The Effect of a Tangential Frictional Force on Rotating Disks: An Experimental Approach

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This paper describes an experiment with two touching rotating disks, whose movement is followed by video analysis. Within the disks’ movements, there are intervals with sliding and intervals without sliding, that is, intervals with frictional forces between the touching surfaces and intervals without frictional forces. This system configuration allows for measurement of the changeable magnitudes and directions of frictional forces (much more difficult to set up with translational motion). This activity may be used to combat students’ misconceptions of “frictional force always opposes the motion” and “between the same two bodies, one gets the same frictional force,” commonly found in the classroom, or just to reinforce the rotational dynamics relationships.

Theoretical background

The following forces act upon the system (depicted in Fig. 1): gravitational forces, vertical, pointing to the center of Earth; action of disks on each other with a tangential component (the tangential friction forces, in the horizontal plane, tangent to the disks, opposing the relative rotational movement, equal in magnitude) and a normal component in the y-direction; reaction forces with a vertical component (canceling the gravitational force) and a horizontal component, exerted by the axles on the disks, canceling the horizontal forces.

If disk 1 (the big disk with radius \( R \) and moment of inertia \( I_1 \)) while rotating around the z-axis is slightly pushed against disk 2 (the small disk with radius \( r \) and moment of inertia \( I_2 \)), initially at rest, frictional forces will appear opposing the relative rotational motion. They will have different effects on the disks: the big disk will be deaccelerated, and the small disk will be accelerated until they rotate with equal tangent speeds. At this instant there is no relative motion between the surfaces, the tangential velocities are equal, and friction disappears. If the velocity of one of the disks suddenly drops, (this can be achieved by using one’s fist to give a quick punch on top of the disk, see Fig. 2, right), the frictional force reappears until a synchronous movement is attained again. This system is therefore an easy showcase to confront the students’ misconceptions: the frictional force does not always act to oppose the motion; indeed, it is the frictional force that causes the small disk to rotate. And within the same situation, two disks rotating in contact, the force can appear, disappear, or even invert its direction.

Starting from the rotational dynamics equations, and noting that all forces (except friction) acting on disk 1 have zero torque with respect to the rotation axis (the same for disk 2), one can write the net torque

\[
\begin{align*}
R \times F_{a1} &= I_1 \cdot \alpha_1 \\
r \times F_{a2} &= I_2 \cdot \alpha_2
\end{align*}
\]

and relate the angular acceleration to the magnitude of frictional force.

Experimental setup

Several components from the rotational system ME-8950A (PASCO, Roseville, CA) were used in the experimental setup. A 22.8-cm diameter disk (\( I = 0.0091 \, \text{kg} \cdot \text{m}^2 \)) and a 7-cm diameter small black disk (intended to be a 900-g counterweight, \( I = 0.0063 \, \text{kg} \cdot \text{m}^2 \), see Gomes et al.\(^4\)) were mounted on two cast iron bases using the rotating axes included in the kit. A ring (12.7-cm outer diameter, 1420 g, \( I = 0.0048 \, \text{kg} \cdot \text{m}^2 \)) was placed on top of the big disk to increase the moment of inertia. Small pieces of white tape were stuck on the periphery of the disks so as to enable rotation tracking through video analysis. Two small photogate heads were placed in such a way that the punch is applied to the smaller disk. A smartphone (Xiao-mi Mi 4c, 30 fps) was placed half a meter above the disks.

The large disk was spun at a certain angular speed (by hand spinning of the axle below the disk) and then the small disk—

![Fig. 1. Forces acting on disk 1 and disk 2. \( P_1 \) is the gravitational force acting on disk 1, and \( F_{a1} \) and \( F_{a2} \) are the tangential and normal forces exerted by disk 2 on disk 1. \( N_{x1}, N_{y1}, \) and \( N_{z1} \) are the components of the normal force exerted by the axle on disk 1. In a similar way for disk 2.](image)

![Fig. 2. Photo of the experimental setup (seen from the top) and scheme (seen from the side). The fist shows how the punch is applied to the smaller disk.](image)
initially at rest—was approached till they touched. At intervals, a punch was given on the small disk, to force an abrupt decrease of its speed (Fig. 2).

The video file was then handled using Tracker—a free open source video analysis and modeling tool—the positions of the white tapes were retrieved, and the data were exported to a CSV file, for further data processing and plotting (in an MS Excel spreadsheet).

