentation of a Market," *International Journal of Industrial Organization* 18(8), 1279-1290.

[6] Shaffer G. and Z. L. Zhang (1995), "Competitive Coupon Targeting," *Marketing Science* 14, 395-416.

[7] Stahl, D. O. (1994), "Oligopolistic Pricing and Advertising," *Journal of Economic Theory* 64, 162-177.

[8] Stegeman, M. (1991), "Advertising in Competitive Markets," *American Economic Review* 81, 210-223.

[9] Tirole, J. (1988), *The Theory of Industrial Organization,* Cambridge, MA: MIT Press.

5. APPENDIX

Proof. **Proposition 1.** Under mass advertising, in $t = 1,2$, firm 1 faces the problem $Max_{p_1^t}\Pi_1^t = p_1^t\left(\frac{p_2^t-p_1^t}{\Delta s} - a\right) - A$, and firm 2 $Max_{p_2^t}\Pi_2^t = p_2^t\left(b - \frac{p_2^t-p_1^t}{\Delta s}\right) - A$. The first order conditions (FOC) are[15] $\frac{p_2^t-p_1^t}{\Delta s} - a - \frac{p_1^t}{\Delta s} = 0$ and $b - \frac{p_2^t-p_1^t}{\Delta s} - \frac{p_2^t}{\Delta s} = 0$, respectively, and the corresponding solution is $\widehat{p}_1^t = \frac{\Delta s(1-a)}{3}$, $\widehat{p}_2^t = \frac{\Delta s(2+a)}{3}$. Going back to $t = 0$, both firms choose the level of quality that maximizes the total intertemporal profit: $Max_{s_1}\Pi_1^T = 2\widehat{p}_1^t\left(\frac{\widehat{p}_2^t-\widehat{p}_1^t}{\Delta s} - a\right) - 2A - \alpha s_1^2$, $Max_{s_2}\Pi_2^T =$

[15] It is straightforward to check that the second-order conditions are always satisfied.

$2\widehat{p}_2^t\left(b-\frac{\widehat{p}_2^t-\widehat{p}_1^t}{\Delta s}\right)-2A-\alpha s_2^2$. Given that $\frac{\partial \Pi_1^T}{\partial s_1}=-\frac{2(1-a)^2}{9}-2\alpha s_1<0$, the solution is $s_1=0$, whereas the equation $\frac{\partial \Pi_2^T}{\partial s_2}=\frac{2(2+a)^2}{9}-2\alpha s_2=0$ determines the equilibrium value of s_2, $s_2^m=\frac{(2+a)^2}{9\alpha}$. Plugging this solution into $\widehat{p}_1^t$, $\widehat{p}_2^t$, Π_1^T and Π_2^T, the remaining results described in the proposition can be readily obtained. □

