Linear-Analog Transformation Approach for Coupled Gas and Power Flow Analysis

Jesus Beyza[a], Jose A. Dominguez-Navarro[a], Jose M. Yusta[a,1,*]

[a]*University of Zaragoza, Department of Electrical Engineering, C/ Maria de Luna 3, 50018, Zaragoza, Spain*

Abstract

In this paper, we present a methodology to analyze the integrated operation of coupled natural gas and electricity networks in steady-state. The interaction of the gas network with the electrical grid is modeled through mathematical equations that represent the energy exchange between the two infrastructures. The joint natural gas and power flow is solved using the linear-analog transformation (LAT) and the Newton-Raphson (NR) algorithm, respectively. Here, a unified solution framework of the two systems is presented using the previous proposed methods. The applicability of the methodology is illustrated using two case studies: IEEE-14 bus test system combined with a 16-node natural gas network and the IEEE-30 bus test system integrated with a 15-node natural gas network with 4 compressors. The proposed methodology proves to be useful for the assessment of coupled natural gas and electricity critical infrastructures.

Keywords: Natural gas network, power systems, power flow analysis, linear-analog transformation.

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[*]Corresponding author

Email addresses: jbbcia4@hotmail.com (Jesus Beyza), jadona@unizar.es (Jose A. Dominguez-Navarro), jmyusta@unizar.es (Jose M. Yusta)

[1]Phone: (0034) 976761922

List of Symbols

i	Index for the upstream node.
j	Index for the downstream node.
$q_{G(i,j)}$	Flow between pipeline i and j.
$C_{(i,j)}$	Conductivity of the pipeline between i and j.
$s_{(i,j)}$	Coefficient of the pipeline elevation.
e	Pipeline roughness.
$p_{(i)}$	Network pressure at point i.
$L_{(i,j)}$	Pipeline conductivity of the linear-analog.
$T_{(i,j)}$	Conductivity transform between the points i and j.
$r_{(i,j)}$	Pressure ratio between the points i and j.
HP	Horsepower of compression station.
n_{st}	Compression steps.
n_p	Polytropic coefficient.
Z_{av}	Compressibility factor of the gas.
η	Efficiency of the compressor station.
$T_{(i)}$	Conductivity transform of the well i.
k_c	Compressor coefficient.
$r_{c(i,j)}$	Compression ratio of the station between i and j.
$C_{c(i,j)}$	Constant of the compressor station between i and j.
g_k	Natural gas taken from the network by the compressor station.
k	Index for each iteration of LAT method.
Cf	Natural gas consumption factor.
S	Gas supplying at the corresponding node.
D	Natural gas demand for domestic use and industrial.
D_g	Natural gas demand for power generation.
$\Delta P_{(i)}$	Active power mismatch of node i.
$P_{gen}^{(i)}$	Active power of generation at node i.
$P_{load}^{(i)}$	Active power of load at node i.
$P_{cal}^{(i)}$	Calculated value of active power injection at node i.

ΔP	Vector of active power mismatch.
$\Delta Q_{(i)}$	Reactive power mismatch of node i.
$Q_{gen}^{(i)}$	Reactive power of generation at node i.
$Q_{load}^{(i)}$	Reactive power of load at node i.
$Q_{cal}^{(i)}$	Calculated value of reactive power injection at node i.
ΔQ	Vector of reactive power mismatch.
$V_{(i)}$	Voltage magnitude at node i.
ΔV	Vector of changes on the voltage magnitudes.
$G_{(i,j)}$	Conductance of the branch between i and j.
$B_{(i,j)}$	Susceptance of the branch between i and j.
$\gamma_{(i)}$	Angle of the voltage at node i.
$\Delta \delta$	Vector of changes on the angle magnitudes.
$e_{(i)}$	Amount of gas required by the generator of node i.
$K_{0(i)}, K_{1(i)}, K_{2(i)}$	Coefficient of gas consumption of generator i.
$P_{G(i)}$	Active power generation at node i.
K	Characteristic matrix of gas network.
P	Vector of gas network pressures.
S	Vector of gas supply and consumption.
$O_{(i)}$	Summation of the off-diagonal entries of matrix K.
s	Elevation coefficient.
SG_G	Specific gravity of the gas.
ΔH	Pipelines elevation.
T_{av}	Average temperature.
L_e	Pipeline equivalent length.
L	Horizontal pipeline equivalent length.
J	Jacobian matrix.
σ_G	Parameter of generalized gas model.
T_{sc}	Absolute temperature at standard conditions.
p_{sc}	Pressure at standard conditions.
f_F	Friction factor.

d	Internal diameter of the pipeline.
ε	Error on the estimation of gas network pressure.
$P_{G,min}$	Minimum power of the generator.
$P_{G,max}$	Maximum power of the generator.

1. Introduction

The growing adoption of renewable energies as an alternative to conventional fuels for power generation has been accompanied by an increasing deployment of natural gas installations due to its capacity to deal with the unexpected variations of intermittent sources such as wind and solar photovoltaic generation [1]. The interdependence between both systems lies mainly in the fact that natural gas-fired generators accounted for more than 40 % of the operating electricity generating capacity in some countries. Thus, these networks cannot be considered as separated systems because operation of one can have an impact on the other.

This situation has established a very close relationship between power and gas systems from both financial and operational perspectives [2]. In this sense, the interaction between these systems for the study of their respective deregulated markets has gained relevance [3, 4], as well as for reliability studies [5], optimal control and scheduling [6–8], and critical infrastructures analysis [9, 10].

The focus of this work relies on the analysis of the integrated power and natural gas systems, which is a crucial process for most of the studies previously mentioned. On the one hand, power system simulation is performed including transmission lines, active and reactive power consumption, reactive power control, among other assets [11, 12]. On the other hand, natural gas networks have been traditionally represented by pipelines and compressors, avoiding the effects of other elements such as valves and regulators [3, 13].

