The 'spinning disk touches stationary disk' problem revisited: an experimental approach

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Abstract

A popular Newtonian Mechanics problem, featured in textbooks, Physics Olympiads and forums alike, concerns two disks with different radii and moment of inertia that rotate differently and that touch each other. Most students struggle to calculate the final angular velocity of the disks, erroneously attempting to use different conservation laws. In this paper we propose a simple experiment that should help Physics teachers explain this challenging exercise in an engaging way for the students. By using a smartphone/tablet and video analysis tools, the angular velocity of both disks can be easily tracked as a function of time, clearly showing the three stages of the interaction (before the collision, with only one disk rotating; the collision of the disks with slippage; and the rotation of the two disks in harmony, without frictional forces in between). Processing and plotting of the data in a spreadsheet immediately shows which quantities are conserved and which are not. Several extensions to the core experiment are also suggested.

Keywords: disks; friction; Newtonian Mechanics; smartphone; video analysis.

1. Introduction

Most engineering degrees start with a Newtonian Physics course, and such introductory Mechanics course aims to provide students with an understanding of the fundamental concepts of Physics and their interrelationships. Concepts like force, torque, or angular momentum and their relationships (including conservation laws) are first introduced when studying a particle, then a rigid body, and finally a system of rigid bodies. An introductory course should also teach students to diagnose which of the conservation laws to apply in a given situation, if any. The problem with two spinning discs that touch (and the calculation of their final velocities), frequently found in textbooks as a challenging problem, establishes a fertile situation for a thorough discussion of what conservation laws to apply. Most students find the problem very difficult and it is common to find in web platforms (i.e., Physics Stack Exchange, PhysicsForum, …) several enquiries about the correct solution. Actually, this problem has been featured in three video posts by Walter H.G. Lewin, former Physics Professor at MIT, in YouTube (1-3).

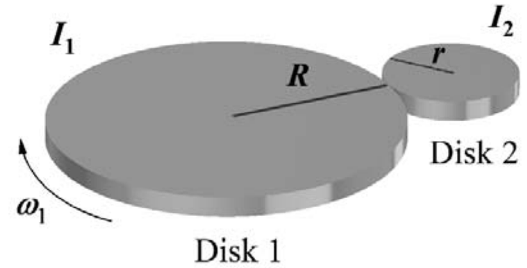
In this paper we show how the problem can be tackled experimentally, in an easy way, using video analysis. In recent years, video analysis has emerged as a very useful tool for slowing down (4-11) or quickening (12, 13) movements that otherwise would be difficult to follow with the naked eye. Video analysis allows the student to monitor the rotation of the two disks, to retrieve hundreds of (t, θ_1, θ_2) measurements, so as to easily identify linear or accelerated movements, and to probe what physical quantities are actually being conserved during the interaction of the disks.

2. Theoretical background

2.1. *Problem formulation*

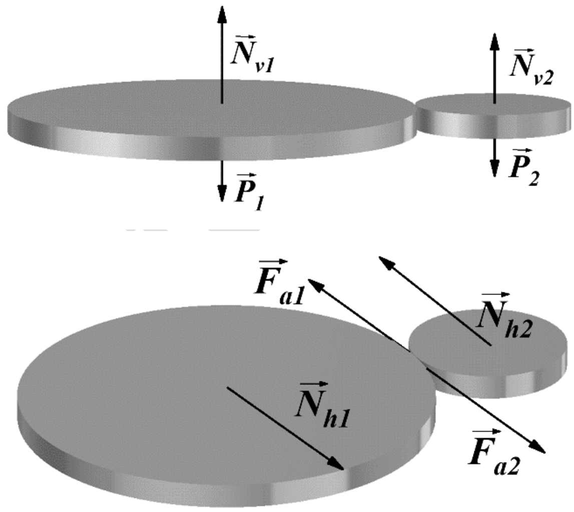
A common phrasing of the problem (Scheme 1) is:

'A disk of radius R and moment of inertia I_1 rotates with angular velocity ω_1. A second disk, of radius r and moment of inertia I_2 is at rest. The axes of the two disks are parallel. The disks are moved together so that they touch. After some initial slipping, the two disks rotate together. Find the final rate of rotation of the smaller disk'.



