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Long-run inflation-growth relationship: nominal rigidities, unemployment and financial frictions

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LONG-RUN INFLATION-GROWTH RELATIONSHIP:
NOMINAL RIGIDITIES, UNEMPLOYMENT AND
FINANCIAL FRICTIONS

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Introduction

The Federal Open Market Committee (FOMC) judges that inflation at the rate of 2 percent ... is most consistent over the longer run with the Federal Reserve's mandate for price stability and maximum employment. Over time, a higher inflation rate would reduce the public's ability to make accurate longer-term economic and financial decisions. On the other hand, a lower inflation rate would be associated with an elevated probability of falling into deflation, which means prices and perhaps wages, on average, are falling—a phenomenon associated with very weak economic conditions. Having at least a small level of inflation makes it less likely that the economy will experience harmful deflation if economic conditions weaken. The FOMC implements monetary policy to help maintain an inflation rate of 2 percent over the medium term.

Board of Governors of the Federal Reserve System

Schmitt-Grohe and Uribe (2010) state that observed inflation targets around the industrial world are concentrated at two percent per year and investigate the extent to which the observed magnitudes of inflation targets are consistent with the optimal rate of inflation predicted by leading theories of monetary non-neutrality.

They find that consistently those theories imply that the optimal rate of inflation ranges from minus the real rate of interest to numbers insignificantly above zero. Furthermore, they argue that the zero bound on nominal interest rates does not represent an impediment for setting inflation targets near or below zero.

They address the question of whether observed inflation targets around the world, ranging from two percent in developed countries to three and a half percent in developing countries, can be justified on welfare-theoretic grounds, showing that the two leading sources of monetary non-neutrality in modern models of the monetary transmission mechanism—the demand for money and sluggish price

adjustment—jointly predict optimal inflation targets of, at most, zero percent per year.

They show that additional reasons frequently put forward in explaining the desirability of inflation targets of the magnitude observed in the real world—incomplete taxation, zero lower bound on nominal interest rates, downward rigidity in nominal wages, and quality bias in measured inflation—are shown to deliver optimal rates of inflation insignificantly above zero.

In this Ph.D. thesis we investigate the long-run relationship between inflation, economic growth, labor market variables and financial variables in order to find the costs associated with different trend inflation targets adopted by central banks. The conclusions are coincident with the previously quoted valuations of Schmitt-Grohe and Uribe (2010).

Some precedents

The first notion about an optimal inflation rate was the Friedman Rule (Friedman, 1969). As money paid no interest, the optimal policy called for setting nominal interest rates on bonds equal to zero. As the nominal interest rate equals the real interest rate plus expected inflation, to set the nominal interest rate to zero implies that the inflation rate must equal minus the real interest rate. This would involve reducing the nominal quantity of money, but this would fall at a slower rate than the price level, and the quantity of real balances would increase. Models of growth with money, e.g. Sidrausky's model (1967), confirm this policy as adequate.

Recent models of endogenous growth, such as Amano et al. (2009), revitalized the interest in the negative optimal inflation rate, but in this case it would be equal to minus the long-run growth rate in a context compatible with a cashless economy.

In contrast to these proposals, Krugman (2014) and a number of respected macroeconomists, most notably Blanchard (2010) and Ball (2013), have argued for a sharply higher target for the inflation rate, say 4 percent, based on the argument of the perils of the zero lower bound.

However, Coibion, Gorodnichenko and Wieland (2012) derive the utility-based welfare loss function taking into account the effects of positive steady-state inflation, and solve for the optimal level of inflation in the model for plausible calibrations, finding large welfare gains and a very low optimal inflation rate consistent with price stability. Their results suggest that raising the inflation target is too blunt an instrument to efficiently reduce the severe costs of zero-bound episodes that can be interpreted as supporting the current regimes, while providing little evidence in favor of raising these targets against the zero-bound constraint on interest rates.

The reasons for a positive inflation rate target

The reasons for a positive inflation target are very well presented in Billi and Kahn (2008) who, after recognizing that inflation nowadays is low in many countries, think that the question of what inflation rate to aim for is central and that most policymakers agree that they should not allow inflation to fall below zero because the costs of deflation are thought to be high. But they add that both policymakers and economists disagree, however, about how much above zero, if at all, central

banks should aim to keep inflation. One reason for keeping inflation above zero stems from the fact that nominal interest rates cannot fall below zero. And a very low-inflation environment limits the extent to which policymakers can respond to an economic slowdown. Once short-term rates fall to zero, conventional monetary policy tools no longer work to stimulate economic activity.

Knowing what inflation rate to aim for is also critically important because many central banks have adopted formal numerical inflation objectives over the last few decades. Setting an appropriate target for inflation requires understanding how alternative inflation objectives impact economic stability and overall economic well-being. Ideally, policymakers should aim for an inflation rate that maximizes the economic well-being of the public, but Billi and Kahn (2008) think that rigorous estimates of such an “optimal inflation rate” have not been available in economics literature.

There is widespread agreement among the public, economists and policymakers that inflation is bad for the economy. As a result, in recent decades, central banks have adopted policies first to fight inflation and then to keep inflation low. But, for a number of reasons, inflation can be too low. Accordingly, while policymakers want to keep inflation low, they have not typically aimed for zero inflation. In fact, the target is around 2%.

Inflation is costly. When it is unanticipated, it arbitrarily benefits debtors and hurts creditors by decreasing the nominal value of outstanding debt. It discourages saving and investment by creating uncertainty about future prices. And, it forces businesses and individuals to spend time and resources predicting future prices and hedging against the risk of unexpected changes in the price level.

Inflation is also costly even when it is fully anticipated. Through its interaction with the tax system, it can increase tax burdens by artificially raising incomes and profits. In addition, inflation causes firms to incur costs of changing prices. And, to the extent firms only infrequently change prices, inflation can distort relative prices and undermine the efficiency of the market's pricing mechanism. Finally, inflation causes individuals to hold less cash and make more trips to the bank because inflation lowers the relative value of money holdings. All of these factors cause the economy to operate less efficiently, hampering economic growth and ultimately reducing standards of living. As a result, policymakers want to keep inflation low.

Although inflation is costly, for a number of reasons, inflation can be too low. First, available measures of inflation are imperfect and tend to overstate "true" inflation. Measurement errors of the available adjustments for improvement in the quality of goods and services are inadequate: there are difficulties in incorporating new goods into the indexes, consumer willingness to substitute cheaper goods and services for similar products that have seen price increases, and changes in consumers' shopping patterns that may favor discount retailers. For all these reasons measures of inflation are imperfect and tend to be biased upward. A measured inflation rate of 0 percent would not correspond to price stability, but rather would imply a decline in the price level over time. Recent estimates suggest price stability would be associated with an inflation rate of just under 1 percent per year, more precisely between 0.4 and 0.6 percent per year.

Second, a little inflation may make it easier for firms to reduce real wages when necessary because of downward wage rigidity in order to maintain employment.

Third, a negative inflation rate deflation could be even more costly than a similar rate of inflation, suggesting that a low rate of inflation might be desirable to insure against falling prices because the cost of deflation is particularly severe compared to inflation. A positive rate of inflation may reduce the risk of the economy ever experiencing deflation and its consequences. Finally, at very low levels of inflation, nominal interest rates may be close to zero, limiting a central bank's ability to ease policy in response to economic weakness.

The contribution of the Ph.D. Thesis

The issue tackled in this thesis is the long-run relationship inflation/growth in neo-Keynesian DSGE models with endogenous growth, considering the coherence of the inflation targets of the central banks.

The results obtained are related to the type of wage considered, the existence of frictions in labor and credit markets and the empirical implications for six advanced countries. They can be summarized in the following four points:

- The consideration of the wage per unit of labor (per worker or per hour) is the reason for obtaining negative optimal trend inflation, while that inflation is zero with wage per unit of human capital. Both results come from a dynamic mechanism that reaches a situation which is equivalent to wage flexibility.
- The same results on optimal inflation are confirmed once unemployment is introduced in the models and it is found an extension of the Friedman critique to the de Phillips curve in the long

run which generalizes the usual version of the mainstream macroeconomic models.

The extension maintains the inflation/unemployment independence (natural rate), adding a protagonist role of employment and labor force participation rates, that are maximal for the optimal inflation rate.

The inflation rate value that coincides with the natural unemployment rate is not indifferent, as in Friedman's critique, because it can be accompanied by different growth, employment and labor force participation rates.

- The frictions of the financial sector confirm the same results on the optimal trend inflation and not always have a negative impact on the achievable economic growth because it depends on the type of friction. Finally, the empirical application explores in what extent the six considered countries could improve their growth, employment and labor force participation rates according to the obtained inflation/growth relationship in every case. The growth gain, after adjusting their inflation targets, would come for the USA, Australia and Spain from an increase of the employment and labor force participation rates, while in the case of Japan, France y Germany it would come from a productivity increase.

I describe below the way these results have been obtained in the three chapters of the thesis.

Chapter 1: *Optimal trend inflation, nominal rigidities and human capital growth*

The current value for the inflation rate target set by many central banks—around 2% in advanced economies— is not currently being reached and the reaction has been the adoption of unconventional measures in order to address the problem posed by the zero lower bound and the menace of deflation. At the same time, there is a great concern about the simultaneous generalization of low growth rates in the advanced economies, which has led once again to discussion about the phenomenon of “secular stagnation.” Consequently, the current concerns about trend inflation and long-run growth has brought to the fore the high value that a well-established relationship between them would have, in order to shed light on the management of these two very important variables.

A point of high concern for central banks is to what extent the existence of a well-established relationship between trend inflation and long-run growth could help in the assessment of monetary policy decisions in setting the inflation rate target. When rigidities, especially in wages, are considered in endogenous growth contexts, non-neutrality is an immediate result in the long run, as an incipient research line has shown. As the stability of a given currency is linked to the credibility of the inflation rate target, this rate can be considered as the trend inflation and a well-established relationship with the long-run growth could help in the choice of the target.

Taking into consideration this economic context and the quoted precedents, especially Amano et al. (2009) and Amano, Carter and Moran (2012), the objective

of this chapter is to know whether the optimal trend inflation rate around -2% to -3% can be generalized for any engine of endogenous growth with sticky prices and wages. In order to answer this question, we have considered four different endogenous growth models: Schumpeterian technological change, as in Aghion and Howitt (1992); physical capital externality, as in Romer (1986); technological change, as in Romer (1990); and human capital, as in Lucas (1988).

After analyzing the impact of price and wage rigidities on the long-run growth rate in the four models, we can conclude that the optimal trend inflation rate is not always negative. Firstly because, with only price rigidity, the long-run relationship between inflation and growth is not relevant, at least for admissible values of quarterly inflation or deflation rates, so the neutrality of trend inflation is the conclusion in this case, regardless of growth engines. Secondly, this result cannot be generalized either when we consider sticky wages, since the model based on human capital accumulation reaches the maximum growth rate for a null inflation. While the annual objective inflation rate is within the interval between -2% and -3% when we consider stickiness in nominal wage per hour (Schumpeterian, physical capital and technological change models), we find that the model of human capital reaches its maximum growth for a null inflation rate as a consequence of its stickiness in nominal wage-per-unit of human capital.

What is the cause of this difference? It lies in the fact that, in the first group of models with wages per hour or worker, the wage-setting process must adjust the nominal value in order to compensate inflation and growth. Consequently, a negative trend inflation rate with an absolute value equal to the long-run growth rate makes a nominal revision unnecessary, so the situation is the same as if there

were wage and price flexibility. The long-run real wage that individuals receive will grow at the same rate as with flexibility, thanks to the falling trend of prices.

However, the wage-setting process in the human capital model must not compensate the effect of growth because wages respond without lag to the human capital accumulation process carried out by individuals. That is, wage rigidity does not affect the productivity component of labor contracts, only the wage-per-unit of human capital because wage contracts consider the skill aspects separately and revise them with flexibility. Then, as nominal wages grow in the long run at the rate of trend inflation, the compensation for inflation is sufficient to recover the equilibrium real value of the wage-per-unit of human capital. Then the maximum growth rate is reached with null trend inflation in a situation that is also equivalent to flexibility. The long-run real wage that individuals receive will grow, thanks to long-run human capital accumulation.

On the basis of the above analysis, we identify that the ultimate reason behind negative or null optimal trend inflation is the attainment of a situation that is equivalent to wage flexibility, with the result depending on the type of wage unit considered in the wage settlements. This finding is a clear contribution to showing the mechanism that clarifies the meaning and the costs of nominal wage rigidities in the long-run when the inflation target set by central banks is not the optimal one.

An additional and outstanding aspect of the models with wages per hour is that trend inflation has a very small effect on long-run growth. However, the results of the human capital model show a much more important non-neutrality phenomenon, given that the distortion directly affects not only the labor demand,

but also the effort devoted to human capital accumulation and, hence, the growth rate. This result suggests the convenience of taking into account the role played by human capital (or job skill) when studying the influence of wage rigidity on growth in the long run. All the previous studies have considered the influence of nominal rigidities on wages per hour, ignoring the important role played by human capital in the wage-setting process.

Chapter 2: Labor force participation and growth in the long run: a New-Keynesian extension of Friedman's Phillips curve revision

Through the second chapter we will continue studying the relationship between trend inflation and long-run economic growth, but integrating new variables that provide a more general perspective, which, except for a few rare exceptions, are not usually considered in macroeconomic models. Specifically, we analyze what is the impact on the quoted results in Chapter 1 of considering that labor supply is no longer equal to labor demand and, therefore, unemployment appears in the economy. The objective is to know how a distortion in the labor market, which leads to unemployment, affects the value obtained for the optimal trend inflation.