The computational simulation of both systems requires the solution of algebraic non-linear equations due to their complex behavior. In this regard, most of the methodologies available in the literature are based on Newton-Raphson

4

method [14]. In the case of power system analysis, voltages across the transmission or distribution systems are generally close to their nominal values, so that in many cases convergence of Newton method is observed [15]. However, in the case of natural gas networks, the solution of non-linear equations strongly depends on the initial approximation. These phenomena were extensively studied by Li et al. [16], who discussed the problem formulation in two different ways, taking into account the nodes (Newton-nodal method) and the loops (Newton-loop method) of the corresponding topology. The Newton-nodal formulation is based on the sum of flows at each node, while Newton-loop method uses the sum of the pressure drops around each loop. As loop formulation is almost quadratic due to the quadratic behavior of flow rates, Newton-loop method offers good convergence when compared to its counterpart. However, the loop to be analyzed has to be adequately selected, thus the formulation of the problem using loops becomes difficult. Otherwise, the use of numerical methods for the calculation of Jacobian matrix can ease the computational implementation of both methods by avoiding the required mathematical derivation in analytical form.

In other proposals that consider technical aspects in the operation of the coupled gas and electricity network, Martínez-Mares and Fuerte-Esquivel [11] incorporated the influence of the changes on natural gas temperature over the pipeline, which is an aspect that is not frequently considered to maintain the simplicity of the model. Erdener et al. [17] implemented a technique based on Newton-loop-node formulation combined with the Breadth First Search (BFS) algorithm. As the equation set under study is highly non-linear, BFS algorithm was used to find an initial solution to the algebraic problem. With the high interest on natural gas resources, specifically in urban areas, Shabanpour-Haghighi and Seifi [12] developed a model to incorporate a district heating network to the integrated power and gas system, taking into account the hydraulic as well as the thermal circuits on the balance equation. Wang et al. [18] developed the decoupled implicit method for efficient network simulation (DIMENS), which uses divide-and-conquer technique to increase the computational speed. DIMENS di-

vides the gas network into sub-systems that are later fully analyzed. Dyachenko et al. [13] proposed the operator splitting technique based on the solution of a non-linear hyperbolic partial differential equation set to describe the hydrodynamic behavior of the gas system. As the most important characteristic, the technique is stable and accurate, as well as computationally efficient.

Recently, Ayala and Leong [19, 20] proposed the linear-analog concept in order to reduce the complexity of the algebraic equations used to evaluate the performance of natural gas networks. This method is based on applying linear-analog transformation (LAT) approach to the model of each asset of the gas network. Contrary to other proposals, in the LAT approach initial assumptions of pressures and flow rates are not necessary and only a set of algebraic equations that can be solved by standard numerical methods are required.

It can be noted that the works discussed above show that electricity and natural gas networks are well studied separately, however, a scope for improvement is observed in the development of techniques for analyzing the integrated operation of both critical infrastructure systems. This paper presents a methodology that allows to obtain the steady-state solution of coupled electricity and natural gas flows using the novel linear-analog method to solve the gas-flow problem. In the latter, the advantages of Newton's traditional approach using only nodal equations are preserved, but the formulation is simplified, the calculation of derivatives is eliminated and the computational cost is reduced [19, 20].

The rest of this paper is organized as follows: Section 2 describes in detail the mathematical models of the generalized gas-flow equation, pipes and compressors of the natural gas infrastructure using the linear-analog approach. Then, Section 3 presents the steady-state power system modeling with Newton-Raphson method. The simulation of the joint system is formulated and performed in Section 4, to be later implemented in two case studies presented in Section 5. Finally, the sum of the main conclusions of this work are discussed in Section 6.

2. Modeling of the natural gas network

Fig. 1 presents a natural gas network with the components considered for the simulation, including supply and demand nodes, as well as the suction and discharge nodes of the compressor station.
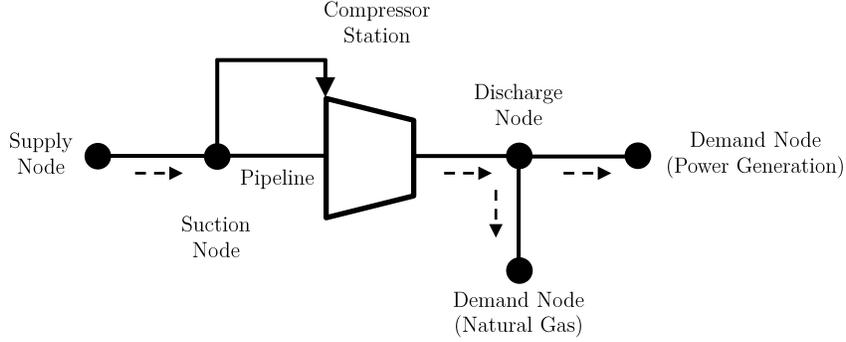

Figure 1: A simple natural gas network.

The reasoning behind LAT approach consists on substituting the non-linear formulation of system dynamics by an alternative model, simplifying according to some specific assumptions related to the behavior of the gas under laminar conditions. Therefore, this method has proved to be an ideal approach to solve the gas-load flow problem for natural gas infrastructures composed of pipes, compressors, supplies, among others. To find the solution of the gas system, the LAT method only uses nodal equations simplifying the computational calculation [19, 20].

2.1. Linear-analog of the pipeline system

Analyzing the behavior of natural gas network starts by establishing the expression that estimate the flow rate $q_{G(i,j)}$ through a pipeline (i, j). A general form of such expression is presented in Eq. (1),

$$q_{G(i,j)} = C_{(i,j)}\sqrt{p_{(i)}^2 - e^{s_{(i,j)}}p_{(j)}^2} \qquad (1)$$

It can be noted the non-linear characteristic of Eq. (1), which increases the difficulty to accurately determine the pressure and flow across the system.

7

In order to avoid this problem, LAT method uses a similar expression that is presented in Eq. (2).

$$q_{G_{(i,j)}} = L_{i,j}\left(p_{(i)} - e^{\frac{s_{(i,j)}}{2}}p_{(j)}\right) \tag{2}$$

Where the inclination of the pipelines are considered by means of the elevation coefficient $s_{(i,j)}$, while the pressure conductivity is included by using $L_{(i,j)} = T_{(i,j)}C_{(i,j)}$, which depends on the conductivity transform $T_{(i,j)} = \sqrt{1 + \frac{2}{r_{(i,j)}-1}}$, obtained from the generalized gas flow model, and on the conductivity of the pipeline $C_{(i,j)}$.