Scheme 1. Nomenclature used in the problem formulation.

In this situation, the following forces act upon the system (depicted in Scheme 2): the gravitational forces ($\vec{P}_1$ and $\vec{P}_2$), vertical, pointing to the centre of Earth; friction forces ($\vec{F}_{a1}$ and $\vec{F}_{a2}$), in the horizontal plane, tangent to the disks, opposing the rotational movement, equal in magnitude; normal reaction forces with a vertical component (cancelling the gravitational force, $\vec{N}_{v1}$ and $\vec{N}_{v2}$) and an horizontal component, exerted by the axles into the disk, cancelling the frictional force ($\vec{N}_{h1}$ and $\vec{N}_{h2}$).



Scheme 2. Forces and moments involved.

2.2. *Popular misconceptions*

Those students who think of the problem as a collision of two bodies try to use the conservation of momentum first (often an extremely powerful tool in solving problems): in fact, no net external force acts on the system of the two disks, and therefore the total linear momentum of the system cannot change. Nonetheless, this reasoning soon gets to a dead end: the centre of mass of either disks, or of the system, is not moving, before or after the collision, and therefore the value for the rotation of the small disk cannot be retrieved by using of this law.

Other students try to approach the problem by using the conservation of energy, but –for a system of bodies– the law states: the work done by all forces, internal and external, applied to a system of bodies in a certain time interval equals the variation of kinetic energy of the system, during the same interval. Since there is work done by the frictional forces, the kinetic energy is not conserved.

When the two aforementioned laws fail, students turn their attention to the conservation of the angular momentum of the system, which can be applied in the case of a falling disk into a rotating disk, because in that situation no external net torque acts on the system along a specific (the rotation) axis, and the angular momentum of the system remains constant. In the present situation, however, for any of the two axis that we could choose, there is a non-zero torque from the $\vec{N}_{\text{horizontal}}$ components.

2.3. *Correct solution*

Since aforementioned conservation laws fail, the problem has to be solved by rotation dynamics equations, applied to both disks, and bearing in mind that the two frictional forces, $\vec{F}_{a1}$ and $\vec{F}_{a2}$, are a third-law force pair.

Therefore, for each disk, the torque exerted by the frictional force will cause an angular acceleration

$$\vec{F}_{a1} \times \vec{R} = I_1 \cdot \vec{\alpha}_1 \quad [1]$$

$$\vec{F}_{a2} \times \vec{r} = I_2 \cdot \vec{\alpha}_2 \quad [2]$$

Because the frictional forces are a third law pair, they are equal in moduli and using the definition of angular acceleration, one gets

$$\frac{I_1 \cdot \alpha_1}{R} = \frac{I_2 \cdot \alpha_2}{r} \Longrightarrow \frac{I_1}{R} \cdot \frac{d\omega_1}{dt} - \frac{I_2}{r} \cdot \frac{d\omega_2}{dt} = 0 \quad [3]$$

$$\frac{d}{dt}\left(\frac{I_1}{R} \cdot \omega_1 - \frac{I_2}{r} \cdot \omega_2\right) = 0 \Longrightarrow \frac{I_1}{R} \cdot \omega_1 - \frac{I_2}{r} \cdot \omega_2 = \text{constant} \quad [4]$$

that is, $\frac{I_1}{R} \cdot \omega_1 - \frac{I_2}{r} \cdot \omega_2$ is a conserved quantity.

If so, it will remain constant before and after the collision, being ω_1 and ω_2 the angular velocity of disk 1 and disk 2, respectively, before the collision and ω_1' and ω_2', the velocities of disks 1and 2 after the collision.

$$\left(\frac{I_1}{R} \cdot \omega_1' - \frac{I_2}{r} \cdot \omega_2'\right) = \frac{I_1}{R} \cdot \omega_1 - \frac{I_2}{r} \cdot \omega_2 \quad [5]$$