The results of this chapter come to confirm Friedman's criticism of the Phillips curve in the long run in 1967, introducing some additional endogenous labor variables and a distortion in the labor market. We consider as endogenous variable not only employment, but also unemployment and labor force participation. This shift in the focus provides important and apparently groundbreaking conclusions, given the more general perspective that it is able to provide for the macroeconomic dynamics. A first result confirms the irrelevance of unemployment rate as a long-run key macroeconomic variable in the labor market, being replaced

by the employment and labor force participation rates. In fact, we can confirm by means of simulations using Dynare that the optimal trend inflation rate is independent of the unemployment rate but, by contrast, it maximizes simultaneously employment and labor force participation rates.

These results contain significant promise, beyond the confirmation of Friedman's hypothesis of the independence between trend inflation and unemployment in the long run, because it seems that they could represent a relevant New-Keynesian extension of Friedman's Phillips curve critique. Effectively, the constant long-run unemployment rate is compatible with many values of the labor force participation and employment rates, two variables with a great factual economic impact but with hardly any presence in theoretical macroeconomic analysis. In our results, they appear as two key labor market variables in the relationship between trend inflation and long-run growth, from the perspective of the monetary policy summarized by the trend inflation rate as the inflation target and, therefore, the possibilities of monetary policy to affect it.

Moreover, the new labor market context (efficiency wages) has relevant additional consequences. While unemployment rate with wage stickiness is higher than that of flexibility in the Schumpeterian model, we find the opposite in the human capital model. But the more remarkable result is that a sticky average real wage can be higher, equal or lower than a flexible one and, unlike chapter 1, the value of the achievable growth rate will be respectively lower, equal or higher with wage stickiness than with wage flexibility. Consequently, the unemployment caused by the labor market distortion introduced can lead to a "growth loss" or a "growth premium" in the case of wage stickiness, as well as a loss or a gain in employment

and labor force participation rates. Nevertheless, from all these possibilities, the more likely combinations of efficiency wage parameters in the two models are those leading to a “growth loss.”

Chapter 3: Financial frictions, unemployment and long-run inflation–growth relationship: empirical implications

Throughout Chapter 3 we complete our analysis with the introduction of a financial sector. We can find different precedents in the existing literature on the link between financial system and long-run economic growth, where we are interested in the long-run relationship between leverage ratio and growth. The results in the literature to date appear conclusive in that there is not a relation generally applicable to all the possible situations. According to the literature, there is not a unique relation between the leverage ratio and the growth rate, with any direction of causality and even the absence of causality being possible when the sample pools cross-country and time series data.

The first objective of this chapter is to know how a distortion in the financial market impacts on the quoted conclusions of the two previous chapters, taking into consideration that monetary policy is closely related to interest rates and, hence, to financial activity. The results obtained from this analysis allow us to confirm that the incorporation of financial frictions has not impact on the main results from chapters 1 and 2. On the other hand, the consequences of introducing financial frictions cannot be generalized regardless of the friction type, since we have found that the costly verification model has no impact on the long-run inflation–growth relationship if we consider nominal wage stickiness, unlike flexibility. Moreover, we confirm the previously quoted results about the non-

conclusive influence of the leverage ratio on the growth rate given that, in the two considered models of financial frictions, the behavior of the relation between the two variables is contrary once trend inflation is considered.

Our approach involves intermediate goods producers' or retailers' need for external resources to fund their R&D activity or working capital, respectively, because their internal funds are no longer enough. But in addition to that, we will consider the existence of asymmetric information in the financial market: asymmetric information in favor of financial entities in the Schumpeterian model, *financial intermediation model* according to Gertler and Karadi (2011), and asymmetric information in favor of borrowers in the Lucas human capital model, *costly verification* according to Bernanke, Gertler and Gilchrist (1999) and Gertler (2009).

The second objective of this third chapter is to explore the empirical implications of the models for six developed countries governed by different central banks (United States, Australia, Japan and European Monetary Union countries –EMU– France, Spain and Germany) in order to conclude to what extent they could improve their long-run growth, employment and labor force participation rates. The conclusions from the Schumpeterian model are that the two countries with more potential increase in long-run growth are Japan and Germany. The USA and France are situated at an intermediate level of improvement, while Australia and Spain are the two countries with the lowest level of growth gain. In the Lucas human capital model, France is added to the first group, Australia and Spain would be in the intermediate group and the USA would have the lowest improvement.

The way to achieve these gains would be a change in trend inflation (inflation target). The single country with a quarterly positive change is Japan (+0.21%), while the rest of the countries should decrease the quarterly target by at least -0.27% Germany, -0.36% France, -0.76% Spain, and -0.88% the USA and Australia. For these last two countries the gain in the employment and labor force participation (LFP) rates would be, at most, one percentage point, three quarters of a percentage point in Spain and near zero in Japan, Germany and France. So, the growth gain in the first three countries would come from the improvement in the LFP rate, while in the case of the last three it would come from a change in the allocation of resources leading to an increase in total factor productivity (TFP) growth. In other words, the growth gain would come from the increase in the TFP growth in the second group while in the first one the growth gain would come from the LFP rate.

The desirability of low inflation rate target

As a consequence of the foregoing considerations, although certain economic vicissitudes may have led to the justification of high inflation targets, under the current economic environment and from a theoretical point of view, the decisions of monetary policy should not only avoid increasing the target, but they should be aimed at reducing it.

The results of this thesis provide substantial grounds to conclude the suitability of a low rate target of near zero in terms of economic growth and welfare. According to the contribution of this thesis, and following authors such as Schmitt-Grohe and Uribe (2010) and Coibion, Gorodnichenko and Wieland (2012), upward moves of

the target represent an opportunity cost not only in terms of economic growth and welfare, but also of employment and labor participation force rates.

Chapter 1

Optimal trend inflation, nominal rigidities and human capital growth¹

Abstract

A wage-setting process defined in terms of wage per hour is the key factor for obtaining negative optimal trend inflation in a closed economy. However, this inflation will be zero if the process is established on the wage-per-unit of human capital. The origin of both results is a dynamic mechanism that, with some differences, makes possible the attainment of a situation equivalent to wage flexibility. Finally, while the effect of trend inflation on the long-run growth rate is tiny in the first case, it is much greater in the second, highlighting the relevance of this approach.

1.1. Introduction

After the Great Recession, central banks have assumed a leading role to revitalize credit, consumption and growth in many economies, especially in the advanced ones. This role focuses interest on the consequences of their monetary policy decisions, not only in the short but also in the long run. Given that the current monetary policy is relatively new, these consequences, especially those related to the long run, are not well known.

Among these consequences, a point of high concern is to what extent the existence of a well-established relationship between trend inflation and long-run growth could help in the assessment of monetary policy decisions. When rigidities,

¹ An article with the results of this chapter was accepted in April 2018 by *Macroeconomic Dynamics* and is currently pending publication.

especially in wages, are considered in endogenous growth contexts, the non-neutrality of this monetary policy is an immediate result in the long run, as an incipient research line has shown. As the stability of a given currency is linked to the credibility of the inflation rate target, this rate can be considered as the trend inflation, and a well-established relationship with the long-run growth could help in the choice of the target.

There exists a vast literature on the optimal inflation rate that has been revitalized recently as a consequence of the limitations posed by the zero lower bound of the interest rate to the monetary policy. As a consequence, the suggestion of increasing the target to separate it from zero has emerged (Krugman, 2014). Although it has received some support, the proposal has been predominantly contested from the academic world because the results on the optimal inflation rate point to values around zero or even clearly negative.

The question to pose, then, is what would be the explanation for the very common 2% target value of many central banks? A complete set of reasons has been offered to support this value and the non-consideration of its increase. But this is not a central question of interest in this chapter. The fact is that this value has not been reached for a long time and the reaction has been the adoption of unconventional measures in order to address the problem posed by the zero lower bound and the menace of deflation. At the same time, there is great concern about the simultaneous generalization of low growth rates in the advanced economies that has led to the reopening of the discussion about the phenomenon of “secular stagnation.” As can be seen, the current concerns about trend inflation and long-run growth brings to the fore the high value that a well-established relationship

between the two would have, in order to shed light on the management of these very important couple of variables.

Research regarding this relationship requires economic models that connect their short and long-term interactions from the most convenient perspective. Ignoring these interactions might be causing a misunderstanding of interesting aspects of macroeconomic behavior. Dynamic stochastic equilibrium models (DSGE) used for the analysis of monetary policy have, until recently, avoided the introduction of trend inflation and long-run growth and, consequently, their implications are still not well known. The long-run implications of trend inflation have been studied by Ascari (2004), Hornstein and Wolman (2005), Kiley (2007), Levin and Yun (2007), Amano, Ambler and Rebei (2007), Ascari and Ropele (2007), and Coibion and Gorodnichenko (2011), while the study of the interactions of long-run growth and monetary policy have been initiated by Amano et al. (2009), Coibion, Gorodnichenko and Wieland (2012), and Amano, Carter and Moran (2012), partly as a consequence of the introduction of trend inflation into DSGE models.

Amano et al. (2009) and Coibion, Gorodnichenko and Wieland (2012) provide clear conclusions that show the steady optimal inflation rate is negative with nominal wage rigidity. But these two works have a clear limitation. They assume an exogenous growth rate and conclude the optimal inflation rate from simulations with price and wage rigidities. This assumption of growth exogeneity would be admissible provided monetary policy is neutral in the long run, or even if its non-neutrality were quantitatively insignificant as some models seem to point out. But things are very different if alternative models are able to suggest a significant enough effect of trend inflation on long-run growth. The contribution of Amano,

Carter and Moran (2012) extended the conclusion to the endogenous growth context and confirmed the result in a model of technological change, as suggested by Romer (1990).

The main conclusion of Amano et al. (2009) and Amano, Carter and Moran (2012) is that the value of the trend inflation rate that maximizes welfare and the log-run growth rate is clearly negative (respectively, -1.8% and -3%). From the endogenous growth point of view, this is a result requiring confirmation because it has been obtained through the introduction of a particular type of growth engine, technological change as in Romer (1990), into a DSGE model with trend inflation and wage and price rigidities with Taylor contracts.

Throughout this paper we analyze whether this result, the maximization of the long-run growth rate for a negative long-run inflation rate of around 2%–3%, can be generalized to any other engine of growth with sticky prices and wages. In order to answer this question, we introduce alternative growth engines into the same framework of Amano et al. (2009), Coibion, Gorodnichenko and Wieland (2012) and Amano, Carter and Moran (2012). Specifically, we will introduce four different growth engines: physical capital externality as in Romer (1986); Schumpeterian technological change according to Aghion and Howitt (1992); technological change in Romer's model (1990) (reformulating the version presented by Amano, Carter and Moran (2012)) and human capital as in Lucas (1988). The steady-state properties of these models will be developed to proceed subsequently to their calibration and simulation in order to replicate the type of results required to answer the question posed. The calibration and simulation of these models will be made by means of the software platform Dynare.

After analyzing the impact of price and wage rigidities on the long-run growth rate, we can conclude that the trend inflation rate that makes the long-run growth maximum is not always negative. The conclusion of Amano et al. (2009) and Amano, Carter and Moran (2012) cannot be generalized. Firstly, because with only price rigidity the long-run relationship between inflation and growth is not relevant, at least for admissible values of inflation or deflation, so the neutrality of trend inflation in this case is the conclusion for the four growth engines.

But, if we consider sticky wages, this result cannot be generalized either. While three models confirm a result similar to the one obtained in Amano et al. (2009) and Amano, Carter y Moran (2012), the model based on human capital accumulation reaches the maximum growth rate for a null inflation. That is, the inflation rate that maximizes the growth rate is within the interval $[-1,5\% \text{ to } -3\%]$ when we consider sticky wages in models with physical capital externality, as in Romer (1986), Schumpeterian growth as in Aghion and Howitt (1992) or technological change as in Romer (1990). But this result is not general since we find a growth engine that reaches its maximum growth for a null inflation, which corresponds to the model of human capital based on Lucas (1988).

What is the cause of this difference? It lies in the fact that in the first three models the wage-setting process must adjust the nominal value in order to compensate inflation and growth. A negative trend inflation rate with absolute value equal to long-run growth rate makes a nominal modification unnecessary, and the situation would be as if there were wage and price flexibility. As a consequence, the maximum growth is the same as in the case of flexibility. Any deviation of the trend inflation rate from this negative value has the effect of a distortion, leading to a

lower growth rate because it has the effect of a negative shock of productivity. In this way the decrease in prices implied by the optimal trend inflation value avoids the negative distortion on growth introduced by wage stickiness. The long-run real wage individuals receive will grow as with flexibility, thanks to the falling trend of prices.

Wage setting in the human capital model does must not compensate the effect of growth because wages respond without lag to the human capital accumulation process carried out by the individuals. Then the compensation of the inflation is sufficient to recover the equilibrium real value of the wage-per-unit of human capital. Given the distortion inflation introduces with price and wage rigidity, growth is maximum with null trend inflation. The long-term real wage individuals receive will grow thanks to long-term human capital accumulation.

An additional and outstanding aspect of the first three models is that trend inflation has a very small effect on growth in steady state. But, once again, the model of human capital is the exception in the quantitative importance of the effect. This is the model whose results indicate an important non-neutrality phenomenon.