Proof. **Proposition 2.** Solving the game backwards, if, in $t=2$, both firms use database advertising, then, given the first-period prices (p_1^1,p_2^1) and the quality levels (s_1,s_2), firm 1 and firm 2 face a demand $x_1^2=Min\left[\frac{p_2^1-p_1^1}{\Delta s}-a;\frac{p_2^1-p_1^1}{\Delta s}-\frac{p_1-v}{s_1}\right]$ and $x_2^2=Min\left[b-\frac{p_2^1-p_1^1}{\Delta s};b-\frac{p_2-v}{s_2}\right]$, respectively. Let us assume that $Min\left[\frac{p_2^1-p_1^1}{\Delta s}-a;\frac{p_2^1-p_1^1}{\Delta s}-\frac{p_1-v}{s_1}\right]=\frac{p_2^1-p_1^1}{\Delta s}-\frac{p_1-v}{s_1}$ and $Min\left[b-\frac{p_2^1-p_1^1}{\Delta s};b-\frac{p_2-v}{s_2}\right]=b-\frac{p_2-v}{s_2}$. Under these conditions, the profit-maximization prices are $p_1=\frac{v\Delta s+(p_2^1-p_1^1)s_1}{2\Delta s}$ and $p_2=\frac{v+bs_2}{2}$ and the corresponding demands are $x_1^2=\frac{v\Delta s+(p_2^1-p_1^1)s_1}{2\Delta s\, s_1}$, $x_2^2=\frac{v+bs_2}{2s_2}$. We focus the model on the case in which v is sufficiently high so that, under monopoly pricing, there are no quantity distortions, i.e. the market is covered. In the case of firm 1, this means that $Min\left[\frac{p_2^1-p_1^1}{\Delta s}-a;\frac{p_2^1-p_1^1}{\Delta s}-\frac{p_1-v}{s_1}\right]=\frac{p_2^1-p_1^1}{\Delta s}-a$ so, in equilibrium, the condition $\frac{v\Delta s+(p_2^1-p_1^1)s_1}{2\Delta s\, s_1}>\frac{p_2^1-p_1^1}{\Delta s}-a$ must hold. In the case of firm 2, this means that $Min\left[b-\frac{p_2^1-p_1^1}{\Delta s};b-\frac{p_2-v}{s_2}\right]=b-\frac{p_2^1-p_1^1}{\Delta s}$ so, in equilibrium, the condition $\frac{v+bs_2}{2s_2}>b-\frac{p_2^1-p_1^1}{\Delta s}$ must also hold. If $x_1^2=Min\left[\frac{p_2^1-p_1^1}{\Delta s}-a;\frac{p_2^1-p_1^1}{\Delta s}-\frac{p_1-v}{s_1}\right]=\frac{p_2^1-p_1^1}{\Delta s}-a$, i.e. $p_1\leq v+as_1$, the firm maximizes $\Pi_1^2=p_1\left(\frac{p_2^1-p_1^1}{\Delta s}-a\right)-\beta\left(\frac{p_2^1-p_1^1}{\Delta s}-a\right)$ and the FOC is $\frac{d\Pi_1^2}{dp_1}=x_1^2>0$, so the optimization problem has a corner solution $\widehat{p}_1^2=v+as_1$. If $x_2^2=Min\left[b-\frac{p_2^1-p_1^1}{\Delta s};b-\frac{p_2-v}{s_2}\right]=b-\frac{p_2^1-p_1^1}{\Delta s}$, i.e. $p_2\leq v+s_2\left(\frac{p_2^1-p_1^1}{\Delta s}\right)$, the firm maximizes $\Pi_2^2=p_2\left(b-\frac{p_2^1-p_1^1}{\Delta s}\right)-\beta\left(b-\frac{p_2^1-p_1^1}{\Delta s}\right)$ and the FOC is $\frac{d\Pi_2^2}{dp_2}=x_2^2>0$, so the optimization problem also has a corner solution $\widehat{p}_2^2=v+s_2\left(\frac{p_2^1-p_1^1}{\Delta s}\right)$. Given these prices, in $t=1$, firms 1 and 2 choose p_1^1 and p_2^1 to maximize the intertemporal profit:

$$\Pi_1^{T1}=p_1^1\left(\frac{p_2^1-p_1^1}{\Delta s}-a\right)-A+(v+as_1-\beta)\left(\frac{p_2^1-p_1^1}{\Delta s}-a\right) \tag{2}$$

$$\Pi_2^{T1}=p_2^1\left(b-\frac{p_2^1-p_1^1}{\Delta s}\right)-A+\left(v+s_2\left(\frac{p_2^1-p_1^1}{\Delta s}\right)-\beta\right)\left(b-\frac{p_2^1-p_1^1}{\Delta s}\right) \tag{3}$$

and the corresponding FOCs are:

$$\frac{d\,\Pi_1^{T1}}{dp_1^1} = \frac{p_2^1 - p_1^1}{\Delta s} - a - \frac{p_1^1}{\Delta s} - \frac{(v + a s_1 - \beta)}{\Delta s} = 0, \tag{4}$$

$$\frac{d\,\Pi_2^{T1}}{dp_2^1} = b - \frac{p_2^1 - p_1^1}{\Delta s} - \frac{p_2^1}{\Delta s} - \frac{\left[(v-\beta)\Delta s + s_2(p_2^1 - p_1^1)\right]}{(\Delta s)^2} + \frac{s_2(p_1^1 - p_2^1 + b\Delta s)}{(\Delta s)^2} = 0, \tag{5}$$

which yields

$$\widehat{p}_1^1 = \frac{-(1+a)s_1^2 + s_2\left[-2s_2(1-a) + 5(v-\beta)\right] + s_1\left[(3+a)s_2 - 3(v-\beta)\right]}{3s_1 - 5s_2}, \tag{6}$$

$$\widehat{p}_2^1 = \frac{-2(1+a)s_1^2 + s_1\left[s_2(6+5a) - 3(v-\beta)\right] - s_2\left[(4+a)s_2 - 5(v-\beta)\right]}{3s_1 - 5s_2}. \tag{7}$$