When the slippage is over, the frictional forces become zero, and $R \cdot \omega_1' = -r \cdot \omega_2'$

$$-\frac{I_1}{R} \cdot \left(\frac{\omega_2' \cdot r}{R}\right) - \frac{I_2}{r} \cdot \omega_2' = \frac{I_1}{R} \cdot \omega_1 - 0 \quad [6]$$

$$\Longrightarrow \omega_2' \cdot \left(\frac{I_1 \cdot r}{R^2} + \frac{I_2}{r}\right) = -\frac{I_1}{R} \cdot \omega_1 \quad [7]$$

$$\Longrightarrow \omega_2' = -\frac{r \cdot R \cdot I_1}{R^2 \cdot I_2 + r^2 \cdot I_1} \cdot \omega_1 \quad [8]$$

3. Experimental

A complete rotational system ME-8950A (Pasco, Roseville, CA, USA) was adapted for the experimental procedure presented herein. Furniture wheels may be used instead as a cheap alternative, as discussed in the extensions to the activity (see section 5).

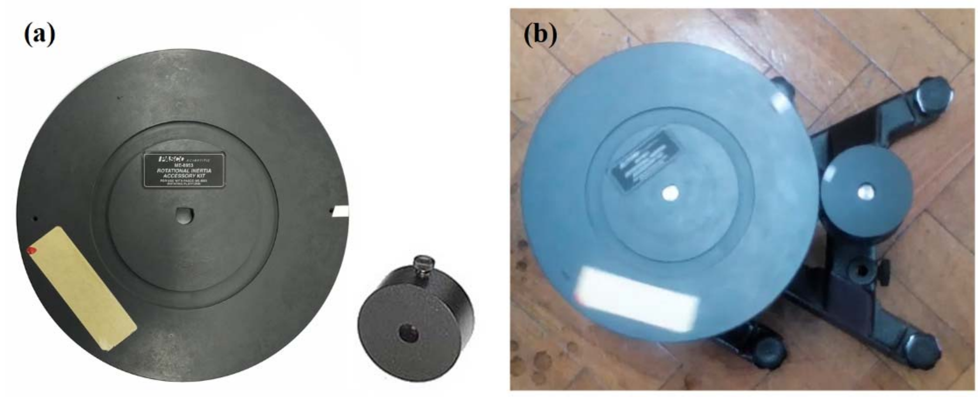


Figure 1. *Left*: the two disks used for the experiment (the screw was removed from the smaller disk); *right*: experimental setup.

The 22.8 cm diameter disk (dark grey, **¡Error! No se encuentra el origen de la referencia.**) and the 7 cm diameter small black disk (intended to be a 900 g counterweight) were mounted on two cast iron bases using the rotating axles included in the kit. The large disk was spun at a certain angular speed and then the small disk –initially at rest– was approached till they touched. Small pieces of white tape were stuck on their periphery so as to enable rotation tracking through video analysis.

A tablet's camera was used to record the experiment (a Lenovo Tab A10, 5 MPx rear camera) from a zenith position (see **¡Error! No se encuentra el origen de la referencia.**). The video file was then processed using Tracker –a free open source video analysis and modelling tool–, and the data obtained was exported to a .csv file, so as to complete the data processing and plotting steps in a MS Excel spreadsheet. OpenOffice Calc may be used instead, but the former may be more ubiquitous and easily understandable (14).

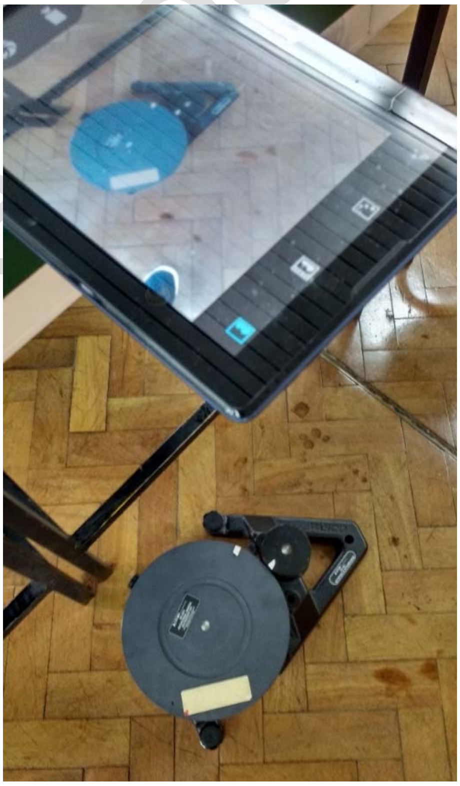


Figure 2. Experiment recording with a tablet.