This result indicates the convenience of taking into account separately the evolution of the dynamic of human capital or labour skill when studying the influence of wage rigidity on growth in the long run. All the previous studies have considered the influence of nominal rigidities in wages per unit of labour. In the case of the human capital model, the effect of nominal rigidity in the wage-per-unit of human capital is studied. In this case, the nominal evolution of the wage-per-unit of labour contains the dynamics of the human capital or, in other words, the

growth rate of the economy. In steady state the real wage-per-unit of human capital must be constant without taking into account the growth of the economy.

Before proceeding with the rest of the chapter, we must emphasize that we are only going to talk about growth to identify the optimal trend inflation, although the ultimate goal of the individuals is welfare maximization. The reason for this is that, given our interest in the steady state, talking in terms of growth is equivalent to talking in terms of utility, as in Gomme (1993) and Amano, Carter and Moran (2012).

Section 2 contains the presentation of the four models and concludes with the steady-state systems of equations that are systematically collected in Appendix A.2. Section 3 analyzes the effects of nominal rigidities on growth rate. Section 4 summarizes the main impacts described in Section 3 and makes a comparison between the behavior of the different models. The transmission mechanisms of the models and some outstanding results are interpreted in Section 5. Finally, Section 6 summarizes the main findings.

1.2. DSGE models with endogenous growth and staggered wage and price setting

Four growth engines are analyzed in this paper: physical capital externality as in Romer (1986); Schumpeterian technological change, according to Aghion and Howitt (1992); the type of technological change introduced by Romer (1990); and human capital as characterized in Lucas (1988).

The technological change and human capital models require a special mention. The

first is the model used by Amano, Carter and Moran (2012); however, we will develop it in greater detail in order to be able to analyze more thoroughly the impact of nominal rigidities on growth.

Human capital accumulation raises the productivity of both labour and physical capital. The basic idea of this model is that people divide their time between work and training. So there is a trade-off, since when taking part in training people do not receive work income, but their future productivity will increase and, consequently, their future wages. It is a question of postponing income today (and hence consumption) for income tomorrow.

The elements of the four models will be presented throughout this section. Firstly, the behavior of the main agents in the economy will be described. Secondly, the source of growth will be explained in greater detail. Hereafter, we will obtain the mechanisms of price and wage setting. Finally, we will conclude with the system of equations that characterizes the steady state in each model.

The assumption is made that there is no money, following the “cashless economy” hypothesis (Woodford, 2003; Galí, 2008) typically adopted in New-Keynesian macroeconomic models.

1.2.1 Agents

Households

Household members offer labor to intermediate or final goods producers depending on the model, consume the final goods and hold bonds. Households are composed of infinite-horizon individuals and are uniformly distributed in a

continuum $[0, 1]$.

Their expected utility takes the form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{1}{1+\nu} \int_0^1 L_{st}^{1+\nu} ds \right) \quad (1.1)$$

where $\beta \in (0,1)$ is the utility discount factor, $\nu (>0)$ the disutility of the labor parameter, C is consumption, L_s represents the supply of labor service s with $s \in [0,1]$ and $L_t \left(= \int_0^1 L_{st}^{\frac{(\sigma-1)}{\sigma}} ds \right)^{\sigma/\sigma-1}$ the composite supply of labor services, and σ being the elasticity of substitution.

Furthermore, households must satisfy their budget constraint, which prevents the present value of the expenditure exceeding the income and the value of their initial assets.

The expression of the budget constraints for the model with capital externality is:

$$C_t + \frac{B_t}{P_t} = \frac{B_{t-1}}{P_t} R_t^{st} + D_t + \int_0^1 \frac{W_{st}}{P_t} L_{st} ds \quad (1.2.a)$$

while the only difference in the budget constraints for the Schumpeterian and technological change models is the inclusion of the variable $R\&D_t$:

$$C_t + \frac{B_t}{P_t} + R\&D_t = \frac{B_{t-1}}{P_t} R_t^{st} + D_t + \int_0^1 \frac{W_{st}}{P_t} L_{st} ds \quad (1.2.b)$$

and that corresponding to the human capital model differs by including the dynamics associated with the variable K :

$$C_t + \frac{B_t}{P_t} + K_{t+\tau+1} = \frac{B_{t-1}}{P_t} R_t^{st} + D_t + \int_0^1 \frac{W_{st}}{P_t} L_{st} ds + (1 + R_{t+\tau} - \delta) K_{t+\tau} \quad (1.2.c)$$

The variables that define these budgets constraints are the following: B_t , the nominal value of the stock of one-period life bonds that households hold in their portfolios; R_t^{st} , the nominal gross interest rate; R_t the real gross interest rate; W_{st} , the nominal wage for labor service s ; D_t , firms' dividends; C_t , consumption; $R\&D_t$, investment in "research and development" (R&D); and K_t the stock of capital owned by the household.

Moreover, we must consider the following restriction to avoid Ponzi schemes (Galí, 2008), in the four models:

$$\lim_{T \rightarrow \infty} E_t(B_T) \geq 0 \quad (1.3)$$

Regarding the human capital model, as in Christiano, Eichenbaum, and Evans (2005), the representative household holds a stock of physical capital, rents it to the intermediate goods producers, and decides how much physical capital to accumulate. For simplicity, it is assumed in this model that there are no adjustment costs of investment. Then, the law of motion of physical capital is given as follows:

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (1.4)$$

where δ represents a depreciation rate of physical capital and I_t gross investment.

In addition, human capital requires a special mention related to the effective supply of every labor service s . Individuals are supposed to make two decisions. First, each individual chooses the total time devoted to non-leisure activities, that is, production activity plus accumulation of human capital, N_{st} . Second, each household member also chooses the fraction of every time unit that will be devoted to the production activity, u_{st} ($u_{st} \in [0,1]$), and the fraction devoted to

human capital accumulation, $1 - u_{st}$. Therefore, the effective labor supply is defined as follows:

$$L_{st} = u_{st}N_{st}h_{st} \quad (1.5)$$

It is assumed that human capital accumulation has the following technology

$$h_{st+1} = [1 + \xi(1 - u_{st})N_{st}]h_{st} \quad (1.6)$$

where ξ is a productivity parameter of the accumulation process. The law of motion for the economy's total human capital is then given by:

$$h_{t+1} = \int_0^1 h_{st} di = \left\{ \int_0^1 [1 + \xi(1 - u_{st})N_{st}] \frac{h_{st}}{h_t} ds \right\} h_t \quad (1.7)$$

Capital goods producers

Capital goods producers are agents present only in the physical capital externality model. In this model, capital stock is accumulated through the investment process subject to the adjustment costs function. The main relationships of the capital accumulation process in this model are:

$$K_{t+1} = K_t + I_t^n \quad (1.8)$$

$$I_t = I_t^n \left(1 + f \left(\frac{I_t^n}{K_t} \right) \right) \quad (1.9)$$

$$g_t = \frac{K_{t+1}}{K_t} = 1 + \frac{I_t^n}{K_t} \quad (1.10)$$

where I_t^n and I_t are net and gross investment at t , respectively, and $f \left(\frac{I_t^n}{K_t} \right)$ is the adjustment costs. At the beginning of each period, capital producers convert the

used capital into new capital and resell it to the intermediate goods producers, along with the newly created capital. Unlike net investment, refurbished capital does not entail adjustment costs.

The capital producers will determine the capital price Q_t that maximizes the value of their net investment. So the investment decision problem, which is common to all capital producers, is the following:

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t \left[Q_t I_t^n - \left(I_t^n + f\left(\frac{I_t^n}{K_t}\right) I_t^n \right) \right] \quad (1.11)$$

from which the price of the new capital can be obtained:

$$Q_t = 1 + f\left(\frac{I_t^n}{K_t}\right) + \frac{I_t^n}{K_t} f'\left(\frac{I_t^n}{K_t}\right) \quad (1.12)$$

We assume the following functional form of the adjustment cost:

$$f\left(\frac{I_t^n}{K_t}\right) = \frac{\varsigma}{2} \left(\frac{I_t^n}{K_t} - \frac{I^n}{K} \right)^2 \quad (1.13)$$

where $\varsigma > 0$, $\frac{I^n}{K}$ is steady-state net investment–capital ratio and $f\left(\frac{I^n}{K}\right) = f'\left(\frac{I^n}{K}\right) = 0$. Consequently, Q is 1 in steady state.

Intermediate goods firms

There are two possible types of model, depending on the behavior of the intermediate goods firms. In the first type intermediate goods firms have the same technology as the final good, while in the second intermediate goods are produced by means of a differentiated production function. The first group

includes the Schumpeterian and technological change models and the second the model with physical capital externality and the human capital model.

Schumpeterian and technological change models

Under the Schumpeterian and technological change models, monopolistically competitive firms obtain intermediate goods. This sector operates a simple technology that generates one unit of a given intermediate good from one unit of final output. They sell their goods to final goods firms and set the prices according to Taylor contracts for I periods.

Model with physical capital externality

Unlike the previous models, in the model with physical capital externality each intermediate goods producer is indexed by $j \in [0, 1]$ and has a Cobb–Douglas production function of the type:

$$Y_{jt}^i = K_{jt}^\alpha \left[K_t \left(\int_0^1 L_{s jt}^{\frac{\sigma-1}{\sigma}} ds \right)^{\sigma/\sigma-1} \right]^{1-\alpha} = K_{jt}^\alpha [K_t L_t]^{1-\alpha} \quad 0 < \alpha < 1 \quad (1.14)$$

where Y_{jt}^i is the production obtained by firm j with a capital stock K_{jt} , which is acquired from capital producers at the end of period $t-1$. The index $K_t = \int_0^1 K_{jt} dj$ is the stock of knowledge generated by capital accumulation, which firms take as given, and will be the source of economic growth (Romer, 1986). Intermediate goods firms are perfectly competitive.

From profit maximization, the demand for labor of the firm j can be expressed as:

$$L_{jt} = \frac{(1 - \alpha) Y_{jt}^i}{\Delta_{wt}^i} \quad (1.15)$$

$$\Delta_{wt}^i = \left[\int_0^1 \left(\frac{W_{st}}{P_t^i} \right)^{1-\sigma} ds \right]^{1/1-\sigma} \quad (1.16)$$

This labor demand function is common for all producers because $\frac{Y_{jt}^i}{L_{jt}}$ only depends on market elements (Δ^w , average real wage). Consequently, aggregating the production functions of all intermediate goods producers, assuming that they are identical and that the capital-labor ratio is common across them, we have the following expression for the output of intermediate goods:

$$Y_t^i = K_t L_t^{1-\alpha} \quad (1.17)$$

Considering capital market is competitive, the profitability rate (r_t^q) can be obtained from the profit maximization problem:

$$r_t^q = \frac{\alpha P_t^i \frac{Y_{jt}^i}{K_{jt}} + (Q_{t+1} - \delta)}{Q_t} \quad (1.18)$$

Therefore, r_t^q determines the allocation of capital to produce each intermediate good. As it depends only on market factors, we can conclude that it is common for all producers too.

Moreover, given that we suppose absence of financial frictions, we must have:

$$r_t^q = R_t \quad (1.19)$$

Human capital model

This model assumes also that there is a representative perfectly competitive intermediate goods producer $j \in [0, 1]$ with technology:

$$Y_{jt}^i = AK_{jt}^\alpha L_{jt}^{1-\alpha} \quad (1.20)$$

where Y_{jt}^i is the output of a homogeneous intermediate good, A total factor productivity, K_{jt} stock of physical capital, and L_{jt} the composite index of differentiated labor services.

With regard to the labor demand, from profit maximization, we obtain the demand for labor service s of the firm j :

$$L_{sjt} = [(1-\alpha)AK_{jt}^\alpha]^\sigma \left(\frac{W_{st}}{P_t^i} \right)^{-\sigma} L_{jt}^{1-\sigma\alpha} \quad (1.21)$$

where L_{sjt} is the demand of the differentiated labor service s . The aggregated demand for labor is as follows:

$$L_t = \left[\frac{(1-\alpha)A}{\Delta_{wt}^i} \right]^{\frac{1}{\alpha}} K_t \quad L_t = \int_0^1 L_{jt} dj \quad K_t = \int_0^1 K_{jt} dj \quad (1.22)$$

where Δ_{wt}^i again represents average real wage. The intermediate goods producers' optimal conditions can be rewritten as follows:

$$L_t = \left[\left(\frac{\varepsilon-1}{\varepsilon} \right) \frac{(1-\alpha)A}{\Delta_{wt}} \right]^{\frac{1}{\alpha}} K_t \quad (1.23)$$

$$R_t = \alpha \left[A \left(\frac{\varepsilon-1}{\varepsilon} \right) \right]^{\frac{1}{\alpha}} \left[\frac{1-\alpha}{\Delta_{wt}} \right]^{\frac{1-\alpha}{\alpha}} \quad (1.24)$$

$$\Delta_{wt} = \left[\int_0^1 \left(\frac{W_{st}}{P_t} \right)^{1-\sigma} ds \right]^{1/1-\sigma} \quad (1.25)$$

Retail firms or final goods producers

Models with capital externality and human capital

In the models where intermediate goods firms have a differentiated production function and operate in a competitive market (models where the source of growth is physical capital externality or human capital), there are an infinite number of retail firms over the continuum $[0,1]$, which repackage the homogeneous intermediate goods and sell them to households. It is assumed that they have the same simplified production technology that converts one unit of homogeneous intermediate good into one unit of differentiated final good. Consequently, the final output Y_t is composed of a continuum of retail final goods:

$$Y_t = \left(\int_0^1 Y_{rt}^{(\varepsilon-1)/\varepsilon} dr \right)^{\varepsilon/\varepsilon-1} \quad (1.26)$$

where Y_{rt} is the output of retailer r . If users of the final output minimize costs, the demand for each differentiated final good r is:

$$Y_{rt} = \left(\frac{P_{rt}}{P_t} \right)^{-\varepsilon} Y_t \quad (1.27)$$

$$P_t = \left(\int_0^1 P_{rt}^{1-\varepsilon} dj \right)^{1/1-\varepsilon} \quad (1.28)$$

where P_{rt} is the price of Y_{rt} and P_t is the price index of the final output. They sell their goods to households and set the price according to Taylor contracts for each interval of I periods.