Given, $(\widehat{p}_1^1, \widehat{p}_2^1)$, $(\widehat{p}_1^2, \widehat{p}_2^2)$, in $t = 0$, each firm sets the quality level that maximizes the total intertemporal profit, Π_j^T. Some algebraic manipulations yield that $\frac{d\,\Pi_1^T}{ds_1}_{|s_1=0} = \frac{4(1-a)(9a-4)}{125}$ so, if $a < \frac{4}{9}$, it holds that $\frac{d\,\Pi_1^T}{ds_1}_{|s_1=0} < 0$, so the equilibrium level of quality is $s_1 = 0$. Given this result, the equation $\frac{d\,\Pi_2^T}{ds_2} = 0$ yields[16] $s_2 = \frac{(3+2a)^2}{25\alpha}$. Plugging this solution into $\widehat{p}_1^t$, $\widehat{p}_2^t$, Π_1^T and Π_2^T, the remaining results described in the proposition can be readily obtained. We finally note that, in equilibrium, the condition $\frac{v\Delta s + (p_2^1 - p_1^1)s_1}{2\Delta s\, s_1} > \frac{p_2^1 - p_1^1}{\Delta s} - a$ always holds and the condition $\frac{v + bs_2}{2s_2} > b - \frac{p_2^1 - p_1^1}{\Delta s}$ implies $v > \frac{(1-a)(3+2a)^2}{125\alpha}$.

In order to confirm that these pricing-advertising strategies are an equilibrium, we must check that firms have no profitable deviations. Firms can only deviate by launching a mass advertising campaign in $t = 2$ and competing for the segment of fully informed consumers. Let us assume that firm 2 uses database

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[16] We note that, given $s_1 = 0$, it holds that $\frac{\partial^2\,\Pi_2^T}{d(s_2)^2} = -2\alpha < 0$, whereas, given $s_2 = \frac{(3+2a)^2}{25\alpha}$, we find that $\frac{\partial^2\,\Pi_1^T}{d(s_1)^2} = \frac{1}{25[(3+2a)^2 - 15\alpha s_1]} 2\alpha(-4(3+2a)^6(58 + a(79+13a)) + 75(3+2a)^4(181 + 16a(15+4a))s_1\alpha - 33750(3+2a)^4 s_1^2\alpha^2 + 337500(3+2a)^2 s_1^3\alpha^3 - 1265625 s_1^4\alpha^4)$, and extensive simulations of the model indicate that, for all $s_1 < s_2 = \frac{(3+2a)^2}{25\alpha}$ and $a < \frac{4}{9}$, the condition $\frac{\partial^2\,\Pi_2^T}{d(s_1)^2} < 0$ always holds, so the second-order conditions are fulfilled.