4. Results and discussion

Several trials were performed changing the initial velocity of the big disk, disk 1, or changing its moment of inertia. Figure 3 shows the results of the video analysis of one of such trials, in which a point on each of the disks periphery was followed with time. Their angular position times their radius is plotted in Figure 3(a) and its derivative in Figure 3(b). The first region of the graph corresponds to the interval before the collision, with only disk 1 rotating; the second interval (with a light grey background) corresponds to the collision of the disks with slippage; and in the third region the disks rotate in harmony, without frictional forces in between.

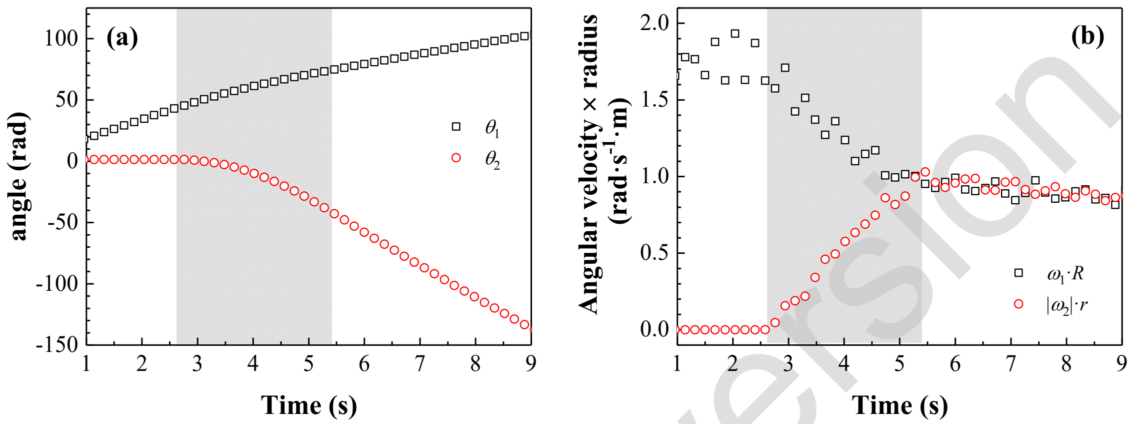


Figure 3. (*a*) Angle as a function of time for the two disks; (*b*) angular velocity×radius plot. 1 out of 3 points has been plotted for clarity reasons.

The rotation kinetic energy of each of the disks and their sum are depicted in Figure 4(a). As predicted above, the energy decreased while the slippage occurred. For the angular momentum (see Figure 4(b)), there was a decrease for the big disk and an increase (in module) for the small disk. A careless sum of their values (as the one that students would attempt) would also show a decrease while friction occurs.

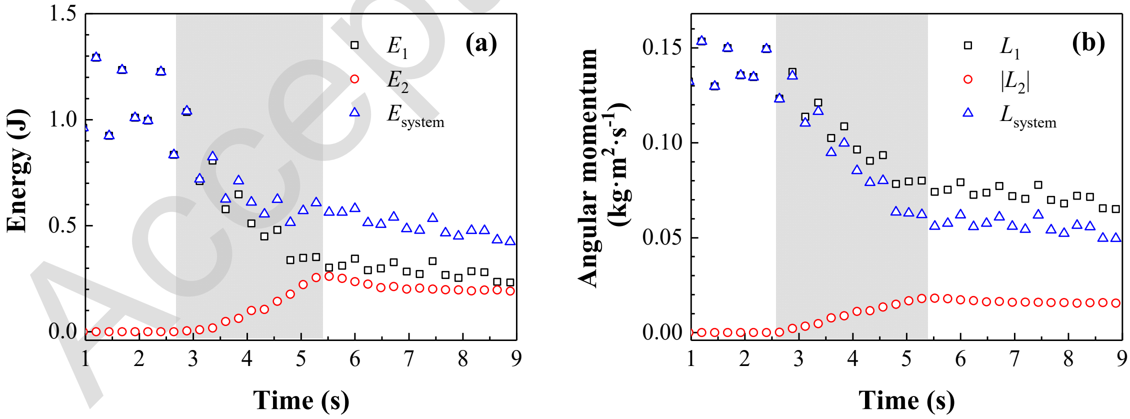


Figure 4. (*a*) Energy and (*b*) angular momentum plots, for each of the disks and their sum. 1 out of 4 points is plotted for clarity reasons.