Now, equating Eq. (1) and Eq. (2) as shown in Eq. (3) [19, 20],

$$L_{i,j}\left(p_{(i)} - e^{\frac{s_{(i,j)}}{2}}p_{(j)}\right) = C_{(i,j)}\sqrt{p_{(i)}^2 - e^{s_{(i,j)}}p_{(j)}^2} \tag{3}$$

and introducing the pressure ratio defined as $r_{(i,j)} = \frac{p_{(i)}}{e^{\frac{s_{(i,j)}}{2}}p_{(j)}}$, the linear representation of the quadratic expression shown in Eq. (3) is obtained in Eq. (4) [19, 20]:

$$L_{(i,j)}^2(r_{(i,j)} - 1) = C_{(i,j)}^2(r_{(i,j)} + 1) \tag{4}$$

Then, the corresponding linear equation system can be solved by means of standard techniques available in the literature. Here, $(T_{(i,j)}) > 1$ due to the pressure ratio $r_{(i,j)} = \frac{p_{(i)}}{e^{\frac{s_{(i,j)}}{2}}p_{(j)}}$, which leads to the pipeline conductivity $L_{(i,j)} > C_{(i,j)}$. Therefore, the calculation of gas flow through the network is an iterative process, starting with the assumption that $L_{(i,j)} = C_{(i,j)}$ or $2C_{(i,j)}$, in order to obtain a first solution. Once the pressures have been initially estimated, the value of pressure ratio is then calculated, as well as the updated value of the pipeline conductivity of the respective linear-analog (the value of $L_{(i,j)}$ for the next iteration).

2.2. The linear-analog of the compressor station

Compression stations are conveniently installed to move the gas through the network and compensate the effects of friction of the pipelines, among other factors. Some compressor stations take gas from the network according to the

8

amount of horsepower required, while others take power from an electrical system to boost the pressure.

The relationship between the power demanded by the station to increase the gas pressure, the compression stages, compressor efficiency and natural gas properties is presented in Eq. (5),

$$HP = 0.0857 \frac{n_{st} \cdot n_p}{n_p - 1} q_{G_{(i,j)}} T_{(i)} Z_{av} \left(\frac{1}{\eta}\right) \left[\left(\frac{p_{(j)}}{p_{(i)}}\right)^{\frac{n_p - 1}{n_{st} \cdot n_p}} - 1 \right] \tag{5}$$

This expression can be simplified by introducing the compressor coefficient shown in Eq. (6),

$$k_c = 0.0857 \left(\frac{n_{st} \cdot n_p}{n_p - 1}\right) T_{(i)} \left(Z_{av}\right) \left(\frac{1}{\eta}\right) \tag{6}$$

Thus, gas flow through the compressor station can be obtained by re-ordering Eq. (5) and Eq. (6), which results in Eq. (7),

$$q_{G(i,j)} = \frac{HP}{k_c \left[\left(r_{c_{(i,j)}}\right)^{\frac{n_p - 1}{n_{st} \cdot n_p}} - 1 \right]} \tag{7}$$

The compression ratio of the compressor station located between the nodes i and j is defined according to Eq. (8),

$$r_{c_{(i,j)}} = \frac{p_{(j)}}{p_{(i)}} \tag{8}$$

Once the compression ratio has been defined, gas flow can be simplified and overwritten as presented in Eq. (9) [19, 20],

$$q_{G_{(i,j)}} = C_{c_{(i,j)}} HP \tag{9}$$

Where the variable ($C_{c_{(i,j)}}$) is defined according to Eq. (10),

$$C_{c_{(i,j)}} = \frac{1}{k_c \left[\left(r_{c_{(i,j)}}\right)^{\frac{n_p - 1}{n_{st} \cdot n_p}} - 1 \right]} \tag{10}$$

Regarding compressor stations based on gas absorption, the amount of gas drawn from the network ($g_{(k)}$) can be estimated using Eq. (11),

$$g_{(k)} = (Cf) HP_{(k)} \tag{11}$$

The factor (Cf) is strongly related to the natural gas calorific value. Regarding compressor stations powered by electricity, the corresponding amount of gas drawn is neglected.

2.3. Balance of the natural gas system

The balance of natural gas on each node of the system is shown in Eq. (12), which establishes that the total sum of gas entering and leaving the node must be equal to zero.

$$\sum_{\substack{j \neq i \\ j \in i}} (L_{ij} \cdot (p_i - p_j)) + S_i - D_i - D_{g_i} - g_{(k)} = 0 \tag{12}$$

Where $(L_{ij} \cdot (p_i - p_j))$ is the flow rate of the pipelines adjacent to the node i, S_i represents the natural gas inputs, D_i are the domestic and industrial loads, D_{g_i} are the natural gas demands for power generation and $g_{(k)}$ is the natural gas consumption by compressors. The latter is used only in the balance equation of the compressor input node.

3. Modeling of the transmission power system

As in the case of natural gas networks, power system behavior is modeled by using a set of non-linear equations represented in Eqs. (13) to (16) that are frequently solved using Newton-Raphson approach [11]. Power balance of the system is determined through Eqs. (13) and (14) considering the respective value of voltage and angle at each node, as well as the power flows through the transmission lines.

$$\Delta P_{(i)} = P_{gen}^{(i)} - P_{load}^{(i)} - P_{cal}^{(i)} = 0 \tag{13}$$

$$\Delta Q_{(i)} = Q_{gen}^{(i)} - Q_{load}^{(i)} - Q_{cal}^{(i)} = 0 \tag{14}$$

$$P_{cal}^{(i)} = \sum_{j \in i} \left\{ V_{(i)}^2 G_{(i,i)} + V_{(i)} V_{(j)} [G_{(i,j)} \cos{(\theta_{(i)} - \theta_{(j)})} + B_{(i,j)} \sin{(\theta_{(i)} - \theta_{(j)})}] \right\} \tag{15}$$

$$Q_{cal}^{(i)} = \sum_{j \in i} \left\{ -V_{(i)}^2 B_{(i,i)} + V_{(i)} V_{(j)} [G_{(i,j)} \sin{(\theta_{(i)} - \theta_{(j)})} - B_{(i,j)} \cos{(\theta_{(i)} - \theta_{(j)})}] \right\} \tag{16}$$

Next, the slack generator is chosen and the voltages and angles are calculated in order to solve the power flow problem. Finally, the non-linear equation system is solved by using Newton-Raphson approach.

4. Modeling of the coupled gas and power grid

Once the mathematical model of natural gas and power systems has been described, the interdependency model to study the interaction between them can be created. In this sense, the connection points of both infrastructures are on the gas fired power plants (GFPPs) and on the compressor units.