Schumpeterian model

In the Schumpeterian model according to Aghion and Howitt (1992), the final goods production function is the following:

$$Y_t = L_t^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^\alpha di \quad (1.29)$$

where x_{it} is the intermediate good i used at t , $0 < \alpha < 1$, L_t , the composite demand of labor services and A_{it} is its productivity (or quality level). The productivity evolves according to an innovation process, which will be explained later.

The final goods producing sector is perfectly competitive, with firms choosing their inputs to maximize their profits. Consequently, the final goods producers' profits can be represented as follows:

$$F_{Yt} = P_t \int_0^1 (A_{it} L_t)^{1-\alpha} x_{it}^\alpha di - \int_{s=0}^1 W_{st} L_{st} ds - \int_{i=0}^1 P_{it} x_{it} di \quad (1.30)$$

where P_{it} is the price of the intermediate good i .

Once the demand function for labor service s is obtained, the demand function for L_t is:

$$L_t = \frac{(1-\alpha)Y_t}{\Delta_{wt}} \quad (1.31)$$

where $\Delta_{wt} = \left[\int_{s=0}^1 \left(\frac{W_{st}}{P_t} \right)^{1-\sigma} ds \right]^{\frac{1}{1-\sigma}}$ represents average real wage, in this case with respect to the final goods price, unlike what happened in the models with physical capital externality and human capital.

Technological change model

The final goods producers' technology in the technological change model is similar to that of the Schumpeterian model, but slightly different. Specifically, the production function is as follows:

$$Y_t = L_t^{1-\alpha} X_t^\alpha \quad (1.32)$$

where the term:

$$X_t = \left(\int_0^{Z_t} x_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (1.33)$$

is a Dixit–Stiglitz aggregate over a range of intermediate goods between 0 and Z_t . Z_t represents the variety of this type of goods in period t ; ε the elasticity of substitution across varieties; and x_{it} the output of intermediate good i . L_t represents the composite demand of differentiated labor services.

The intermediate goods demand function is:

$$X_t = Z_t^{\frac{1}{(\varepsilon-1)(1-\alpha)}} \left(\frac{\alpha}{\Delta_t^P} \right)^{\frac{1}{1-\alpha}} L_t \quad (1.34)$$

where:

$$\Delta_t^P = \left[\frac{1}{Z_t} \int_0^{N_t} \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \quad (1.35)$$

measures the average of the relative prices $\frac{P_{it}}{P_t}$ at which intermediate goods are sold (P_t is the intermediate goods price index). The labor demand function is similar to that of the Schumpeterian model:

$$L_t = \frac{1 - \alpha}{\Delta_{wt}/Y_t} \quad (1.36)$$

Central bank

The central bank is responsible for implementing monetary policy. It takes decisions about the short-term nominal interest rate (R_t^{st}) in each period following a Taylor rule of the type:

$$R_t^{st} = R\Pi\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \quad (1.37)$$

where R is the intercept reflecting structural factors in the reaction function of the central bank (which can be interpreted as the steady-state real interest rate), Π is the steady-state gross inflation (or target) and ϕ_π is the parameter that measures the central bank's reaction to inflation deviations from the target.

Finally, the relationship between real and nominal interest rate follows the Fisher equation:

$$R_t^{st} = R_t E_t \Pi_{t+1} \quad (1.38)$$

We only introduce equation (1.37) in the model with physical capital externality, while in the rest we assume trend inflation is given, as would be the case if this equation were present.

1.2.2 Growth and innovation

Every one of the four models is characterized by a different source of growth. Consequently, each growth engine must be explained individually.

Model with capital externality

The gross growth rate of the economy (Y_{t+1}/Y_t) coincides with the growth rate of the capital stock:

$$g_t = \frac{K_{t+1}}{K_t} = 1 + \frac{I_t^n}{K_t} \quad (1.39)$$

Schumpeterian model

This model displays Schumpeterian growth because growth occurs by increasing the quality of intermediate goods (Aghion and Howitt, 1992). By *quality* we must understand technological (or productivity) level of the capital goods.

According to the intermediate goods demand function, the profit of the intermediate goods producer i in t will be:

$$F_{it} = \alpha^{\frac{1}{1-\alpha}} \left(\frac{P_t^i}{P_t} - 1 \right) \left(\frac{P_t^i}{P_t} \right)^{-\frac{1}{1-\alpha}} A_{it} L_t \quad (1.40)$$

So that, taking into account price rigidity during I periods, the average expected profit in a period t for the intermediate producers after having had success in innovation is equal to:

$$VF_{it} = \alpha^{\frac{1}{1-\alpha}} A_{it} L_t \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{t-s}^*}{P_t} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{t-s}^*}{P_t} - 1 \right) \quad (1.41)$$

We assume the following diminishing returns probability function for the success of the innovation:

$$\phi(n_{it}) = n_{it}^{\chi} \quad 0 < \chi < 1 \quad (1.42)$$

with $\phi'(n_{it}) = \chi n_{it}^{\chi-1} > 0$ and $\phi''(n_{it}) = \chi(\chi - 1)n_{it}^{\chi-2} < 0$

If innovation is successful, expected profits will be

$$\phi(n_{it})VF_{it}^* \quad (1.43)$$

where $n_{it} = R_{it}/A_{it}^*$, R_{it} being the quantity of final goods devoted to innovation and A_{it}^* the intermediate goods productivity achieved if innovation is successful. Consequently, the expected profit of the R&D activity that can provide an innovation is:

$$\phi\left(\frac{R_{it}}{A_{it}^*}\right)VF_{it}^* - R_{it} \quad (1.44)$$

The optimal value of n_{it} will be common to all entrepreneurs, due to the fact that n only depends on market elements:

$$n_{it} = n = \left[\chi \alpha^{\frac{1}{1-\alpha}} L_t \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{t-s}^*}{P_t} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{t-s}^*}{P_t} - 1 \right) \right]^{\frac{1}{1-\chi}} \quad (1.45)$$

According to the law of large numbers, the proportion of successful innovators will be $\mu = \phi(n)$. Consequently, the technological level of economy will be

$$A_t = \mu \gamma A_{t-1} + (1 - \mu) A_{t-1} \quad (1.46)$$

$$A_t = \int_0^1 A_{it} di$$

The gross growth rate can be written as follows:

$$g_t = \frac{A_t}{A_{t-1}} = \frac{Y_t}{Y_{t-1}} \quad (1.47)$$

$$g_t = \mu(\gamma - 1) + 1 \quad (1.48)$$

Considering $\mu = \phi(n) = n^\chi$, the gross growth rate in the steady state is

$$g = \left[\chi \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_s^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_s^*}{P} - 1 \right) \right]^{\frac{\chi}{1-\chi}} (\gamma - 1) + 1 \quad (1.49)$$

Technological change model

This section follows Amano, Carter and Moran (2012). Household members design new varieties of intermediate goods using an R&D technology, according to Evans Honkapohja and Romer (1998). In period t , this technology enables the innovator to develop intermediate good i at real cost ηi^ρ . R&D thus causes the range of varieties Z_t to rise over time, which drives the growth process. When a variety has been designed, this design is patented and sold to a prospective intermediate goods producer. In return, the innovator receives a rent.

We assume that the innovator is unaware of the pricing cohort and new patents are uniformly distributed across cohorts. As a consequence, a patent designed in period t will yield

$$\mathbb{P}_s = \frac{1}{Z_s} \int_0^{Z_s} \mathbb{P}_{is} di \quad (1.50)$$

in real dividends in each period $s \geq t+1$, where \mathbb{P}_{is} denotes the real profits that producer i generates in period s .

If we balance these dividends against the upfront cost of R&D, the investment

$$R\&D_t = \eta \int_{Z_t}^{Z_{t+1}} i^\varepsilon di \quad (1.51)$$

must satisfy the following zero-profit condition

$$\eta Z_{t+1}^\rho = \sum_{\tau=1}^{\infty} \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right) \mathbb{P}_{t+\tau} \quad (1.52)$$

If we substitute for profits in this condition, we obtain the following expression in the steady state

$$\eta Z^\rho = \left(\frac{\beta}{g - \beta} \right) \left(\frac{Z_t}{(\Delta^p)^{\varepsilon-1}} \right)^{\frac{1-\varepsilon(1-\alpha)}{(\varepsilon-1)(1-\alpha)}} \alpha^{\frac{1}{1-\alpha}L} \left[\frac{1}{I} \sum_{\tau=0}^{I-1} \left(\frac{P_\tau^*}{P} \right)^{-\varepsilon} \left(\frac{P_\tau^*}{P} - 1 \right) \right] \quad (1.53)$$

Balanced growth requires ε be chosen such that costs rise (or fall) to offset the effect of the demand externality. This is achieved through the following parameter restriction, according to Evans, Honkapohja and Romer (1998):

$$\rho = \frac{1 - \varepsilon(1 - \alpha)}{(\varepsilon - 1)(1 - \alpha)} \quad (1.54)$$

Final output is allocated across its various uses, with both the share of investment in R&D and of production of intermediate goods being constant to output

$$\frac{R\&D_t}{Y_t} = \frac{\eta(g-1)}{1+\rho} \left(\frac{\Delta_t^p}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} \frac{1}{L_t} \quad (1.55)$$

and

$$\frac{\int_0^{Z_t} x_{it} di}{Y_t} = \alpha(\Delta_t^P)^{\varepsilon-1} \left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} \left(\frac{1}{I} \sum_{\tau=0}^{I-1} \Pi^{\varepsilon\tau}\right) \quad (1.56)$$

Human capital model

The growth process in the human capital model is derived from the solution of a dynamic optimization problem that is recorded in Appendix A.1 for price and wage flexibility and for staggered wage and price setting. As a consequence, final output Y , intermediate goods production Y^i , physical capital stock K and effective labor L , grow at the same rate in steady state, which is the growth rate of average human capital h . Let $g(\cdot)$ be the growth rate of a variable at steady state, this situation implies the following relationships:

$$g(Y) = g(Y^i) = g(K) = g(L) = g(h) \quad (1.57)$$

$$g(h) = \begin{cases} [1 + \xi(1 - u_{ss})N_{ss}] & \text{Wage Flexibility} \\ [1 + \xi(1 - u^1)N^1](1 + g(h^1))\left(\frac{J-2}{J}\right) + [1 + \xi(1 - u^{01})N^1](1 + g(h^{01}))\left(\frac{1}{J}\right) + \\ + [1 + \xi(1 - u^0)N^0](1 + g(h^0))\left(\frac{1}{J}\right) & \text{Wage rigidity} \end{cases}$$

where u_{ss} y N_{ss} are steady-state values with wage flexibility, while u^1 , h^1 and N^1 are the decisions for labor services with constant nominal wage for $s \in [0, J-2)$, u^{01} , h^{01} and N^1 for $s \in [J-2, J-2]$, and u^0 , h^0 and N^0 for $s \in [J-1, J-1]$ for the corresponding labor services that will reset nominal wage in the following period.

1.2.3 Wage and price setting

We assume that the existence of price and wage rigidities leads to a Taylor-type process of staggered price and wage setting. In both cases this process takes into

account the preferences of the agents involved. In the case of wages these are the workers, given that we assume the equality between supply and demand for labor services, and in the case of prices, they are the corresponding firms in their profit maximization strategies.

Wages setting

It is the intermediate goods producers who set wages for J periods in the models with physical capital externality and human capital, while in the Schumpeterian and technological change models it is the final goods firms. In both cases they set the wage W^* at t for J periods according to households' preferences, given the equality between labor supply and demand. The optimum wage for any type of labor service will be obtained from the maximization of the total discounted utility for every interval of J period.