Figure 5 represents $\frac{I_1}{R} \cdot \omega_1$, $\frac{I_2}{r} \cdot |\omega_2|$, and $\frac{I_1}{R} \cdot \omega_1 - \frac{I_2}{r} \cdot \omega_2$ as a function of time. One can see that while the former two vary linearly during the slippage interval, the later remains constant throughout the interaction.

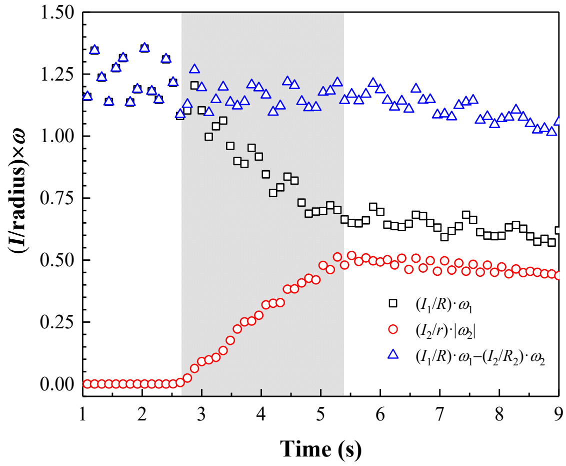


Figure 5. (I/radius)·ω for each of the disks and conserved quantity. 1 out of 3 points is plotted for clarity reasons.

To make calculations from the experimental data retrieved from the video analysis, the value of the moment of inertia for disk 1 was taken from the PASCO manual ($I_1 = 0.0091$ kg·m^2) and that of the counterweight (disk 2) was experimentally determined by coiling a string around it and letting a weight, tied to the other end of the string, fall. The moment of inertia was found to be $I_2 = 6.3 \cdot 10^{-4}$ kg·m^2.

By fitting the curves in Figure 3(a) with a second order polynomial function in the slippage stage, one can retrieve the value of the angular acceleration, and –from that– the modulus of the frictional force. For the run shown above, the retrieved values were α_1=2.513 rad·s^{-2} and α_2=10.829 rad·s^{-2}, and the calculated frictional forces were 0.20 N and 0.195 N, respectively. This is a convenient way for the students to confirm the goodness of their data.

Likewise, fitting of the curves in the third interval with a linear fit yields the values of ω_1=7.515 rad·s^{-1} and ω_2=24.866 rad·s^{-1}. By multiplying by their respective radii, one gets the value of the velocity of a point at the periphery (0.86 m·s^{-1} in both cases), confirming that the video analysis approach is accurate enough.

5. Alternative procedures and suggested extensions to the proposed experiment

5.1. *Different speeds*

If the experiment is repeated spinning disk 1 at different speeds, one can probe the relationship in equation 8 by plotting the observed final angular velocity of disk 2 and that calculated from the initial angular velocity of disk 1. Such plot is shown in Figure 6, together with its linear fit. The slope of such fit, as expected, was close to 1 (0.93, with R^2=0.9993).

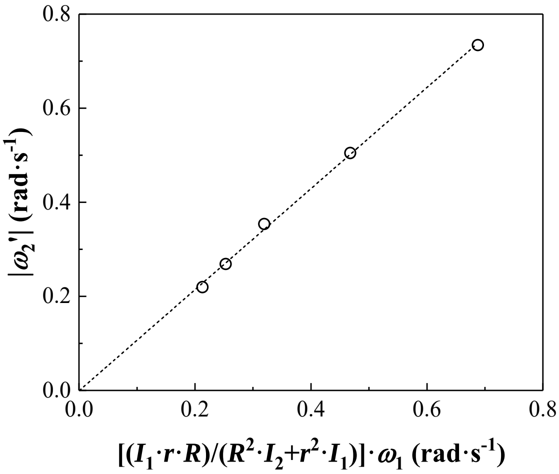


Figure 6. Plot of $|\omega_2'|$ as a function of ω_1, using equation 8.