4.1. Gas fired power plants (GFPPs)

Energy conversion process of GFPP can be briefly described using Eq. (17) [21].

$$e_{(i)} = K_{2_{(i)}} P_{G_{(i)}}^2 + K_{1_{(i)}} P_{G_{(i)}} + K_{0_{(i)}} \tag{17}$$

This expression defines the amount of gas demanded by the generation system to satisfy corresponding amount of power generation P_G. In other words, the gas consumption of each GFPP is related to the real power generation using Eq. (17).

4.2. Compressor units

Power consumption of compressor units is defined in Eq. (5), while those compressor units based on natural gas are modeled according to Eq. (11).

4.3. Unified solution of natural gas and power flows

The unified solution of the coupled electricity and natural gas flow is obtained when the electric and hydraulic models of both infrastructure systems

11

are combined as described in the previous sections. The two networks are coupled with interdependent links that represent the flow exchange between the two systems. The power grid has gas fired power plants connected to their buses that are fed from determined nodes of the gas network. Similarly, the gas network has compressors that operate with external power supplied from certain buses of the electrical grid.

Fig. 2 presents the algorithm proposed to obtain the steady-state solution of coupled natural gas and electricity flow. In the first step of the algorithm of Fig. 2, the information required to simulate the power and natural gas infrastructures is collected assuming the bidirectional links described above. The study starts by solving the load flow problem in the electrical network using the NR method in order to calculate the voltages magnitudes and phase angles at all the buses in the infrastructure according to the defined power generations and loads. Next, active and reactive power flows through transmission lines are obtained. Since the power transmission losses in the electrical grid cannot be determined without first knowing the power flow, a slack generator with its defined voltage magnitude and phase angle is assigned to supply the power generation mismatch. In some instances, the slack generator could be coupled with the gas infrastructure. The power produced by the GFPPs are used to estimate the amount of natural gas required through the Eq. (17). Then, the demand for natural gas is incorporated as a load on the gas network. Next, the gas-load flow study is carried out using the LAT method incorporating the linear-analog equation of the compression stations. Here, consider that the amount of gas required for the power generation and the behavior of the mechanical and electrical compression stations are used to estimate the corresponding values of gas supplies and power demands, respectively. Finally, the process ends once the error become lower than a predefined tolerance.

All of the above is executed within an iterative framework to evaluate the joint operation of both coupled infrastructures. That is to say, in each step the mismatch of power generation of the slack bus is determined according to the gas supply and the electrical energy demanded by the coupled compressors. In

12

this way, a more realistic and accurate simulation of the two interdependent infrastructures is obtained.

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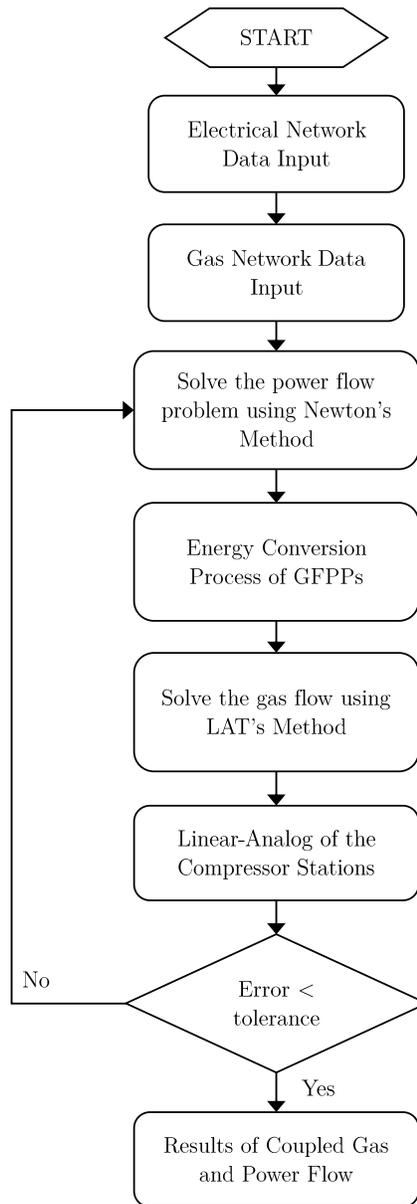
Figure 2: Flowchart of the proposed LAT-NR method.

Power flow study is carried out by following the steps presented as follow:

Step 1. Create the Jacobian matrix of the system.

Step 2. Determine the changes on the voltage magnitudes $(V_{(i)})$ and angles $(\theta_{(i)})$, and update the corresponding values using Eq. (18),

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = [J]^{-1} \cdot \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{18}$$

Step 3. Using the updated values of voltages, calculate the values of current $(I_{(i)})$, active $(P_{(i)})$ and reactive power $(Q_{(i)})$. Then, calculate the corresponding active and reactive power mismatch.

Step 4. Repeat the steps previously explained until the active and reactive power mismatches becomes lower than a tolerance.

Step 5. At this step, natural gas required to supply the power generation can be calculated using Eq. (17).

Natural gas flow study can be carried out by following the steps presented as follow [19, 20]:

Step 1. Using the information related to the structure of the gas system, the coefficients of the pipeline elevation (s) are calculated using Eq. (19),

$$s = 0.0375 \left(\frac{SG_G \cdot \Delta H}{Z_{av} \cdot T_{av}} \right) \tag{19}$$

While the equivalent length (L_e) is calculated using Eq. (20),

$$L_e = \left(\frac{e^s - 1}{s} \right) L \tag{20}$$

Note that $L_e = L$ for horizontal pipelines.

Step 2. The friction factor (f_F) estimated according to a determined model (Weymouth, Panhandle, etc.) is used to calculate the conductivity $(C_{(i,j)})$ as shown in Eq. (21),

$$C_{(i,j)} = \frac{\sigma_G}{\sqrt{SG_G T_{av} Z_{av}}} \left(\frac{T_{sc}}{p_{sc}} \right) \sqrt{\frac{1}{f_F}} \left(\frac{d^{2.5}}{L_e^{0.5}} \right) \tag{21}$$

Step 3. The compressor coefficients (k_c and $C_{c_{(i,j)}}$) are calculated using Eq. (6) and Eq. (10), while the conductivity of the linear-analog is estimated by means of $L_{(i,j)} = T_{(i,j)} C_{(i,j)}$.