Experimental results

The values obtained from preliminary runs from the video analysis and from photogate and PASCO software matched perfectly, so that in subsequent runs only video analysis was used.\(^5\)

Figure 3 shows the variation with time of the angular position of both disks. Both lines exhibit linear sections that correspond to constant tangential speed without slippage and parabolic sections where there is a tangential acceleration (acceleration in opposite directions as evidenced by the opposite nature of the curvature of the two lines in Fig. 3). By fitting the curve with a second-order polynomial function in the slippage stage, one can extract the value of the angular acceleration, and—from that—the magnitude of the frictional force. By fitting of the curves in the synchronous, non-slippage stages, with a linear fit, one retrieves the values of the angular velocities. Table I summarizes the results obtained for the several stages, using Eqs. (1) and (2) and, as expected, the modulus of the pair of frictional forces is identical during slippage (as required by Newton's third law) and the tangential speed of both disks is equal and constant during the synchronous non-slippage stages.

The data in Table I can be visualized in Fig. 4, where the variation of the tangential velocity of both the disks is plotted. Once the disks touched, there was an acceleration of the small disk and a deacceleration of the big disk until the contact points moved with the same velocity.

At that instant, the friction force disappeared, and the two disks stayed with constant velocities. When the first punch disturbed the system, the frictional force reappeared causing new opposing accelerations.

In conclusion, this activity, as described above, allows the teacher to set an example that challenges common student misconceptions about friction. The use of the video analysis allows the user to eschew sensors, interfaces, and dedicated software, and proves again to be a reliable tool in mechanics physics courses.\(^6-13\)

<table>
<thead>
<tr>
<th>magnitude of</th>
<th>Slipping disks after touching</th>
<th>synchro- nous</th>
<th>First punch</th>
<th>synchro- nous</th>
<th>Second punch</th>
<th>synchro- nous</th>
<th>Third punch</th>
<th>synchro- nous</th>
<th>Fourth punch</th>
<th>synchronous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular acceleration (rad/s(^2))</td>
<td>(\omega_{\text{final}} = 3.0(1)) (\omega_{\text{big}} = 4.4(1))</td>
<td>(\omega_{\text{final}} = 18(1)) (\omega_{\text{big}} = 2.6(1))</td>
<td>(\omega_{\text{final}} = 18(1)) (\omega_{\text{big}} = 2.2(1))</td>
<td>(\omega_{\text{final}} = 20(1)) (\omega_{\text{big}} = 2.7(1))</td>
<td>(\omega_{\text{final}} = 18(1)) (\omega_{\text{big}} = 2.6(1))</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Frictional Force (N)</td>
<td>(F_{\text{final}} = 0.55(2)) (F_{\text{big}} = 0.54(1))</td>
<td>(F_{\text{final}} = 0.35(2)) (F_{\text{big}} = 0.32(1))</td>
<td>(F_{\text{final}} = 0.33(2)) (F_{\text{big}} = 0.27(1))</td>
<td>(F_{\text{final}} = 0.35(2)) (F_{\text{big}} = 0.33(1))</td>
<td>(F_{\text{final}} = 0.33(2)) (F_{\text{big}} = 0.31(1))</td>
<td></td>
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</tr>
<tr>
<td>Peripheric velocity (m/s)</td>
<td>(v_{\text{final}} = 2.7(2)) (v_{\text{big}} = 2.7(1))</td>
<td>(v_{\text{final}} = 1.7(2)) (v_{\text{big}} = 1.8(1))</td>
<td>(v_{\text{final}} = 1.02(5)) (v_{\text{big}} = 1.03(4))</td>
<td>(v_{\text{final}} = 0.52(3)) (v_{\text{big}} = 0.52(3))</td>
<td>(v_{\text{final}} = 0.22(2)) (v_{\text{big}} = 0.21(2))</td>
<td></td>
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Table I. Magnitudes of velocities, accelerations, and frictional forces (experimental uncertainties in round brackets).

![Fig. 3. Plot of \(\phi\) as a function of \(t\) for the two disks. The angular position of the big disk, disk 1, is plotted with thin orange markers/lines. The angular position of the small disk, disk 2, is plotted with thick blue markers. The intervals in which there is slippage between the disks are highlighted in grey. A dashed line shows the projected angular position for the big disk if there were no frictional forces. Some points are missing due to the obstruction of the video camera by the punching fist.](image)

![Fig. 4. Plot of angular speed \(\times\) radius as a function of \(t\) for the two disks, in the 0–9-s interval (disk 1- thin orange markers, disk 2- thick blue markers). The numerical values were obtained by smoothing the numerical derivative of \(\phi\) (t). The intervals in which there was slippage between the disks are highlighted in grey.](image)

Acknowledgments

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References


And the Survey Says…

What are typical starting salaries for new physics PhDs?

We have seen that physics PhD recipients follow a variety of paths upon completing their degree. Those who accept potentially permanent positions in the private sector typically report higher starting salaries. Among postdocs, those working at government labs typically report higher starting salaries than those working at universities and UARI (University Affiliated Research Institutes). It is possible that some of the difference there owes to 9/10-month contracts vs. 11/12-month contracts.

In May, we will look at where these physics PhD recipients hope to be working in 10 years.

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All the reports from the SRC are available at www.aip.org/statistics.

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