Model with capital externality

The optimal nominal wage can be expressed in the following form:

$$W_t^* = \left(\frac{\sigma}{\sigma - 1} \frac{\sum_{\tau=0}^{J-1} \beta^\tau L_{t+\tau}^{(1+\nu)} P_{t+\tau}^{i\sigma(1+\nu)} \Delta_{wt+\tau}^{\sigma(1+\nu)} P_{t+\tau}^{\sigma(1+\nu)}}{\sum_{\tau=0}^{J-1} \frac{\beta^\tau}{C_{t+\tau}} P_{t+\tau}^{i\sigma} \Delta_{wt+\tau}^\sigma L_{t+\tau} P_{t+\tau}^{\sigma-1}} \right)^{1/(1+\nu\sigma)} \quad (1.58)$$

While steady-state real wage normalized by capital stock will be:

$$\frac{W^*}{KP} = \left(\frac{\sigma}{\sigma - 1} \frac{C \sum_{\tau=0}^{J-1} \beta^\tau L^{(1+\nu)} P^{i\sigma(1+\nu)} (g^\tau \Delta_w^K)^{\sigma(1+\nu)} \Pi^{\tau\sigma(1+\nu)}}{K \sum_{\tau=0}^{J-1} \beta^\tau L P^{i\sigma} (g^\tau \Delta_w^K)^\sigma \Pi^{\sigma-1}} \right)^{\frac{1}{1+\sigma\nu}} \quad (1.59)$$

$$\Delta_w^K = \frac{\Delta_W}{K} \quad (1.60)$$

Human capital model

Optimal nominal wage:

$$W_t^* = \left(\frac{\sigma}{\sigma - 1} \right) \frac{E_t \sum_{\tau=0}^{J-1} \beta^\tau P_{t+\tau}^\sigma K_{t+\tau}^{\sigma\alpha} L_{t+\tau}^{1-\sigma\alpha} N_{t+\tau}^v (u_{t+\tau} h_{t+\tau})^{-1}}{E_t \sum_{\tau=0}^{J-1} \beta^\tau C_{t+\tau}^{-1} P_{t+\tau}^{\sigma-1} K_{t+\tau}^{\sigma\alpha} L_{t+\tau}^{1-\sigma\alpha}} \quad (1.61)$$

Steady-state real wage:

$$\frac{W^*}{P} = \left[\left(\frac{\sigma}{\sigma - 1} \right) \left(\left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{\Delta_w^{1-\alpha\sigma}}{(1-\alpha)A} \right)^{\frac{1}{\alpha}} \frac{C \sum_{\tau=0}^{J-1} \beta^\tau N_t^{1+v}}{K \sum_{\tau=0}^{J-1} \beta^\tau \Pi^{(\sigma-1)\tau}} \right]^{\frac{1}{1-\sigma}} \quad (1.62)$$

$$N_\tau = N^1 \text{ for } \tau = 0, 1, 2, \dots, J-2$$

$$N_\tau = N^0 \text{ for } \tau = J-1$$

Schumpeterian model

Optimal nominal wage:

$$W^* = \left[\frac{\sigma}{\sigma-1} \frac{\sum_{\tau=0}^{J-1} \beta^\tau L_{st+\tau}^{1+v}}{\sum_{\tau=0}^{J-1} \lambda_{t+\tau} L_{st+\tau} P_{t+\tau}^{-1}} \right]^{\frac{1}{1+\sigma v}}$$

Steady-state real wage normalized by final good output:

$$\frac{W^*}{PY} = \left(\frac{\sigma(1-\alpha)^v}{\sigma-1} \frac{C \sum_{\tau=0}^{J-1} \beta^\tau (g^\tau \Delta_w^Y)^{(\sigma-1)(1+v)} \Pi^{\sigma(1+v)\tau} g^{(1+v)\tau}}{Y \sum_{\tau=0}^{J-1} \beta^\tau (g^\tau \Delta_w^Y)^{(\sigma-1)} \Pi^{(\sigma-1)\tau}} \right)^{\frac{1}{1+\sigma v}} \quad (1.63)$$

$$\Delta_w^Y = \frac{\Delta_w}{Y} \quad (1.64)$$

Technological change model

Optimal monetary wage:

$$W_t^* = \left(\frac{\sigma(1-\alpha)^{\sigma\nu} \sum_{\tau=0}^{J-1} \beta^\tau P_{t+\tau}^{\sigma(1+\nu)} Y_{t+\tau}^{\sigma(1+\nu)} L_{t+\tau}^{(1+\nu)(1-\sigma)}}{\sigma-1 \frac{\sum_{\tau=0}^{J-1} \beta^\tau}{\bar{C}_{t+\tau}} P_{t+\tau}^{\sigma-1} Y_{t+\tau}^\sigma L_{t+\tau}^{(1+\nu)(1-\sigma)}} \right)^{\frac{1}{1+\sigma\nu}} \quad (1.65)$$

Steady-state real wage normalized by final good output:

$$\frac{W^*}{PY} = \left(\frac{\sigma(1-\alpha)^{\sigma\nu} \bar{C}}{\sigma-1 \bar{Y}} \frac{\sum_{\tau=0}^{J-1} (\beta \Pi^{\sigma(1+\nu)} g^{\sigma(1+\nu)})^\tau L^{\nu(1-\sigma)}}{\sum_{\tau=0}^{J-1} (\beta (\Pi g)^{\sigma-1})^\tau} \right)^{\frac{1}{1+\sigma\nu}} \quad (1.66)$$

Price setting

According to the previous paragraphs, in the models with capital externality and human capital as sources of growth, it is the retail firms who set the price that maximizes their expected profits for every I period, while it is the intermediate goods producers who do so in the Schumpeterian and technological change models.

Model with capital externality

Retail firms set the price P_t^* through the solution to the following maximization problem:

$$Max_{P^*} \sum_{\tau=0}^{I-1} E_t \left[\frac{\lambda_{t+\tau}}{\lambda_t} Y_{t+\tau}(P^*) \left(\frac{P_t^*}{P_{t+\tau}} - P_{t+\tau}^i \right) \right] \quad (1.67)$$

where $Y_{t+\tau}(P^*) = \left(\frac{P_t^*}{P_{t+\tau}} \right)^{-\varepsilon} Y_{t+\tau}$ and λ is the wealth marginal utility of consumers, identified as the Lagrange multiplier of the utility maximization. We use the quotient between two periods as discount factor.

Solving this problem, we obtain the optimal price to be set in t :

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{\tau=0}^{I-1} \beta^\tau (P_{t+\tau})^\varepsilon \frac{Y_{t+\tau}}{C_{t+\tau}} P_{t+\tau}^i}{E_t \sum_{\tau=0}^{I-1} \beta^\tau (P_{t+\tau})^{\varepsilon-1} \frac{Y_{t+\tau}}{C_{t+\tau}}} \quad (1.68)$$

The relative price in steady state will be:

$$\frac{P^*}{P} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{\tau=0}^{I-1} \beta^\tau \Pi^{\varepsilon\tau} P^i}{\sum_{\tau=0}^{I-1} (\beta \Pi^{\varepsilon-1})^\tau} \quad (1.69)$$

As the problem to be solved is the same as in the other models, we present only the final expressions below.

Human capital model

Optimal price to be set in t :

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{\tau=0}^{I-1} \beta^\tau (P_{t+\tau})^\varepsilon \frac{Y_{t+\tau}}{C_{t+\tau}} P_{t+\tau}^i}{E_t \sum_{\tau=0}^{I-1} \beta^\tau (P_{t+\tau})^{\varepsilon-1} \frac{Y_{t+\tau}}{C_{t+\tau}}} \quad (1.70)$$

Relative price in steady state:

$$\frac{P^*}{P} = \frac{\sum_{\tau=0}^{I-1} (\beta \Pi^\varepsilon)^\tau}{\sum_{\tau=0}^{I-1} (\beta \Pi^{\varepsilon-1})^\tau} \quad (1.71)$$

Schumpeterian model

Optimal price to be set in t :

$$P_t^* = \frac{1}{\alpha} \frac{\sum_{\tau=0}^{I-1} \frac{\lambda_{t+\tau}}{\lambda_t} x_{it+\tau}(P_t^*)}{\sum_{\tau=0}^{I-1} \frac{\lambda_{t+\tau}}{\lambda_t} \frac{x_{it+\tau}(P_t^*)}{P_{t+\tau}}} \quad (1.72)$$

where $x_{it+\tau}(P_t^*)$ is the demand of the intermediate good i in $t + \tau$ with the price fixed in the value P_t^* .

Relative price in steady state:

$$\frac{P^*}{P} = \frac{1 \sum_{\tau=0}^{l-1} (\beta(\Pi)^{1/1-\alpha})^\tau}{\alpha \sum_{\tau=0}^{l-1} (\beta(\Pi)^{\alpha/1-\alpha})^\tau} \quad (1.73)$$

Technological change model

Optimal price to be set in t:

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \left(\frac{\sum_{\tau=0}^{l-1} \beta^\tau \frac{C_t}{C_{t+\tau}} L_{t+\tau}^{\varepsilon(1-\alpha)} X_{t+\tau}^{1-\varepsilon(1-\alpha)} P_{t+\tau}^\varepsilon}{\sum_{\tau=0}^{l-1} \beta^\tau \frac{C_t}{C_{t+\tau}} L_{t+\tau}^{\varepsilon(1-\alpha)} X_{t+\tau}^{1-\varepsilon(1-\alpha)} P_{t+\tau}^{(\varepsilon-1)}} \right) \quad (1.74)$$

Relative price in steady state:

$$\frac{P^*}{P} = \frac{\varepsilon}{\varepsilon - 1} \left(\frac{\sum_{\tau=0}^{l-1} \left[\beta g^{\frac{(1-\varepsilon)(1-\alpha)}{\alpha}} \Pi^\varepsilon \right]^\tau}{\sum_{\tau=0}^{l-1} \left[\beta g^{\frac{(1-\varepsilon)(1-\alpha)}{\alpha}} \Pi^{(\varepsilon-1)} \right]^\tau} \right) \quad (1.75)$$

1.2.4 Equilibrium conditions

The aggregate equilibrium of the economy in the four models is the equality between final output and the sum of consumption and gross investment. We assume, for the sake of simplicity, that there are neither public expenditures nor an external sector. However, we must also consider some specific characteristics of each model.

Model with physical capital externality

The final goods output of the economy weighted by the price dispersion is equivalent to the intermediate goods firms' output:

$$Y_t^i = \Delta_t^P Y_t \quad (1.76)$$

where Y_t is final good output of the economy, Y_t^i the output of the intermediate goods firms and Δ_t^P the price dispersion which can be represented as follows:

$$\Delta_t^P = \frac{1}{I} \sum_{\tau=0}^{I-1} \left(\frac{P_{t-\tau}^*}{P_t} \right)^{-\varepsilon} \quad (1.77)$$

Schumpeterian model

Final good output is equal to the sum of consumption, R&D investment and intermediate goods production:

$$Y_t = C_t + R\&D_t + \int_{i=0}^1 x_{it} di \quad (1.78)$$

Consequently, we obtain the following expression for consumption in steady state:

$$\frac{C}{Y} = 1 - \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \frac{A}{Y} - \left[\chi \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1 \right) \right]^{\frac{1}{1-\chi}} \frac{A}{Y} \quad (1.79)$$

Technological change model

With regard to the technological change model, final good output is also composed of consumption, investment in R&D and intermediate goods production. As a result, consumption follows the next expression in steady state:

$$\frac{C}{Y} = 1 - \alpha(\Delta^P)^{\varepsilon-1} \left(\frac{P^*}{P}\right)^{-\varepsilon} \left(\frac{1}{I} \sum_{\tau=0}^{I-1} \Pi^{\varepsilon\tau}\right) - \frac{\eta(g-1)}{1+\rho} \left(\frac{\Delta^P}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{L} \quad (1.80)$$

Human capital model

Final good output is composed of consumption and investment, and the steady-state consumption to physical capital ratio in steady state, C/K , is determined as follows:

$$\frac{C}{K} = \frac{Y}{K} - g(K) - \delta \quad (1.81)$$

Since the right-hand side is constant over time in steady state, consumption and capital grow at the same rate, and therefore:

$$\frac{C}{K} = A^{\frac{1}{\alpha}} \left[\left(\frac{\varepsilon-1}{\varepsilon} \right) \frac{1-\alpha}{\Delta_t^W} \right]^{\frac{1-\alpha}{\alpha}} - g(C) - \delta \quad (1.82)$$

$$g(Y) = g(Y^m) = g(K) = g(L) = g(C) \quad (1.83)$$

1.2.5 Steady state

Considering that our objective is to analyse the long-term behavior of the economy, we must define the steady state and the system of equations that determine the values of the endogenous variables in this situation. Since our

models incorporate economic growth and some variables grow in steady state, these variables must be normalized.

In the model with capital externality, economic growth is represented by the gross growth rate of capital $g_{t+1} = \frac{K_{t+1}}{K_t}$ ($= \frac{Y_{t+1}}{Y_t}$). Consequently, the normalization of all the growing variables must be done through the capital stock. The system of equations is presented in section A2.1 of Appendix A. The endogenous variables are $g, I^k, I^{nk}, r^q, R, R^{st}, L, Y^{i,k}, Y^k, \Delta^P, \frac{P^*}{P}, \frac{P_{-s}^*}{P}, \Delta_W^K, P^i, \frac{W^{*k}}{P}, \frac{W_{-s}^{*k}}{P}$ and C^k .

Regarding the Schumpeterian and technological change models, the normalization of all the growing variables is carried out dividing them by the production level of the final good Y_t . The systems of equations are respectively presented in sections A2.2 and A2.3 of Appendix A. The endogenous variables are $\frac{P^*}{P}, \frac{P_{-s}^*}{P}, g, L, \Delta_W^Y, \frac{W^*}{P}, \frac{W_{-s}^*}{P}, C, A$ and R in the Schumpeterian model and $\frac{P^*}{P}, \frac{P_{-s}^*}{P}, \Delta^P, L, \Delta_W^Y, \frac{W^*}{P}, \frac{W_{-s}^*}{P}, \eta$ and C in the technological change model.