Step 4. The characteristic matrix (K) is built in order to create the algebraic system of Eq. (22), using the vector of gas supply and demand (S).

$$\underbrace{\begin{bmatrix} -O_{(1)} & e^{\frac{s_{(1,j)}}{2} L_{(i,j)}} & \cdots \\ L_{(2,j)} & -O_{(2)} & e^{\frac{s_{(2,j)}}{2} L_{(2,j)}} \\ L_{(i,n)} & \cdots & -O_{(n)} \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} p_{(1)} \\ \vdots \\ p_{(n)} \end{bmatrix}}_{P} = \underbrace{\begin{bmatrix} s_{(1)} \\ \vdots \\ s_{(n)} \end{bmatrix}}_{S} \qquad (22)$$

Step 5. The pressure and compressor ratios ($r_{(i,j)}$ and $r_{c_{(i,j)}}$) are calculated in order to later obtain the values of the conductivity transforms using Eq. (23),

$$T_{(i,j)} = \sqrt{1 + \frac{2}{r_{(i,j)} - 1}} \qquad (23)$$

Step 6. This process is repeated until the error on the calculation of all the pressures around the network become lower than a predefined value (ε). The amount of electrical power required by the compressor stations estimated as a result of the gas network analysis is used during the study of transmission power system, which is carried out according to [22].

To enhance the comprehension of the proposed method, two different systems are analyzed in the next section.

5. Case studies

To show the performance of the proposed model in the solution of coupled electricity and gas flow problem, in this paper two case studies are applied as described below:

1) a combined system consisting of the IEEE-14 bus network and a 16-node gas network.

2) a combined system consisting of the IEEE-30 bus network and a 15-node gas network with 4 compressors.

15

Note that the case study 1) corresponds to a natural gas network that includes only pipes, while case study 2) corresponds to a natural gas network that considers pipes and compressors. The results are compared to the traditional Newton-Raphson approach.

5.1. Case study 1

The LAT-NR methodology is applied to analyze the coupled natural gas and electricity system consisting of a 16-node gas network and the IEEE-14 bus test system [23], shown in Fig. 3. The natural gas infrastructure is composed of 9 gas demands, 4 supplies and 21 pipes as shown in Tables 1 and 2. The diameters and lengths of pipes can be consulted in Table A.1. Node 1 of the gas network is selected as a slack node with a pressure equal to 547 PSIA. In the simulation an average gas temperature of 495 °R is considered and the Weymouth generalized gas-flow equation with a compressibility factor of 0.9 and a specific gravity of 0.69 is used.

On the other hand, the electrical network has the power generations and demands presented in Table 1, where two GFPPs are considered operating on buses 2 and 3 fed from nodes 3 and 13 of the natural gas infrastructure, respectively. Similar to the gas network, Table 2 presents the connections of the transmission lines. The operation characteristics of the GFPPs with their gas consumption coefficients are presented in Table A.2. Additionally, the mismatch tolerances are 1^{-3} in the gas infrastructure and 1^{-8} in the electrical infrastructure.

The results in Table 1 indicate that the GFPPs located on buses 2 and 3 of the power system require a total gas flow of 33.7 MMSCFD to generate 178 MW. The reference node of the natural gas network provides 121 MMSCFD to satisfy the total demand of the system. Table 2 reports the flow rates in the pipes and power flows in the transmission lines obtained with the proposed methodology. Here, in order to validate the results achieved with the LAT-NR methodology, Table 2 shows the results of flow rates in the pipes obtained by the Newton-Raphson method. Likewise, the last column of Table 2 indicates the differences between the NR traditional method and the LAT-NR approach.

16

In this case study, the total mean absolute deviation is 0.61, which confirms the viability of the results achieved.

Fig. 4 shows the convergence of the method during the calculation of the flow rates through the pipes for Case 1. The iterative process was initialized under the assumption that $L_{(i,j)} = C_{(i,j)}$, which results in an overestimation of the gas flow during the first iterations of the iterative process (k). However, it can be observed how the error decreases as the number of iterations increases.
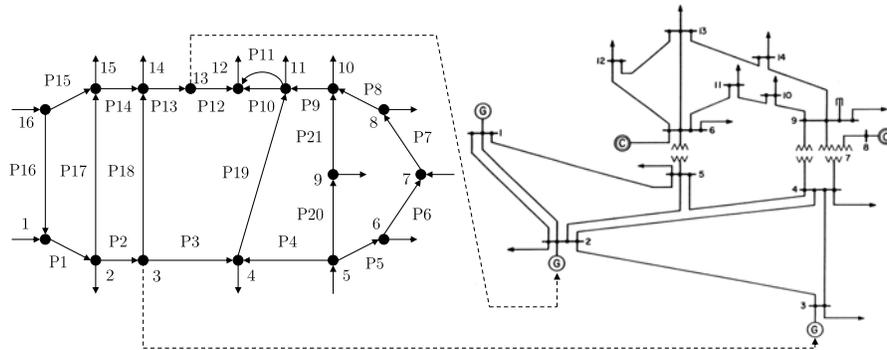

Figure 3: Natural gas network of 16 nodes coupled to power system of 14 buses.

Table 1: Results of supplies, gas demands and power generation for Case 1.

	ELECTRICITY SUPPLY AND DEMAND				NATURAL GAS SUPPLY AND DEMAND		
Bus	Generation		Load		Node	Supply	Demand
	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)		(MMSCFD)	(MMSCFD)
1	85.63	17.72	0.00	0.00	1	121.00	0.00
2	95.00	8.91	21.70	12.70	2	0.00	4.70
3	83.00	-5.24	94.20	19.00	3	0.00	15.10 (GFPP)
4	0.00	0.00	47.80	-3.90	4	0.00	8.90
5	0.00	0.00	7.60	1.60	5	151.80	0.00
6	0.00	12.67	11.20	7.50	6	0.00	20.10
7	0.00	0.00	0.00	0.00	7	192.60	0.00
8	0.00	17.41	0.00	0.00	8	0.00	83.60
9	0.00	0.00	29.50	16.60	9	0.00	11.20
10	0.00	0.00	9.00	5.80	10	0.00	57.80
11	0.00	0.00	3.50	1.80	11	0.00	60.80
12	0.00	0.00	6.10	1.60	12	0.00	80.80
13	0.00	0.00	13.50	5.80	13	0.00	18.60 (GFPP)
14	0.00	0.00	14.90	5.00	14	0.00	64.20
					15	0.00	50.70
					16	11.10	0.00
TOTAL	263.30	51.47	259.00	73.50		476.50	476.50