5.2. *Estimation of the frictional effect of the axles*

Since the axles are not completely frictionless, another possible extension would consist in tracking the movement of the disks until they completely stop. Since the friction is small, so that the video to be processed is not very long, it is convenient to spin the disk at a small/medium initial speed. A fit of the curve $\theta(t)$ after the slippage interval until the disk stops with a second order polynomial yielded for our setting a value of -0.482 rad·s^{-2} angular acceleration.

5.3. *Using the small disk to move big one*

In a classroom, some of the groups may also try spinning the smaller disk instead of the larger one, and should reach similar conclusions.

5.4. *Low-cost option with furniture wheels*

If the laboratory is not equipped with nice disks and their almost friction-free rotation axles, it is possible to use a cheap alternative. A set of 5 different wheels would cost less than 20 euros, and they stand up by their own while their axles remain horizontal. The initial rotation and the collisions are easy to perform, as well as the video analysis of a point in the periphery. As noted above, the moment of inertia may be determined by a simple experiment in which a string is rolled up around the disk, and a weight at its free end is left to fall. Figure 7 shows one of these setups and the corresponding results. The major advice, if this setup is to be used, is to look for smooth rotations at the DIY shop.

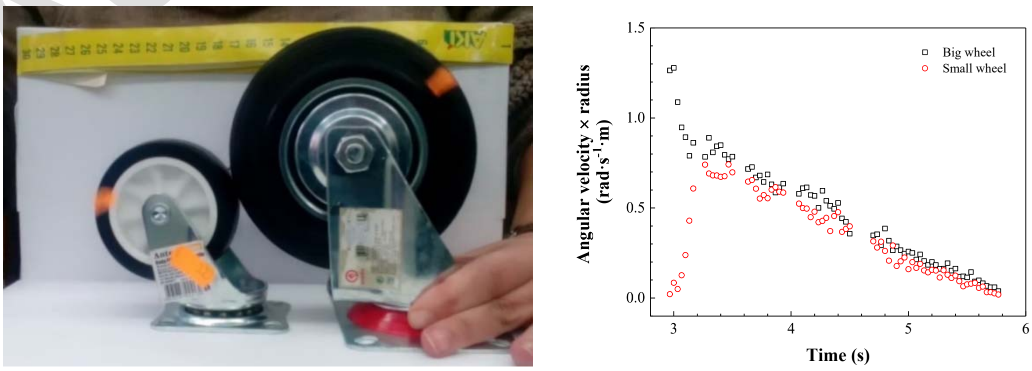


Figure 7. *Left*: alternative experimental setup; *right*: angular velocity × radius plot for the furniture wheels.

5.5. *Tracking in the smartphone/tablet with VidAnalysis or Vernier Video Physics apps*

If the teacher prefers to conduct the video analysis directly in the students' smartphones/tablets, VidAnalysis (15) free app can be installed in Android devices. This intuitive and easy-to-use app requires the setting of the x-y axes, a length scale and the tracking of the position of the marker through screen touching, frame by frame. The app can export the data to a .csv, in a similar fashion to how Tracker does. Vernier Video Physics (16) paid app would offer similar features for iOS devices.

It should be clarified that a tablet would be the preferred option if any of these apps is to be used instead of Tracker, given that the larger size of the screen –as compared to a smartphone– would improve the precision when touching with the finger on the tape in the different frames.

5.6. *Using the in-built smartphone's accelerometer instead of video tracking*

If the teacher prefers to avoid video analysis, it should also be possible to use the in-built accelerometer of the smartphone and apps such as Physics Toolbox Gyroscope (17) to obtain the angular velocity as in references (18-21).

6. Summary and conclusions

Video tracking is a powerful tool for analysis the movements difficult to follow with the naked eye. By applying it to a challenging Newtonian Mechanics problem, it can help the students retrieve real data and, by its subsequent processing and plotting, gain insight on the actual physical parameters involved. Several possible extensions, including a low-cost alternative to commercial equipment, are also suggested, making it adaptable to students of different ages.

7. Acknowledgments

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