Finally, taking into account the representative household's optimal control problem of human capital model developed in Appendix A.1, the steady-state system of equations is different depending on the existence or not of wage rigidity. If wages are flexible, the system is characterized by 6 unknowns: $W_s^*/P, C/K, g(C), N_{ss}, P^*/P_t$ and ΔP . If there is wage rigidity, it is characterized by 10 unknowns, according to Appendix A2.2: $W_s^*/P, \Delta_t^W, C/K, g(C), N^0, N^1, u^0, u^1, P^*/P$ and Δ^P . The system of equations is presented in Section A2.4.

1.3. Nominal rigidities and the relationship inflation-growth in the long run

In this section the models will be simulated in order to obtain the values of the different variables in steady state and their responses to changes in trend inflation, depending on the kind of rigidity. The values of the parameters for each model are presented in Table 1.1. These values are appropriate for quarterly data and are common when they appear in more than one model in order to analyze comparable economies.

Table 1.1: The choice of parameter values

Parameter	Description	Model with physical capital externality	Schumpeterian model	Technological change model	Human capital model
δ	Capital depreciation rate	0.048			0.048
α	Output elasticity with respect to capital	0.332	0.332	0.332	0.332
β	Utility discount factor	0.999	0.999	0.999	0.999
ε	Elasticity of substitution among retail or intermediate goods	1.40		1.40	1.40
ϕ_π	Coefficient of inflation reaction in the Taylor rule	2.05	2.05	2.05	2.05
σ	Elasticity of substitution among labor services	10	10	10	10
ν	Relative utility weight of labor	1	1	1	1
I	Periods it takes to reset prices	1, 2	1, 2	1, 2	1, 2
J	Periods it takes to reset wages	1, 4	1, 4	1, 4	1, 4
γ	Productivity upgrade after every innovation		1.009		
χ	Elasticity of the probability of success in the innovation with respect to relative investment		0.1		
ρ	Innovation costs elasticity			0.1	
η	Unit cost of innovation			10	
ξ	Productivity parameter of human capital accumulation				0.018
A	Constant total factor productivity				1

The values chosen for parameters α , β and ϕ_π are usually found in simulations of DSGE models. The value of capital depreciation δ has been chosen in order to obtain plausible values of annual growth rates (around the interval 2% – 3%). The elasticity of substitution for retail or intermediate goods (ε) and differentiated labor services (σ) are set at 1.4 and 10, respectively. The first value responds also to the search of plausible growth rates. The second value is consistent with the findings reported in Basu (1996) and Basu and Fernald (1997). The disutility of the labor parameter, v , is assigned a value of one, as in Hornstein and Wolman (2005). The length of price contracts I is set to 2 when there is no price flexibility ($I=1$), based on results reported in Bils and Klenow (2004). The length of wage contracts J is set equal to 4 when there is no wage flexibility ($J=1$), as in Erceg Henderson and Levin (2000) and Huang and Liu (2002). Taylor (1999) provided a review of the empirical literature, concluding that the average frequency of wage changes is about one year. The rest of the parameters ($\gamma, \chi, \rho, \eta, \xi, A$) are present only in one model and the values are very plausible from the perspective of every one of them.

1.3.1 Model with physical capital externality

Firstly, in order to analyze the impact of the different types of rigidity on the steady-state relationship between trend inflation and log-run growth, it is convenient to start from the model with price and wage flexibility ($I=J=1$). Figure

1.1 shows with a blue line how the quarterly growth rate remains constant at 0.541% whatever the inflation rate.

Secondly, if we calibrate the model with only price rigidity ($I=2, J=1$) and simulate it for different values of trend inflation, we obtain the same relationship between inflation and growth as for flexibility (dotted green line). As a consequence, price rigidity does not have any impact on growth rate.

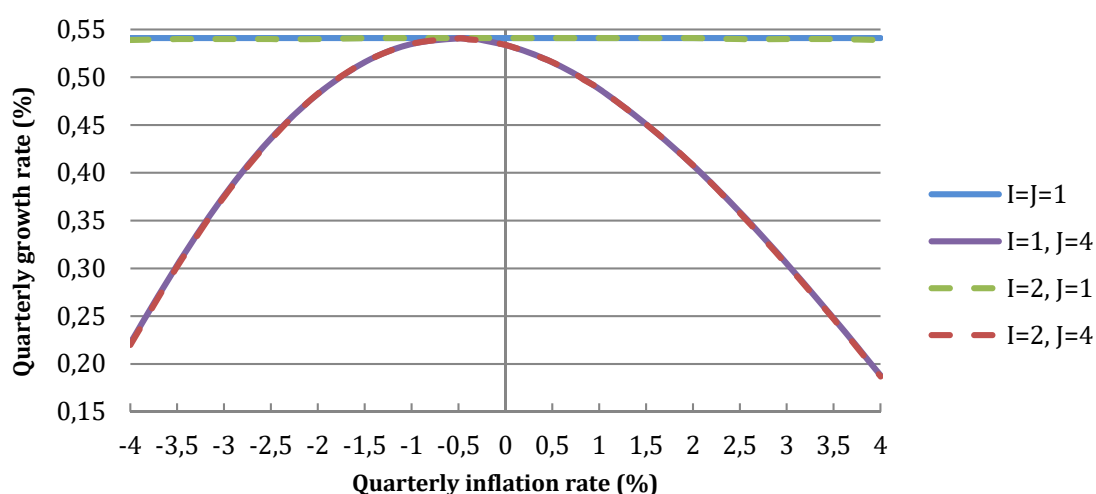


Figure 1.1: Long-term inflation–growth relationship for different types of rigidity – Model with physical capital externality.

Regarding only wage rigidity ($I=1, J=4$), if we simulate the calibrated model for different values of inflation, we obtain the relationship between inflation and growth displayed in Figure 1.1 as the continuous purple line with a maximum of 0.541% (at the same value as when wages are flexible). However, this value is reached for a deflation rate of -0.541% (-2.18% annual). When inflation or deflation rate is different from this value, the greater the difference the lower long-run growth is. Consequently, the long-run relationship between inflation and growth is an inverted-U shape. In other words, there exists a distortion in the

allocation of resources for values of the trend inflation rate different from -0.541%, which is the same rate as the growth rate with flexibility with the negative sign. If we calibrate the model with simultaneously both kinds of rigidity, wage rigidity dominates price rigidity and the outcome is the same as with only wage rigidity (an inverted-U shape with maximum growth rate 0.541% for a trend inflation rate of -0.541%).

To end the summary of the simulations of this model we can revise the shape of the lines as well as the importance of the effects that trend inflation has on the growth rate. There are three characteristics to be highlighted. The first is the lack of impact of trend inflation on long-run growth with only price rigidity. The second, the symmetry around the inflation rate value -0.541% in the case of wage rigidity (with or without price rigidity). The third and very important characteristic is the very low effect that inflation rate has on the long-run growth rate under wage rigidity. For example, a change of 4 percentage points in the annual inflation rate from -2.18% affects the growth rate only in less than two tenths of a percentage point. Really it is a very low effect.

1.3.2 Schumpeterian model

The long-run inflation–growth relationship in the Schumpeterian model of endogenous growth with flexible wage and price is shown as the horizontal blue line in Figure 1.2, so that the long-run growth rate is independent of the inflation rate at the quarterly value of 0.511%. As in the previous model, when only price rigidity is present ($J=1$, $I>1$), the result is the same as in the case of flexibility (dotted green line).

When only wage rigidity is present we also find an inverted-U-shaped relationship. The growth is maximum at a quarterly rate of 0.511% when $\Pi = 0.99489$. That is, if prices fall at the quarterly rate of -0.511% (-2.06% annual) the economy reaches the maximum growth rate, but this rate diminishes as inflation moves further away from -0.511%. Once again, the maximum growth rate takes place for an inflation rate of the same value and negative sign and this is the value corresponding to price and wage flexibility.

Taking into account that price rigidity does not affect the growth rate of flexibility, it is a direct result that when price and wage rigidity simultaneously exist (dotted red line), the long-run relationship between inflation and growth is the same as when only wage rigidity exists.

We can also appreciate the existence of symmetry around the inflation rate value -0.511% in the case of wage rigidity (with or without price rigidity). Finally, with wages rigidity, we find again a very low effect of trend inflation on the long-run growth rate. In this model the effect is even lower than in the previous one. A change of 4 percentage points in the annual inflation rate from -2.06% affects the growth rate only in less than one hundredth of a percentage point.

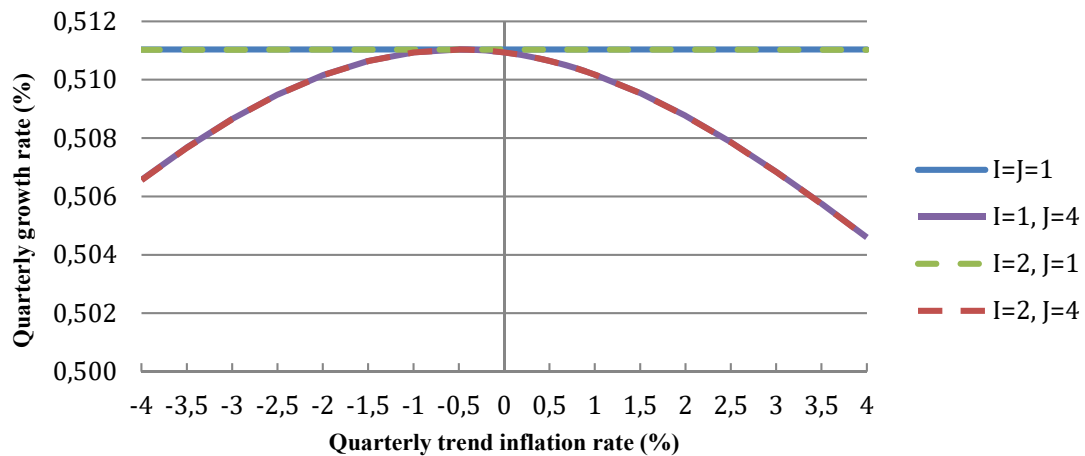


Figure 1.2: Long-term inflation–growth relationship for different types of rigidity – Schumpeterian growth model.

1.3.3 Technological change model

In the model of technological change, the results are very close to the previous ones. The long-run relationship between inflation and growth with flexible prices and wages is a constant line, that is, the quarterly growth rate remains constant whatever the inflation rate at 0.571%. This is shown by the blue line in Figure 1.3. The same happens in the case of price rigidity: growth rate continues being independent of trend inflation and it remains at the same value as flexibility.

If we simulate the model of wage rigidity for different values of trend inflation, we also obtain an inverted U-shaped curve that is represented by the purple line in Figure 1.3. The value of the inflation rate that makes growth maximum is -0.571% (-2.3% annual), and the maximum growth rate is 0.571%, the same value with the opposite sign and the value in the cases of flexibility and only price rigidity. As in the previous models, wage rigidity dominates price rigidity and, hence, the relationship between inflation and growth under both rigidities is the same as only wage rigidity.

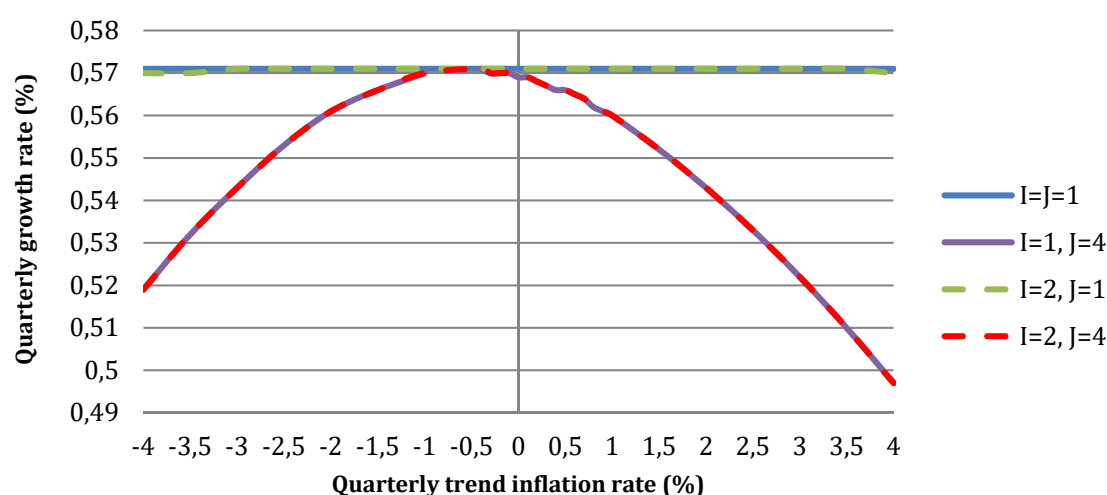


Figure 1.3: Long-term inflation–growth relationship for different types of rigidity – Technological change model.

We can see again the symmetry of the relationship around the inflation rate value -0.571% in the case of wage rigidity (with or without price rigidity). Moreover, as in the two previous models under wage rigidity, trend inflation has a low effect on the long-run growth rate. A change of 4 percentage points in the annual inflation rate from -2.3% affects the growth rate only in two hundredths of a percentage point.

1.3.4 Human capital model

We do not find any difference in the model of human capital (Figure 1.4) compared to the previous ones in terms of flexibility and price rigidity: the long-run inflation–growth relationship is a horizontal line in the value of growth rate 0.787% , which shows the independence between long-run growth and trend inflation.

The biggest difference related to the three previous models is found in the case of wage rigidity: the growth rate is maximized for a null inflation. At that point, the growth rate is the same as in the case of flexible wages (0.787% quarterly, 3.18% annual). As a consequence, if we consider both kinds of rigidities, the growth rate will also be maximum for null inflation and the same value as when only rigidity in wages exists.