Table 2: Power flow and gas flow rate for Case 1.

|			POWER FLOW							GAS FLOW RATE		
Line	From	To	P	Q	P	Q	Pipe	From	To	Gas Flow	Gas Flow	Error
	Bus	Bus	(MW)	(MVAr)	(MW)	(MVAr)		Node	Node	NR	LAT-NR	
										(MMSCFD)	(MMSCFD)	
L1	1	2	42.23	10.46	-41.89	-15.27	P1	1	2	112.91	112.31	0.60
L2	1	5	43.40	7.26	-42.45	-8.65	P2	2	3	83.57	82.40	1.17
L3	2	3	27.12	10.19	-26.74	-13.20	P3	3	4	-25.33	-25.92	0.59
L4	2	4	47.69	0.37	-46.47	-0.30	P4	5	4	142.71	142.12	0.59
L5	2	5	40.38	0.92	-39.52	-2.01	P5	5	6	-75.83	-75.32	0.51
L6	3	4	15.54	-11.04	-15.31	10.31	P6	7	6	95.93	95.42	0.51
L7	4	5	-32.08	4.10	32.22	-3.67	P7	7	8	96.66	97.17	0.51
L8	4	7	29.29	-9.71	-29.29	11.55	P8	8	10	13.06	13.57	0.51
L9	4	9	16.78	-0.49	-16.78	1.91	P9	10	11	28.98	29.57	0.29
L10	5	6	42.15	12.72	-42.15	-8.65	P10	11	12	17.51	17.93	0.42
L11	6	11	6.17	3.88	-6.13	-3.79	P11	11	12	59.15	58.13	1.02
L12	6	12	7.64	2.57	-7.57	-2.42	P12	13	12	4.13	4.72	0.59
L13	6	13	17.14	7.37	-16.94	-6.97	P13	14	13	22.73	23.34	0.61
L14	7	8	0.00	-16.96	0.00	17.41	P14	15	14	-6.87	-5.66	1.21
L15	7	9	29.29	5.41	-29.29	-4.54	P15	16	15	19.18	19.82	0.64
L16	9	10	6.40	3.87	-6.38	-3.83	P16	1	16	8.08	8.72	0.64
L17	9	14	10.17	3.38	-10.04	-3.10	P17	2	15	24.64	25.21	0.57
L18	10	11	-2.62	-1.97	2.63	1.99	P18	3	14	93.80	93.21	0.59
L19	12	13	1.47	0.82	-1.47	-0.82	P19	4	11	108.48	107.29	1.19
L20	13	14	4.91	1.99	-4.86	-1.90	P20	5	9	84.92	84.99	0.07
							P21	9	10	73.72	73.79	0.07
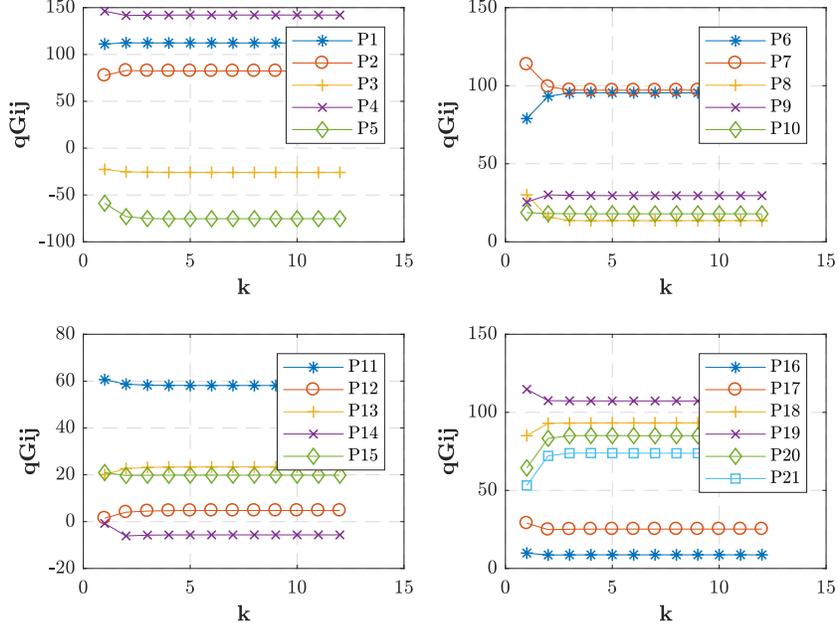
Figure 4: Convergence of flow rate $qGij$ for Case 1.

5.2. Case study 2

The case study corresponds to a 15-node natural gas network with four compressors and the IEEE-30 bus test network [23]. The coupling between both infrastructures occurs through two GFPPs located on buses 1 and 2 as shown in Fig. 5. The gas consumption coefficients of the GFPPs are presented in Table B.1. On the other hand, the natural gas network consists of 15 nodes, 12 pipes, 3 demands, 2 supplies and 4 compressors. The diameters and lengths of the pipes are reported in Table B.2. In the gas system the Weymouth equation is considered with an average gas temperature of 520 °R with a compressibility factor of 0.9. Unlike the case study 1, the efficiency of pipes 9 and 12 is assumed to be 0.85, while the resting pipes are assumed to be 0.9. Node 1 of the gas network is considered as the slack node with a pressure equal to 1000 PSIA. The compression stations are operating with only one stage and a consumption factor of 199.92 SCFD/HP. The tolerances in the iterative process are 1^{-3} in

20

the gas infrastructure and 1^{-8} in the electrical infrastructure.

Table 3 shows the results of simulation of power generation, loads, supplies and gas demands of the coupled electricity and natural gas infrastructure. GF-PPs on buses 1 and 2 produce 25.97 MW and 60.97 MW, respectively. These generators require 16.2002 MMSCFD of natural gas to provide such amount of power. On the other hand, the reference node of gas network injects 174.2916 MMSCFD of natural gas in order to achieve the system balance. In parallel, Tables 4 and 5 show the power flows and flow rates, as well as the natural gas consumption of the compression units, respectively. Moreover, the results are also reported between the LAT-NR methodology and the NR approach. Note that the mean absolute deviations are 0.25, 11.54 and 0.0024, for the flow rates in pipes, horsepower (HP) and gas consumption in compressors. As can be seen, the LAT-NR technique offers a good solution on the simulation results of coupled natural gas and electricity flows.

