Universidad Zaragoza

Year : 2019/20

30602 - Mathematics I

Syllabus Information

Academic Year: 2019/20
Subject: 30602 - Mathematics I
Faculty / School: 109 - Facultad de Economía y Empresa
Degree: 432 - Joint Law - Business Administration and Management Programme
ECTS: 6.0
Year: 1
Semester: First semester
Subject Type: Basic Education
Module: ---

1.General information

1.1.Aims of the course

The general objectives of the mathematical subjects in this Degree are included in the following two main goals: (1) Mathematical education, (2) Training to apply Mathematics to the challenges that the students will encounter in their careers.

In the subject Mathematics I, students are introduced to the rigour, the precision, the capacity for abstraction and the scientific method that characterise most of the subjects in the Degree. Regarding the second goal -to train the students to solve specific problems-, in this subject, the students are taught to model and solve simple problems by using techniques of linear algebra and differential and integral calculus.

In particular, the course Mathematics I is intended to extend mathematical knowledge of the matrix calculation and functions of one variable and introducing the study of functions of several variables, thus preparing the student to assimilate in Mathematics II mathematical tools used in economic analysis, mainly in the field of economic theory and Econometrics. This first course helps the students to take off their knowledge of mainly operational tools of mathematics in high school, to the rigor and abstraction of the scientific field of mathematics, allowing them to deal with other subjects of the degree that use mathematical apparatus and future challenges within their profession. At the end of the course the student will know with some precision the mathematical language, which will allow the students to understand economic concepts and interpret results with some rigor, and meet a set of instruments and methods that will allow them the resolution of simple economic problems.

1.2.Context and importance of this course in the degree

For prospective graduates in Business Administration and Management, Mathematics is a working tool that aids the learning of other subjects such as Microeconomics, Macroeconomics and Econometrics. Special attention will be paid to bringing Mathematics closer to problems in a Management scenario, and a basis for the successful learning of other subjects in the Degree will be established.

The teaching of this subject is given by the Department of Economic Analysis of Zaragoza University that moreover takes charge of other closely related subjects to Mathematics, like Microeconomics, Macroeconomics and Econometrics.

The general aim of these subjects is to study the economic problems from a formal point of view, that is to say, to model the economic reality in order to understand it, and to provide a scientific explanation of what happened and also to try to predict what is going to happen. In this framework, in order to achieve this goal, Mathematics provide

- A language without ambiguity that allows defining economic concepts and economic results with the necessary rigour.
- A set of instruments and calculus methods that ease the solving of economic problems.
- A method of reasoning that allows modeling the economic texts and their interrelations, making precise the initial assumptions and providing validity to the obtained conclusions by deduction.

This subject of the first semester deals with the fundamental parts of a model: variables, constants, parameters, equations, identities, domains, types of functions, solving systems of linear equations by means of linear-algebra techniques, etc. In the second part of the subject, Comparative Statics Analysis will be introduced with the study of calculus, both with one and several variables. This topic will be applied later on (in Mathematics II and Microeconomics) to solve optimization problems.

1.3.Recommendations to take this course

Students should have the level of skills required by the subject ?Mathematics Applied to Social Sciences II? (Spanish Secondary Education) in arithmetic operations, matrix theory, and calculus of real-valued functions of one real-valued variable. A good command of the following topics will significantly facilitate the comprehension of this subject:

- Basic operations with fractions, powers, roots, logarithms, etc.
- Calculation of the roots of a polynomial with real coefficients.
- Solving a system of non-linear equations with two unknowns.
- Matrix operations.
- Working out the rank of a matrix and the solution of a system of linear equations by way of elementary matrix operations.
- Operations with functions. The student should clearly understand the difference between function composition and the product of functions, and between concepts like ?to be defined for? and ?to be continuous in? for a real-valued function of one real-valued variable.
- Derivatives and graphs of a wide variety of functions.
- Calculation of some basic primitive functions.

2.Learning goals

2.1.Competences

After completing the course, the student will be competent in the following skills:

Problem solving.

Analysis and synthesis

Decision-making

Applying knowledge to practice

2.2.Learning goals

The student, in order to pass the course, will have to show her/his competence in the following skills:

1. To have gained a certain ability in using mathematical language, both in comprehension and writing.
2. To be able to distinguish whether the relations between variables in a problem are linear or non-linear, and to be able to represent the different cases by means of a suitable mathematical tool.
3. To be able to use matrix notation to represent a problem of an economic nature with linear relations between variables and to be able to apply matrix algebra to solve the problem.
4. To be able to study a system of linear equations making use of the Rouché-Frobenius theorem.
5. To know how to solve a consistent system of linear equations by the most suitable method and be able to interpret the solutions in accordance with the underlying context.
6. To be able to identify a diagonalisable square matrix.
7. To know how to diagonalise a square matrix when this is possible.
8. To be able to apply matrix diagonalisation to an economic context, such as the study of a dynamical process in the long run.
9. To be able to identify a quadratic form and determine its sign by the most suitable method.
10. To be able to distinguish the endogenous and exogenous variables of an economic system and to know how to use functions to represent the relations between these variables.
11. To understand the concepts of continuous and differentiable function applied to an economic context.
12. To be skilled in calculating partial derivatives and in their interpretation in Economics.
13. To be able to identify a differentiable function and to know the implications of differentiability.
14. To be able to identify the chained dependency between different variables and to know how to calculate the variation in the final variables with respect to any of the initial ones.
15. To be able to distinguish whether a function is written in explicit or implicit form and to know how to obtain the partial derivatives in both cases.
16. To be able to identify a homogeneous function and its implications, in particular, in the scenario of production functions.
17. To know which mathematical tool allows the recovery of a total magnitude from the corresponding marginal magnitude.
18. To understand the concepts of primitive function and indefinite integral.