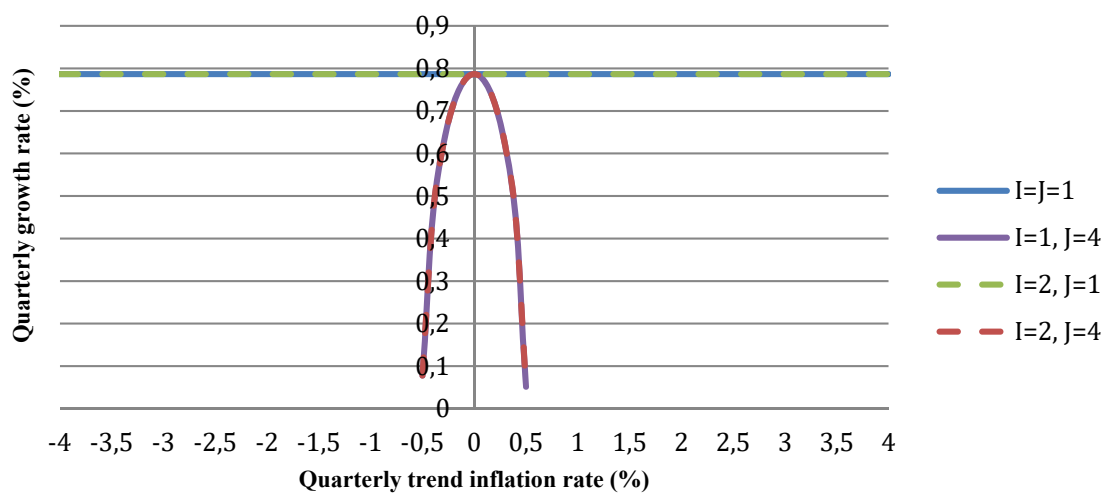


Figure 1.4: Long-term inflation–growth relationship for different types of rigidity – Human capital model.

We can also appreciate the existence of symmetry around null inflation for rigid wages. However, this model not only provides a difference from the three previous models in the value of trend inflation that maximizes the long-run growth rate. It also shows a sharp difference when the effect of trend inflation on the growth rate is considered. Now the units are not hundredths or tenths of a percentage point. A change in the annual inflation or deflation rate of 2 percentage points from 0 is the cause of a decline of 2.75 percentage points in the long-run growth rate.

1.4. Assessment of the main impacts of nominal rigidities on long-run growth-inflation relationship

The independence of long-run growth and trend inflation when only rigidity in prices exists is a result that appears in all four models. Under this type of rigidity, the impact of trend inflation on long-run growth is so limited that the growth rate remains constant for admissible values of trend inflation. Table 1.2 summarizes the inflation rate that maximizes the growth rate depending on the different rigidities and models. It is noteworthy that the behavior when both rigidities exist is always the same as when only wage rigidity exists, but above all, one result in the human capital model stands out: the long-run growth rate is maximum for a null inflation when wages are rigid.

Table 1.2: Quarterly inflation rate with maximum long-run growth. Models and rigidities

	Physical externality model	Schumpeterian model	Technological change model	Human capital model
Prices rigidity (*)	—	—	—	—
Wages rigidity	– 0.541 %	– 0.511 %	– 0.571 %	0 %
Prices and wages rigidity	– 0.541 %	– 0.511 %	– 0.571 %	0 %

(*) Long-run growth and trend inflation are independent.

In contrast to this, wage rigidity shrinks long-run growth for every model, except

for a value of trend inflation where long-run growth rate is the same as for flexibility (Table 1.3). For values differing from this one, the greater the difference between the two inflation rates the lower the growth rate. This value of trend inflation is negative and equal in absolute value to the maximum growth rate, showing that a clear compensation exists between the two rates.

Table 1.3: Maximum quarterly growth rate. Models and rigidities

	Physical capital externality model	Schumpeterian model	Expansion of varieties model	Human capital model
Total flexibility	0.541 %	0.511 %	0.571 %	0.787 %
Prices rigidity	0.541 %	0.511 %	0.571 %	0.787 %
Wages rigidity	0.541 %	0.511 %	0.571 %	0.787 %
Prices and wages rigidity	0.541 %	0.511 %	0.571 %	0.787 %

Moreover, the maximum growth rate with wage rigidity is the same as with price and wage flexibility, indicating that, in fact, the growth rate is reached because the two situations are, for this trend inflation rate, equivalent. Effectively, with this inflation rate the revision of the nominal wage is not necessary because the deflation adjusts real wage in the right amount to obtain the real wage target. When trend inflation is different from this value a distortion is introduced in the demand for labor that reduces the long-run growth rate in a greater amount the greater the difference.

All these results indicate that the revision of wages elevates in excess the average

real wage when trend inflation is different from a rate equal to the growth rate corresponding to price and wage flexibility with negative sign, which decreases the labor demand and affects the long-run growth rate negatively. In fact, inflation acts in this case as a negative productivity shock. When trend inflation is negative at exactly the same value as the growth rate, nominal wage revision is not necessary. This is exactly the same situation as wage flexibility. This is the case of the maximum growth in the first three models. The long-run real wage individuals receive will grow as with flexibility, thanks to the falling trend of prices.

In the case of human capital wage rigidity does not affect the growth component of the variable, it only affects the wage-per-unit of human capital. Wage contracts revise with flexibility the skill components of the contracts. The distortion previously indicated is not present and this is why the maximum growth is reached for null inflation. The long-run real wage individuals receive will grow thanks to long-run human capital accumulation.

1.5. Transmission mechanisms

Having evaluated and compared, through simulations, the impact rigidities have on the long-run relationship between inflation and growth in the four models, it is necessary to identify the main mechanisms in the equations of the steady state that make the relationship between Π and g similar in some cases and different in others, depending on the model and the sort of rigidity. To do so, in this section we consider separately the existence of the distortion in the labor market introduced by wage rigidity as the key factor in the dependence or independence between trend inflation and long-run growth, and the reason why the human

capital model shows two so highly differentiated results related to the other three models: maximum long-run growth rate for null trend inflation, and a significant long-run impact on inflation and growth.

1.5.1 Model with physical capital externality

In order to clarify the mechanisms that take place in the steady state, it is convenient to summarize the main equations that drive the dynamics. In this model, the key variable in the steady state is P^i , the price of the intermediate goods. The behavior of the final good price and investment depends crucially on this variable.

Using expressions (A2.1.3), (A2.1.4), (A2.1.5), (A2.1.9), and (A2.1.10) from Appendix A.2, the behavior of the long-run growth rate can be written as:

$$g/\beta = \alpha P^i L^{(1-\alpha)} + 1 - \delta \quad (1.83)$$

From equations (A2.1.13) y (A2.1.11) in Appendix A.2, it is clear that the steady-state (relative) price of the intermediate good (P^i) is given with price rigidity when Π is known and independent from it with price flexibility.

Once the value of P^i is known, the long-run value of g depends only on the value of L , whose behavior is different for the different types of rigidities. This is the main relationship to be considered in the mechanisms described below.

If we consider flexible prices and wages, neither P^*/P , nor Δ^p , nor Δ^{k_w} , C^k nor P^i depend on Π . So L is also independent of the gross inflation rate. The price of the final good is one, the mark-up is constant at the value $\varepsilon/\varepsilon - 1$ whatever the value of

the gross inflation rate, the price of the intermediate goods is also independent of Π and, as a consequence, the long-run growth rate g .

As we have deduced in the preceding section, if there is price and wage flexibility, changes in the long-run inflation rate do not affect the value of the growth rate of the economy, and the only variable affected is the nominal rate R^{st} .

Regarding price rigidity, although at first sight growth should not remain constant, we cannot observe a decline in growth rate when inflation has values far away from 0. The variations in P^*/P , Δ^p , Δ^{k_w} , C^k and P^i are insignificant for a wide range of plausible values of trend inflation (-4% to 4%, quarterly). For those values, the long-run intermediate good price remains constant as well as L and, consequently, growth rate remains constant.

If we only consider wage rigidity, Δ^{k_w} , C^k and L depend on Π . So g also depends on the gross inflation rate, even though the prices of the final and intermediate goods do not. In this case, the labor demand is distorted by a mark-up depending on Π in the real wage, the key variable being Δw^k . This average wage has a minimum, which implies a maximum for L and, as a consequence of (1.83), a maximum long-term growth rate when Π is less than one with positive growth. This behavior of Δw is the reason why the relationship between long-run growth and trend inflation has an inverted-U shape. Δ^{wk} is then the variable containing the distortion wage rigidity introduces into the labor market and consequently into the long-run economic growth.

Finally, if we analyze the behavior of rigidities in prices and wages, Δ^{k_w} , C^k and L depend on Π , as a result of the presence of rigid wages, so too do, P^*/P , Δ^p , and P^i as a consequence of the presence of price rigidity. But we have already seen the

neutrality of the last set of variables. Consequently, the behavior is the same as in the case of wage rigidity. The relationship between g and Π has an inverted-U shape, with the maximum value at the same point as when only wage rigidity exists, which is exactly the same as in the case of flexibility (when Δ_w^k is minimum) given the independence between g and Π in the case of price rigidity.

1.5.2 Schumpeterian model

The results in the Schumpeterian model are similar to those of the physical capital externality model, in spite of the differences in their economic structures and dynamics. Trend inflation barely has an influence on the long-run growth rate in the model with only price rigidity. Although the terms $\frac{P_{-s}^*}{P}$ change with Π , the effects on $\sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P}\right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1\right)$ and $\left(\frac{1}{I} \sum_{s=0}^{I-1} \frac{P_{-s}^*}{P}\right)^{-\frac{1}{1-\alpha}}$ are negligible while on Δ_w , C/Y and A/Y are null as can be seen in A2.2.7, A2.2.5, A2.2.4 and A2.2.6. The effect on L is also null and, as a result, long-run growth rate remains constant according to A2.2.2.

The situation is different with only wage rigidity for any value of Π because the distortion introduced by the inflation rate in the mark-up of the wage affects Δ_w^Y in A2.2.4 and L in A2.2.3 and, finally, the growth rate in A2.2.2. In fact, as in the previous situation, L has a maximum when Δ_w^Y is minimum, which coincides with a value of Π less than one. Then, according to the expression (A2.2.2), the maximum growth rate occurs for a quarterly deflation rate of -0.5% making L maximum. When rigidity takes place in wages and prices, the result is the same as in the situation with only wage rigidity due to the independence between Π and g

under price rigidity. Once again, we find the distortion in the labor market introduced as a consequence of wage rigidity.

1.5.3 Technological change model

The results under the technological change model are very similar to the previous ones. When only price rigidity exists, the terms $\frac{1}{l} \sum_{\tau=0}^{l-1} \left(\frac{P_{t-\tau}^*}{P_t} \right)^{-\epsilon} \left(\frac{P_{t-\tau}^*}{P_t} - 1 \right)$ and Δ^P change with Π but the effects of the changes in Π are so limited that they are hardly noticeable for fair values of trend inflation, and the same happens with Δ^P , C/Y and Δ^W . As these terms are then independent of Π , as well as L from A2.3.1, the long-run growth rate remains constant from A2.3.6 for values of annual trend inflation from -12% to 12%.

Regarding wage rigidity, Δ^P and P^*/P are independent of Π as well as the term $\frac{1}{l} \sum_{\tau=0}^{l-1} \left(\frac{P_{t-\tau}^*}{P_t} \right)^{-\epsilon} \left(\frac{P_{t-\tau}^*}{P_t} - 1 \right)$. The distortion introduced by the inflation rate in the revision of wages has a clear effect in A2.3.1 on L and, finally, on the growth rate as can be seen in A2.3.6. In fact, as in the previous models, L has a clear maximum when Π is less than one with positive growth and Δ_w^Y is minimum. When L is maximum, long-run growth rate also reaches its maximum value according to the expression (A2.3.6). If we consider both types of rigidities in wages and prices, the behavior is the same as with wage rigidity due to the limited effect of Π on g when only price rigidity exists.

1.5.4 Human capital model

The results in the model of human capital have many aspects in common with the three previous ones but in one key aspect they are much more significant and different. Regarding price rigidity, the effects of Π on Δ^P , P^*/P , C and Δ_w are negligible and on L they are so limited that they are not noticeable until quarterly inflation/deflation rate values are greater than 12–15%. Consequently, according to A2.4.4, $g(C)$ is independent of Π , the same behavior observed with flexibility in prices and wages.

However, if we consider wage rigidity, the effect of Π on g is a consequence of the variation in the wage wage-per-unit of human capital (A2.4.1), since wage contracts revise with flexibility the skill components of the contracts, and in Δ_w (A2.4.2). The distortion in the labor market is only present due to the inflation rate as is reflected in A2.4.2. As A2.4.4 establishes a univocal (and inverse) relationship between Δ_w and $g(C)$, the maximum growth is reached when Δ_w is minimum, that is, for null inflation.

Regarding wage and price rigidity, the situation is similar to previous models. As with price rigidity, the relationship between Π and g is practically null for admissible inflation rates, the behavior under both rigidities being similar to wage rigidity.

1.6. Conclusions

The analysis of four models with different growth engines in order to understand how nominal price and wage rigidities affect long-run growth has been carried out.