In Figs. 6 and 7, the convergence of the iterative process for the calculation of the flow rate, HP and natural gas consumption for case 2 is presented. Identically to Case 1, the process was initialized under the assumption that $L_{(i,j)} = C_{(i,j)}$. Fig. 6 shows the different gas flows calculated in each iterative step (k). Here, it can be observed that during the first iterations the gas flow in certain pipes varies drastically; however, as the number of iterations advances the gas flow stabilizes. In the same way, Fig. 7 a) presents the calculation of HP and Fig. 7 b) shows the gas consumption for the previous calculations. Note that the natural gas consumption is a function of the compressor HP.

21
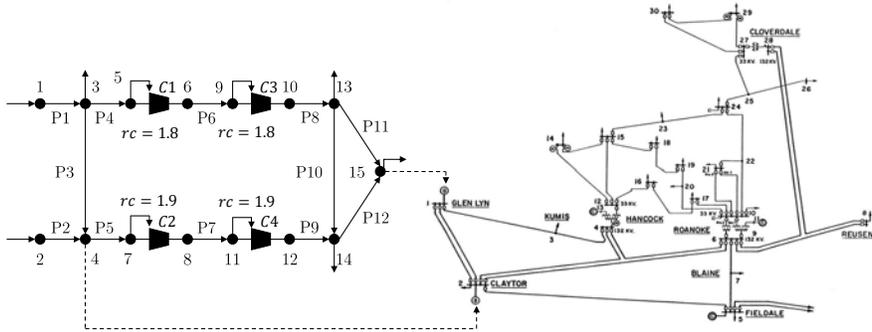
Figure 5: Natural gas network of 15 nodes coupled to power system of 30 buses.

Table 3: Results of supplies, gas demands and power generation for Case 2.

	ELECTRICITY SUPPLY AND LOAD				NATURAL GAS SUPPLY AND DEMAND		
Bus	Generation		Load		Node	Supply	Demand
	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)		(MMSCFD)	(MMSCFD)
1	25.97	-1.00	0.00	0.00	1	174.29	0.00
2	60.97	32.00	21.70	12.70	2	164.80	0.00
3	0.00	0.00	2.40	1.20	3	0.00	92.11
4	0.00	0.00	7.60	1.60	4	0.00	11.11 (GFPP)
7	0.00	0.00	22.80	10.90	5	0.00	0.00
8	0.00	0.00	30.00	30.00	6	0.00	0.00
10	0.00	0.00	5.80	2.00	7	0.00	0.00
12	0.00	0.00	11.20	7.50	8	0.00	0.00
13	37.00	11.35	0.00	0.00	9	0.00	0.00
14	0.00	0.00	6.20	1.60	10	0.00	0.00
15	0.00	0.00	8.20	2.50	11	0.00	0.00
16	0.00	0.00	3.50	1.80	12	0.00	0.00
17	0.00	0.00	9.00	5.80	13	0.00	102.31
18	0.00	0.00	3.20	0.90	14	0.00	102.57
19	0.00	0.00	9.50	3.40	15	0.00	22.89 + 5.09 (GFPP)
20	0.00	0.00	2.20	0.70			
21	0.00	0.00	17.50	11.20			
22	21.59	39.57	0.00	0.00			
23	19.20	7.95	3.20	1.60			
24	0.00	0.00	8.70	6.70			
26	0.00	0.00	3.50	2.30			
27	26.91	10.54	0.00	0.00			
29	0.00	0.00	2.40	0.90			
30	0.00	0.00	10.60	1.90			
TOTAL	191.64	100.41	189.20	107.20		339.09	336.09

22

Table 4: Power flow and gas flow rate for Case 2.

			POWER FLOW							GAS FLOW RATE		
Line	From Bus	To Bus	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)	Pipe	From Node	To Node	Gas Flow NR (MMSCFD)	Gas Flow LAT-NR (MMSCFD)	Error
L1	1	2	10.89	-5.09	-10.86	2.17	P1	1	3	174.29	174.29	0.00
L2	1	3	15.08	4.09	-14.96	-5.57	P2	2	4	164.80	164.80	0.00
L3	2	4	16.07	5.21	-15.89	-6.66	P3	3	4	-42.14	-41.78	0.36
L4	3	4	12.56	4.37	-12.54	-4.30	P4	3	5	124.33	123.96	0.37
L5	2	5	13.79	4.51	-13.68	-6.03	P5	4	7	111.55	111.91	0.36
L6	2	6	20.28	7.42	-19.99	-8.50	P6	6	9	123.56	123.20	0.36
L7	4	6	22.50	11.38	-22.43	-11.12	P7	8	11	110.80	111.16	0.36
L8	5	7	13.68	6.21	-13.56	-6.88	P8	10	13	122.80	122.44	0.36
L9	6	7	9.27	3.17	-9.24	-4.02	P9	12	14	110.06	110.42	0.36
L10	6	8	24.82	24.43	-24.69	-23.92	P10	13	14	5.41	5.11	0.30
L11	6	9	5.79	-3.36	-5.79	3.46	P11	13	15	15.08	15.02	0.06
L12	6	10	3.31	-1.92	-3.31	2.00	P12	14	15	12.90	12.95	0.05
L13	9	11	0.00	0.00	0.00	0.00						
L14	9	10	5.79	-3.36	-5.79	3.51						
L15	4	12	-1.67	-2.02	1.67	2.04						
L16	12	13	-37.00	-9.26	37.00	11.35						
L17	12	14	5.39	0.88	-5.35	-0.80						
L18	12	15	9.48	-1.06	-9.41	1.19						
L19	12	16	9.26	-0.10	-9.18	0.28						
L20	14	15	-0.85	-0.80	0.85	0.80						
L21	16	17	5.68	-2.08	-5.65	2.15						
L22	15	18	9.16	0.76	-9.07	-0.57						
L23	18	19	5.87	-0.33	-5.85	0.38						
L24	19	20	-3.65	-3.78	3.66	3.80						
L25	10	20	5.92	4.62	-5.86	-4.50						
L26	10	17	3.37	8.01	-3.35	-7.95						
L27	10	21	-2.23	-11.67	2.28	11.77						
L28	10	22	-3.75	-8.48	3.82	8.62						
L29	21	22	-19.78	-22.97	19.87	23.16						
L30	15	23	-8.81	-5.25	8.91	5.47						
L31	22	24	-2.10	7.80	2.18	-7.68						
L32	23	24	7.09	0.88	-7.02	-0.75						
L33	24	25	-3.86	1.77	3.89	-1.71						
L34	25	26	3.55	2.37	-3.50	-2.30						
L35	25	27	-7.44	-0.66	7.50	0.78						
L36	28	27	-6.11	-6.08	6.11	6.40						
L37	27	29	6.17	1.68	-6.08	-1.51						
L38	27	30	7.12	1.67	-6.95	-1.35						
L39	29	30	3.68	0.61	-3.65	-0.55						
L40	8	28	-5.31	-6.08	5.34	4.33						
L41	6	28	-0.77	-2.70	0.77	1.75						

Table 5: Consumption of compressor stations for Case 2.