19. To identify whether the indefinite integral of a function can be obtained by basic integration and to be able to work it out by using the table of basic integrals.
20. To be able to choose the most suitable method to calculate the indefinite integral of a function; more specifically, to be able to decide whether this requires a change of variables, integration by parts or integration of rational functions.
21. To understand the geometrical interpretation of the Riemann definite integral.
22. To know how to apply the main properties of the definite integral
23. To be able to relate the concepts of indefinite integral and definite integral.
24. To be able to able to apply the second fundamental theorem of calculus to obtain the value of a definite integral.
25. To know how to make a change of variables in a definite integral.

2.3.Importance of learning goals

The techniques of linear algebra allow the study of both simple models of economic equilibrium and more elaborate theories related to the inter-sector analysis of an economy (input-output models). Differential Calculus, with one and several variables, permits the introduction of the marginalist approach in Economics. In that context, concepts such as function and functional relationship (exogenous and endogenous variable(s)), elasticity, marginal product, marginal rate of substitution, returns to scale, etc. -which will be part of the students' daily jargon- are founded on Calculus (especially on Differential Calculus). Integral Calculus allows the definition of welfare measures (such as consumer surplus) and it is useful for financial model analysis.

In summary, the subject Mathematics I contributes to the comprehension of some theoretical concepts and models which are dealt with in other related subjects in the Degree.

3.Assessment (1st and 2nd call)

3.1.Assessment tasks (description of tasks, marking system and assessment criteria)

The evaluation will be **GLOBAL in both the first and second sittings**. It will consist of a final exam to be taken on the dates determined by the Faculty. The global exam will be written and will assess the proposed learning outcomes through questions that are theoretical, practical, or of a mixed theoretic-practical character and that will be based on the topics taught. It will be worth 10 points.

In addition, in the first sitting it will take place a voluntary intermediate test worth 5 points. This test will assess the student's knowledge of the Chapters 1 and 2 of the subject, and it will take place on the date indicated by the teacher that will be announced in advance in the classroom and/or the virtual teaching platform.

The students who obtain a mark of at least 50% of the maximum value in this test (2.5 points out of 5) will be able to eliminate the corresponding topics from the global exam at the first sitting, and to take the exam of only the rest of the contents (worth 5 points out of 10). In this case, the mark corresponding to the eliminated topics will be added to the mark of the global exam. In order to pass the subject the student should obtain a minimum of 5 points out of 10. If the student obtains a mark of at least 2.5 points out of 5 in the intermediate test and wants to do the whole global exam anyway, the best of the two marks in the first part of the subject will be considered to compute the final mark.

To be eligible for this form of assessment students are required to participate actively and resolve issues, exercises and tests to be carried out in the classroom, according to indications that the teacher in charge of each group of the subject will be exhibiting the same day of the presentation. In such a case, it is necessary to attend and participate in at least 75% of the face-to-face sessions or proposed activities. The student that at the end of the semester does not fulfill this requirement will not be able to be eligible to this procedure of assessment.

It has to be taken into account that the evaluation process closes at the end of the academic year, so it is not possible to claim academic merits from one academic year in a later one.

Students taking their exams at their fifth or sixth opportunity will be marked following the rules established under the Governing Council Agreement on 22 December 2010, which sets out the assessment regulations in the University of Zaragoza.

Evaluation Criteria

Students will be assessed on whether they have acquired the learning outcomes mentioned above. In particular, they will be assessed on the following aspects:

1. Correct mathematical writing.
2. Logical reasoning in the posing and solving of the problems.
3. Reference to the theoretical results used, when relevant.
4. The choice of the most appropriate method for the solving of problems **.**
5. Clarity in the application of mathematical concepts and procedure **s.**
6. Computations carried out with care.
7. The correct expression of the results obtained when solving problems.

4.Methodology, learning tasks, syllabus and resources

4.1.Methodological overview

The learning process that has been designed for this course is based on the following activities:

The objective of this subject is that the students should develop the analytical skills, rigour and intuition needed for using mathematical concepts and results and that they should be able to apply these abilities to the analysis of problems of an economic nature. Therefore, the teaching should aim to provide students with a solid mathematical knowledge and to train them in a way of reasoning that will allow them thereafter to successfully solve a wide variety of questions in an economic scenario. To achieve this, the contents taught will consist of:

4.2.Learning tasks

The programme offered to the students to help them achieve the learning results includes the following activities :

Theoretical lessons which will be based on lectures to present the concepts and results corresponding to the contents. At the same time, some exercises will be solved with the participation of the students to help them comprehend the theoretical concepts presented. These classes are face-to-face and will be given to the full group. Time allotted: 1.2 ECTS credits (30 hours).