advertising and sets $p_2^2 = v + \frac{(3+2a)^2(2+3a)}{125\alpha}$. If firm 1 deviates by using mass advertising and charging a deviation price p_1^d, it faces the following optimization problem $Max_{p_1^d}\ \Pi_1^d = p_1^d\left(\frac{p_2^2-p_1^d}{\Delta s} - a\right) - A$. The solution of this problem yields the optimal deviation price $p_1^d = \frac{v}{2} + \frac{(1-a)(3+2a)^2}{125\alpha}$, so $\frac{p_2^2-p_1^d}{\Delta s} - a = \frac{1-a}{5} + \frac{25v\alpha}{2(3+2a)^2}$. Next, we show that $\frac{p_2^2-p_1^d}{\Delta s} - a < 1$, which implies $v < \frac{2(4+a)(3+2a)^2}{125\alpha}$. To this end, we note that (i) $p_1^1 > 0$ implies $v < \frac{2(1-a)(3+2a)^2}{125\alpha} + \beta$, (ii) $\beta < A$ and (iii) $\Pi_2^T > 0$ implies $A < \frac{(3+2a)^4}{625\alpha}$. Considering these conditions together, we obtain $v < \frac{2(1-a)(3+2a)^2}{125\alpha} + \beta < \frac{2(1-a)(3+2a)^2}{125\alpha} + A < \frac{2(1-a)(3+2a)^2}{125\alpha} + \frac{(3+2a)^4}{625\alpha}$. Finally, some computations yield that $\frac{2(1-a)(3+2a)^2}{125\alpha} + \frac{(3+2a)^4}{625\alpha} < \frac{2(4+a)(3+2a)^2}{125\alpha}$, so the condition $v < \frac{2(1-a)(3+2a)^2}{125\alpha} + \frac{(3+2a)^4}{625\alpha}$ implies $v < \frac{2(4+a)(3+2a)^2}{125\alpha}$ and, therefore, $\frac{p_2^2-p_1^d}{\Delta s} - a < 1$ and the optimal deviation profit is $\Pi_1^d = \frac{\left[2(3+2a)^2(1-a)+125v\alpha\right]^2}{2500\alpha(3+2a)^2} - A$. Firm 1 will not deviate if $\Pi_1^d \leq \Pi_1^2 = (v-\beta)\left(\frac{p_2^1-p_1^1}{\Delta s} - a\right) = \frac{2(v-\beta)(1-a)}{5}$, that is, if $A \geq \frac{\left[2(3+2a)^2(1-a)+125v\alpha\right]^2}{2500\alpha(3+2a)^2} - \frac{2(v-\beta)(1-a)}{5}$. Similarly, given $p_1^2 = v$, if firm 2 deviates, it faces the following optimization problem $Max_{p_2^d}\ \Pi_2^d = p_2^d\left(b - \frac{p_2^d-p_1^2}{\Delta s}\right) - A$. The solution of this problem yields the optimal deviation price $p_2^d = \frac{v}{2} + \frac{(1+a)(3+2a)^2}{50\alpha}$, so $b - \frac{p_2^2-p_1^d}{\Delta s} = \frac{1}{2}\left(1 + a + \frac{25v\alpha}{(3+2a)^2}\right)$. Next, we show that $b - \frac{p_2^2-p_1^d}{\Delta s} < 1$, which implies $v < \frac{(1-a)(3+2a)^2}{25\alpha}$. To this end, we note that (i) $p_1^1 > 0$ implies $v < \frac{2(1-a)(3+2a)^2}{125\alpha} + \beta$, (ii) $\beta < A$ and (iii) $\Pi_1^T > 0$ implies $A < \frac{4(3-a-2a^2)^2}{625\alpha}$. Considering these conditions together, we obtain $v < \frac{2(1-a)(3+2a)^2}{125\alpha} + \beta < \frac{2(1-a)(3+2a)^2}{125\alpha} + A < \frac{2(1-a)(3+2a)^2}{125\alpha} + \frac{4(3-a-2a^2)^2}{625\alpha}$. Finally, some computations yield that $\frac{2(1-a)(3+2a)^2}{125\alpha} + \frac{4(3-a-2a^2)^2}{625\alpha} < \frac{(1-a)(3+2a)^2}{25\alpha}$, so the condition $v < \frac{2(1-a)(3+2a)^2}{125\alpha} + \frac{4(3-a-2a^2)^2}{625\alpha}$ implies $v < \frac{(1-a)(3+2a)^2}{25\alpha}$ and, therefore, $b - \frac{p_2^2-p_1^d}{\Delta s} < 1$ and the optimal deviation profit is $\Pi_2^d = \frac{\left[(3+2a)^2(1+a)+25v\alpha\right]^2}{100\alpha(3+2a)^2} - A$. Firm 2 will not deviate if $\Pi_2^d \leq \Pi_1^2 = \left(v + s_2\left(\frac{p_2^1-p_1^1}{\Delta s}\right) - \beta\right)\left(b - \frac{p_2^1-p_1^1}{\Delta s}\right) = \frac{(3+2a)(18+a(51+4a(11+3a))+125\alpha(v-\beta))}{625\alpha}$, that is, if $A \geq \frac{\left[(3+2a)^2(1+a)+25v\alpha\right]^2}{100\alpha(3+2a)^2} - \frac{(3+2a)(18+a(51+4a(11+3a))+125\alpha(v-\beta))}{625\alpha}$. In sum, firms