Compressor stations	HP NR	HP LAT-NR	Error	Gas consumption NR (MMSCFD)	Gas consumption LAT-NR (MMSCFD)	Error
C1	3821.61	3810.41	11.20	0.7640	0.7617	0.0023
C2	3727.24	3739.19	11.95	0.7451	0.7475	0.0024
C3	3798.13	3787.00	11.13	0.7593	0.7570	0.0023
C4	3702.35	3714.21	11.86	0.7401	0.7425	0.0024
		TOTAL		3.0086	3.0089	
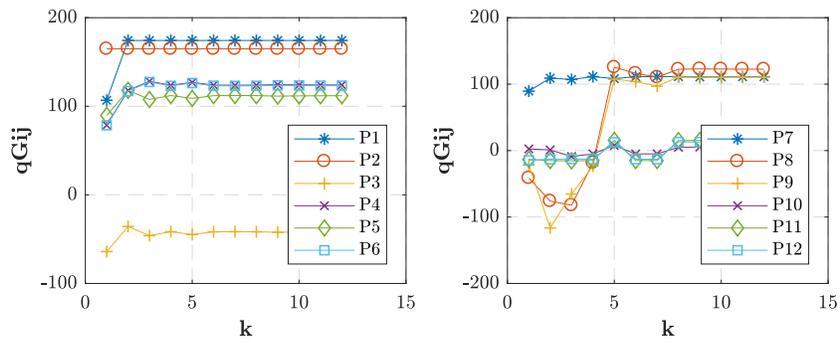
Figure 6: Convergence of flow rate $qGij$ for Case 2.
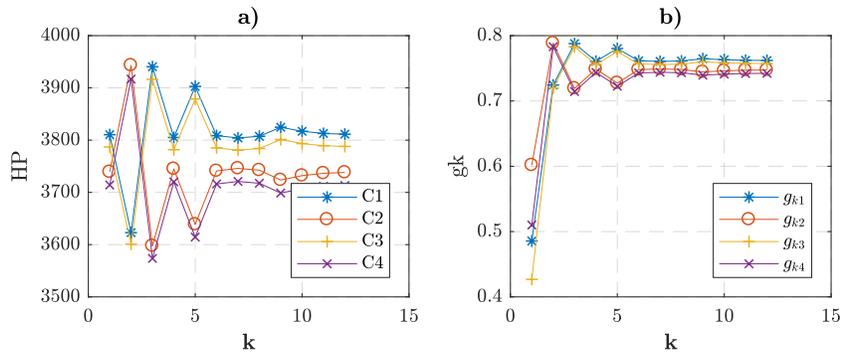
Figure 7: Convergence of HP and $g_{(k)}$ for Case 2.

24

6. Conclusions

In this paper, a methodology has been presented to jointly analyze the coupled electricity and gas flow, where the existence of combined cycle power plants and compressors has been considered. The set of non-linear equations that represent the operation of the power system has been solved using Newton-Raphson (NR) method, while the solution of the gas nodal balance and flow rates in the pipelines and compressors on the gas network have obtained using the linear-analog approach (LAT). Two case studies have been presented to demonstrate the simplicity of the methodology proposed to analyze the interaction between gas and electricity systems. The results obtained with LAT have been verified against the Newton-Raphson method for gas networks, in order to confirm the solution reached, finding a good performance of the joint methodology applied LAT-NR. The application of the proposed approach allows the analysis of vulnerability and resilience of interdependent power and gas infrastructures. The authors are currently working in this area of research using the method described in this paper.

Appendix A. Data for Case 1

Table A.1: Diameters and lengths of pipes for Case 1.

Pipe	Length (miles)	Internal Diameter (in)	Pipe	Length (miles)	Internal Diameter (in)	Pipe	Length (miles)	Internal Diameter (in)
P1	37.49	30.95	P8	13.32	15.5	P15	17.76	12.25
P2	13.88	33.35	P9	15.43	15.5	P16	46.36	12.25
P3	31.26	33.35	P10	10.31	14.18	P17	34.84	15.44
P4	9.13	31.65	P11	19.28	25.17	P18	30.59	25.47
P5	15.99	19.5	P12	21.47	12.25	P19	41.90	25.37
P6	35.52	19.5	P13	11.05	12.25	P20	16.55	23.44
P7	30.18	17.5	P14	5.70	12.25	P21	22.75	23.44

Table A.2: Operation characteristics of GFPPs for Case 1.

Generator	Coefficient of gas consumption			$P_{G_{min}}$	$P_{G_{max}}$
	(MM^3/MW)			(MW)	(MW)
	K_0	K_1	K_2		
2	0	0.00555	0	0	100
3	0	0.00516	0	0	100

Appendix B. Data for Case 2

Table B.1: Operation characteristics of GFPPs for Case 2.

Generator	Coefficient of gas consumption			$P_{G_{min}}$	$P_{G_{max}}$
	(MM^3/MW)			(MW)	(MW)
	K_0	K_1	K_2		
1	0	0.00555	0	0	100
2	0	0.00516	0	0	100

Table B.2: Diameters and lengths of pipes for Case 2.

Pipe	Length (miles)	Internal Diameter (in)	Pipe	Length (miles)	Internal Diameter (in)	Pipe	Length (miles)	Internal Diameter (in)
P1	80.5	19.56	P5	87.9	19.62	P9	97.9	16.69
P2	80.3	19.56	P6	93.5	19.62	P10	86.6	16.69
P3	55.9	19.56	P7	99.7	16.69	P11	79.7	16.69
P4	81.1	19.62	P8	93.5	16.69	P12	83.5	16.69

Acknowledgements

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