Practical lessons, in which the students will apply the theoretical results in order to solve, with the teacher's help, more complete exercises, and problems of an economic nature. Problem sheets will be available for the students and the teacher will announce in advance the problems that will be solved in each practical lesson so that the students can prepare them beforehand. These classes are face-to-face and will be given separately to each subgroup. Time allotted: 1.2 ECTS credits (30 hours each subgroup).

Seminars (practical classes P6), which may consist of a number of different activities designed to support the learning process, including: follow-up of some simple projects that had been assigned to small teams of students and the presentation of these projects; answering questions that students may have regarding some of the contents taught; solving problems of an economic nature by using some of the mathematical tools taught during the classes, etc. These seminars may also be devoted to the teaching of more advanced topics, intended for the students interested in learning some further mathematical tools that would allow them to deal with more general problems. In this way, the students are shown thatboth Mathematics and Economics are vibrant sciences with many facets to be studied.Time allotted:3 hours each subgroup.

Out of class work: 3.6 ECTS credits.

TABLE OF CONTENTS

PART I. MATRICES

Preliminary chapter: Set R^n.

1.- Matrices, determinants, and systems of linear equations.

1.1.- Elementary matrices

1.2.- Determinant of a square matrix.

1.3.- Matrix inversion.

1.4.- Rank of a matrix.

1.5.- Systems of linear equations.

1.5.1.- Rouché-Frobenius theorem.

1.5.2.- Solving systems of linear equations.

2.- Diagonalisation of square matrices.

2.1.- Eigenvalues and eigenvectors of a matrix.

2.2.- Characteristic polynomial.

2.3.- Diagonalisable matrices.

3.- Real quadratic forms.

3.1.- Quadratic forms: definitions.

3.2.- Diagonal form of a quadratic form.

3.3.- Classification of quadratic forms.

3.4.- Restricted quadratic forms.

PART II. DIFFERENTIAL AND INTEGRAL CALCULUS

Preliminary chapter: Basics of Topology in R^n.

1. Real-valued functions of one real-valued variable.

1.1. Real-valued functions of one real-valued variable.

1.2. Limits and continuity.

1.3. Differentiability.

1.4. Higher-order derivatives. Taylor's theorem.

2. Introduction to the theory of integration

2.1. Indefinite integral.

2.1.1. Primitive function and indefinite integral. Properties.

2.1.2. Some techniques of integration: change of variables, integration by parts, and integration of rational functions.

2.2. Definite integral.

2.2.1. Riemann integral. Properties.

2.2.2. Applications.

3. Functions from R*n* to R*m*.

3.1. Scalar-valued and vector-valued functions. Level set.

3.2. Limits and continuity. Properties.

3.3. Derivatives of a function. Properties.

3.4. Differentiable function. Properties.

3.5. Function composition. Chain rule.

3.6. Higher-order derivatives.

3.7. Derivatives of implicit functions.

3.8. Homogeneous functions.

4.3.Syllabus

Chapter 1. Matrices

1.1. Determinants. Applications: calculation of the rank of one matrix, calculation of the inverse matrix and Cramer's Rule

1.2. Rn:Generating systems. Bases.

1.3. Diagonalization of square matrices:

1.3.1. Eigenvalues and eigenvectors of a square matrix: definition and calculation.

1.3.2. Diagonalization of a square matrix.

1.3.3.Application to the calculation of matrix powers.

Chapter 2. Real quadratic forms

2.1. Quadratic forms: definition. Matrix expression and polynomial expression.

2.2. Diagonal expression of a quadratic form.

2.3. Classification of a quadratic form according to its sign.

2.4. Constrained quadratic forms.

Chapter 3. Functions from Rn to Rm

3.1. Preliminaries: topological concepts.

3.2. Functions: domain, range and graph. Level sets of scalar functions.

3.3. Continuity of a function.

3.4. Differentiation of a function. Partial derivatives. Gradient vector. Jacobian matrix.

3.5. Differentiability. Directional derivative of differentiable functions.

3.6. Differentiation of composed functions: Chain's Rule. Tree diagrams.

3.7. Higher order derivatives. Schwarz's Theorem. Hessian matrix. Taylor's Theorem.

3.8. Implicit function Theorem. Differentiation of implicit functions.

3.9. Homogeneous functions. Euler's Theorem.

3.10. Basic integration methods of function of one variable. Barrow's Rule.

4.4.Course planning and calendar

- Presentation of the subject in the first session of the semester, in accordance with the timetable established by the Faculty.
- Continual attendance at, and productive use of, theoretical and practical classes.
- Attendance at practical classes P6.
- Midterm exams, scheduled in accordance with the academic calendar.
- Final exam, on the day established by the Faculty.

4.5.Bibliography and recommended resources