will not deviate if $A \geq Max[\frac{\left[2(3+2a)^2(1-a)+125v\alpha\right]^2}{2500\alpha(3+2a)^2} - \frac{2(v-\beta)(1-a)}{5}; \frac{\left[(3+2a)^2(1+a)+25v\alpha\right]^2}{100\alpha(3+2a)^2} - \frac{(3+2a)(18+a(51+4a(11+3a))+125\alpha(v-\beta))}{625\alpha}] = \overline{A}$. □

Proof. **Proposition 3.** First, it is straightforward to check that $s_2 < s_2^m$, $x_1^t > x_1^m$, $x_2^t < x_2^m$. Second, given that $p_1^2 = v$ is the reservation price of low-quality buyers, it is clear that $p_1^2 > p_1^m$. Third, considering that $v - p_1^m \geq 0$, i.e. $v \geq \frac{(2+a)^2(1-a)}{27\alpha}$ and $\beta < A < \Pi_1^m$ we obtain that $p_1^m - p_1^1 > \frac{2(2+a)^2(1-a)}{27\alpha} - \frac{2(3+2a)^2(1-a)}{125\alpha} - \frac{\left(2-a-a^2\right)^2}{81\alpha} = \frac{(1-a)(1042+a(1056+a(477+125a)))}{10125\alpha}$, which is positive for all $a < 1$, so $p_1^m > p_1^1$. Finally, considering that $\Pi_1^T > 0$, i.e. $A < \frac{4(3-a-2a^2)^2}{625\alpha}$, we have that $\Pi_2^m - \Pi_2^T > \frac{(2+a)^4}{81\alpha} - \frac{(3+2a)^4}{625\alpha} - \frac{4\left(3-a-2a^2\right)^2}{625\alpha} = \frac{(1-a)(523+a(4971+a(6039+1967a)))}{50625\alpha}$, which is positive for all $a < 1$, so $\Pi_2^m > \Pi_2^T$. □

Proof. **Proposition 4.** Starting from the definition of social welfare under mass advertising and direct advertising:

$$W^m = 2\int_a^{\frac{p_2^m - p_1^m}{s_2^m - s_1^m}} (v + x\, s_1^m)\, dx + 2\int_{\frac{p_2^m - p_1^m}{s_2^m - s_1^m}}^{1+a} (v + x\, s_2^m)\, dx - \alpha(s_1^m)^2 - \alpha(s_2^m)^2 - 4A, \tag{8}$$

$$W^{DA} = 2\int_a^{\frac{p_2^1 - p_1^1}{s_2 - s_1}} (v + x\, s_1)\, dx + \int_{\frac{p_2^1 - p_1^1}{s_2 - s_1}}^{1+a} (v + x\, s_2)\, dx - \alpha(s_1)^2 - \alpha(s_2)^2 - 2A - \beta, \tag{9}$$

some computations yield:

$$W^m = 2v + \frac{2(2+a)^3(1+2a)}{81\alpha} - 4A, \tag{10}$$

$$W^{DA} = 2v + \frac{[108 + 2a(189 + 2a(117 + 2a(31 + 6a))]}{625\alpha} - 2A - \beta. \tag{11}$$

Taking into account that consumer surplus under mass and database adverting are $EC^m = W^m - \Pi_1^m - \Pi_2^m$ and $EC^{DA} = W^{DA} - \Pi_1^T - \Pi_2^T$, we have that

$$EC^{DA} - EC^m = \frac{(1-a)^2\,[4271 + 11a(328 + 61a)]}{50625\alpha} - \beta, \tag{12}$$

and considering that $\beta < A$ and $\Pi_1^m > 0$, i.e. $A < \frac{\left(2-a-a^2\right)^2}{81\alpha}$, we obtain that

$$EC^{DA} - EC^m > \frac{(1-a)^2\left[4271+11a(328+61a)\right]}{50625\alpha} - \frac{\left(2-a-a^2\right)^2}{81\alpha} = \frac{(1-a)^2\left[1771+2a(554+23a)\right]}{50625\alpha} > 0. \tag{13}$$

□