The results confirm that monetary policy may be non-neutral in the long run in a context of endogenous growth and non-zero trend inflation. The main conclusions on the relationship between trend inflation and long-run growth are obtained from simulations of the four models using Dynare.

Firstly, the neutrality of trend inflation (monetary policy) on the long-run growth has been confirmed for admissible values of trend inflation in the four models when only price rigidity exists. These values depend on each model. For a model with physical capital externality, long-run growth rate remains constant for values of the quarterly rate of inflation/deflation lower than 4%; however, for the rest of the models, that neutrality remains up to values of 12–15%.

Secondly, when wage rigidity exists, an inverted-U shape is clearly confirmed in the four models for the relationship between the long-run rates of inflation and growth (non-neutrality of the monetary policy). Moreover, the influence of wage rigidity on growth dominates the relationship “trend inflation–long-run growth” when both rigidities coexist, showing the same behavior as when only wage rigidity exists.

A central objective was to confirm whether one of the conclusions of Amano et al. (2009) and Amano, Carter and Moran (2012)— namely, that with price and wage rigidity a negative trend inflation maximizes long-run growth rate—can be generalized regardless the growth engine. Our results lead us to conclude clearly the rejection of the general validity of this conclusion. Firstly, because when only price rigidity exists, long-run growth rate is independent of trend inflation for usual values of inflation or deflation rates. But also, the result cannot be generalized for wage rigidity because, in one of the four models, the model

corresponding to human capital accumulation, the long-run growth rate is maximized for null trend inflation.

It has been shown, in sum, that although there are three growth models for which wage rigidity implies a maximum growth rate for negative trend inflation (deflation), the result cannot be generalized for all the growth engines, because at least the human capital model is an exception.

When trend inflation is non-null, each revision of wages in the first three models when growth is positive elevates in excess their real values. This excess additionally decreases the labor demand in such a way that steady-state growth rate is negatively affected, acting as a greater distortion than when there is not growth. Our results indicate that if the trend inflation rate is negative, at exactly the same absolute value as the growth rate corresponding to price and wage flexibility, long-run growth rate is maximum in exactly the same situation as flexibility in which nominal wage revision is not necessary for recovering productivity growth. Any other value of trend inflation introduces a distortion that is greater, the greater the difference from this negative value.

In the human capital model, wage rigidity does not affect the productivity component of the variable, affecting only the wage-per-unit of human capital. Wage contracts contemplate separately the skill aspects and revise them with flexibility. The distortion due to positive growth indicated in the previous paragraph is not present and this is why the maximum growth is reached for null inflation, again a situation that is equivalent to price and wage flexibility.

This difference is not the only one existing between the human capital model and the rest of the growth engines considered when wage rigidity is present. Moreover,

one of the characteristics of the other three models is the negligible effect that trend inflation has on the long-run growth rate. In fact, we have seen that this effect is less than one hundredth of an annual percentage point for a change of four percentage points of the annual inflation rate in the Schumpeterian model, two hundredths in the technological change model, and less than two tenths in the model of physical capital externality. In contrast to them, in the human capital model this effect is much more significant, given that for a change of two percentage points of the annual inflation rate, the effect on the growth rate is a decline of more than two percentage points. This is an important effect that suggests the convenience of considering labor skill in the analysis of the effects of nominal wage rigidity, especially when they are considered from the economic growth perspective. The reason for this difference is the effect that the distortion in the average wage introduces in the demand for labor, which eventually affects human capital accumulation and hence long-run growth.

Chapter 2

Labor force participation and growth in the long run: a New-Keynesian extension of Friedman's Phillips curve revision

Abstract

Although our objective is to show how the existence of unemployment affects the long-run relationship between inflation and growth in a New-Keynesian model with efficiency wages, endogenous growth and Taylor-type stickiness, we find as a result an extension of Friedman's critique to the Phillips curve that provides a more general long-run perspective than the usual found in mainstream macroeconomic theory. This extension maintains the inflation/unemployment independence (unemployment natural rate hypothesis), but employment and labor force participation rates acquire a protagonist role given that both rates are maximum precisely for the trend inflation rate value that maximizes the long-run growth with sticky wages (per worker as well as per unit of human capital).

The meaning of this result is that the trend inflation rate value for which the natural rate of unemployment takes place is not indifferent, as it does in Friedman's critique to the Phillips curve, because it is associated, when wages are sticky, to different growth, employment, and labor force participation rates. The wealth of interactions and possibilities of the mechanisms through which this association takes place is exposed in two endogenous growth models corresponding to the alternative wage-setting processes.

2.1. Introduction

The conclusions of the first chapter have allowed us to know the relationship between trend inflation and long-run growth for different nominal rigidities and growth engines. On that basis, we can confirm the non-neutrality of the monetary policy as a consequence of the existence of wage stickiness, the behavior of the relationship differing, depending on the model. Specifically, it has been observed

that the value of trend inflation that maximizes long-run growth rate depends on whether the type of wage-setting process is per worker or per unit of human capital. These results have led us to reject the general validity of the results of Amano et al. (2009) and Amano, Carter and Moran (2012), according to which a negative trend inflation rate maximizes long-run growth in a context of price and wage stickiness.

Through this second chapter we will continue studying this relationship, integrating new variables that provide a more general perspective, which, except for a few rare exceptions, are not usually considered in macroeconomic models. Specifically, we analyze what is the impact on the quoted previous results of considering that labor supply no longer equals to labor demand and, therefore, unemployment appears in the economy. The objective is to know how a distortion in the labor market, which leads to unemployment, affects the relationship between trend inflation and long-run growth.

We can find different precedents in the existing literature on the link between unemployment and economic growth in the long run. Bean and Pissarides (1993) were the first to study this relationship. They concluded, using an overlapping-generations model, that adverse labor market institutions raise the unemployment rate and lower the employment and economic growth rates, establishing the existence of a negative relation between unemployment and growth rates in the long run. The same result was obtained by Eriksson (1997) in a model that was basically the same except for infinitely lived households. Both references assumed an exogenous labor force, which means that all the agents are either employed or unemployed.

Chen, Hsu and Lai (2016) take these results as a point of departure but, given that the labor force participation (LFP) has changed substantially across member countries of Organisation for Economic Co-operation and Development (OECD), they consider endogenous labor force participation. From this perspective, the changes in labor market institutions may cause increases or decreases in the long-run economic growth, depending on the effects on the employment rate. In fact, these changes may also affect the unemployment rate in a non-monotone way, which is consistent with the data.

Likewise, Schubert and Turnovsky (2018) delve into the long-run relationship between growth and unemployment, considering the role of job search during unemployment and wage bargaining. They conclude that, while the short-run trade-offs between unemployment and growth are substantial, the long-run trade-offs are much weaker. According to them, an increase in total factor productivity would lead to an immediate significant increase in growth, accompanied equally by a decline in unemployment. During the subsequent period, the unemployment rate would return almost totally to its initial equilibrium value. As a result, although we can find an immediate strong negative relationship, after a short period this relation switches to a strong positive one that neutralizes all the effects on unemployment. This conclusion is much closer than the previous references to a traditional and generally accepted macroeconomic result as it is the critique Friedman made of the Phillips curve 50 years ago.

The results of this chapter come to confirm Friedman's criticism of the Phillips curve in the long run in 1967, introducing some additional endogenous labor variables and a distortion in the labor market. We consider as endogenous variable

not only unemployment, but also employment and LFP. This shift in the focus provides important and apparently groundbreaking conclusions, given the more general perspective that it is able to provide for the macroeconomic dynamics. A first result confirms the irrelevance of unemployment rate as a long-run key macroeconomic variable in the labor market, being replaced by the employment and labor force participation rates. In fact, we can confirm by means of simulations using Dynare that the trend inflation rate that maximizes long-run growth rate is independent of the unemployment rate but, by contrast, it maximizes simultaneously the employment and labor force participation rates.

These results contain significant promise, beyond the confirmation of Friedman's hypothesis of the independence between trend inflation and unemployment in the long run, because it seems that they could represent a relevant New-Keynesian extension of Friedman's Phillips curve critique. Effectively, this critique is confirmed as certain, but this cannot be the end of the labor market story in the long run. The constant long-run unemployment rate is compatible with many values of the labor force participation and employment rates, two variables with a great factual economic impact but with hardly any presence in theoretical macroeconomic analysis. In our results they appear as two key labor market variables in the relationship between trend inflation and long-run growth from the perspective of the monetary policy summarized by the trend inflation rate as the inflation target.

Some authors have recently focused their attention on the evolution of the LFP. Van Zandweghe (2012) and Bullard (2014) admit the endogeneity of the LFP and try to find the reasons for its drop in the USA during the Great Recession of the

period 2007–2009. While the first author admits that half of the decline in LFP is accounted for by trend factors and the other half by cyclical factors, the second attributes almost all the decline to trend factors. However, none of these authors explain what moves the long-run LFP rate if unemployment rate is constant and demographic factors are stable in the long run. Could any monetary policy measure be adopted? Our results confirm the relationship between trend inflation and LFP and, therefore, the possibility of adopting monetary policy decisions to affect it. In fact, given the great concern about the LFP rate in the USA, the new Chair of the Federal Reserve System was alerted in 2017 about the relevance of this magnitude by the editors of Bloomberg (Saraiva and Matthews, 2017).

The economic tradition has accepted for 50 years that trend inflation does not affect unemployment rate (Friedman's Phillips curve critique), although we cannot always find a clear alignment with this macroeconomic core element if we review the literature. In fact, almost all the previous quoted contributions do not take it into account, or they contradict it.

Blanchard (2017), by contrast, in his speech to the American European Association (AEA), when Friedman's contribution celebrated its 50th birthday, admitted to the concerns about the low LFP rate and suggested keeping an open mind about adding some weight to alternative monetary policy measures. He pointed out that if the USA output were allowed to exceed the potential for some time, some of the workers who left the labor force during the previous ten years could be reintegrated. The mechanisms through which this reintegration could be permanent were not indicated, but at least Blanchard's reflection admits the importance of this variable as part of the macroeconomic options and its

sensitivity to the monetary policy.

But Blanchard was making his proposal using an argument that could be inappropriate from the perspective of our contribution. He was suggesting that a higher inflation target could be helpful in encouraging more entries in the labor market but, according to the results we obtain below, the effect of a greater inflation target would be the opposite to that indicated in his suggestion.

This discrepancy brought to the fore the relevance of the matter at hand. Our results confirm the macroeconomic relevance of the LFP rate in the long run beyond the independence between long-run unemployment rate and trend inflation, as well as its nonlinear response to trend inflation and the coincidence between its best behavior and the best for output growth and welfare when wages are sticky.

To address the analysis of this chapter, we will use two of the four growth models analyzed in the previous chapter: the Schumpeterian growth model of Aghion and Howitt and the Lucas human capital model. We will start from these models as they have been characterized in Chapter 1, introducing in detail the specific features corresponding to the considerations of unemployment. Efficiency wages are set according to Shapiro and Stiglitz (1984), which allow the introduction of unemployment and labor force participation rates as endogenous variables. This theory involves incentive problems, which reduce labor demand and generate unemployment that acts as a discipline mechanism for the workers. Moreover, we must carefully select the variables that play the role of labor supply and demand in the models, taking into account that New-Keynesian models introduce leisure in the utility function and differentiate between labor services. We also need to adapt

the Taylor staggered price and wage-setting mechanisms to the asymmetric information distortion introduced. All these features are explained in Section 2.

Section 3 presents the relationship between inflation, growth, unemployment, employment and labor force participation rates, which confirms the relevance of the last two rates in the maximization of the long-run growth rate. Section 4 contains the assessment of the main effect of considering wage stickiness on the long-run growth rate and how the value of the parameters can influence the final results. Section 5 contains the sensitivity analysis of economic and labor market variables to changes in efficiency wage parameters. Finally, section 6 summarizes the main findings.

In spite of having added many additional results, the extension contained in this second chapter maintains the validity of the main conclusions of the first chapter, that is, the non-neutrality of monetary policy under wage stickiness and the different behavior of the trend inflation–growth relationship depending on the wage-setting process (per hour or per unit of human capital). In the same way, the higher influence of the monetary policy on the long-run growth rate when wages are set per unit of human capital compared with the wage per hour is also maintained.

2.2. Two DSGE models with endogenous growth, efficiency wages and staggered wage and price setting

On the basis of two of the models analyzed in Chapter 1, the Schumpeterian and human capital models, we introduce a labor market friction. This friction involves, on the one hand, adapting the households' budget constraints to integrate the

labor supply (labor force participation) and, on the other hand, redefining the wage-setting process, since we no longer consider labor supply equals labor demand and, hence, the approach used in Chapter 1 to apply Taylor staggered mechanism must be discarded and adequately substituted. Efficiency wages have been considered to introduce the distortion causing unemployment, according to which the steady-state wage makes consistent the workers' incentives with the firms' objectives. These modifications allow us to obtain the labor supply and demand and the corresponding unemployment rate.

Regarding the rest of the equations describing the behavior of the agents, for example those describing price stickiness, no changes have been introduced and the expressions of Chapter 1 are maintained.

Price stickiness has been again considered for two periods, and wage stickiness for four. The expression for the sticky wage has been obtained through cumulative probabilities of the efficiency wage-setting process throughout these periods.

2.2.1 Agents

Households

As in the previous chapter, household members offer labor to intermediate or final good producers, depending on the model, consume the final good and hold bonds. However, unlike the first chapter, we assume that supply and demand for labor are no longer equal. Consequently, expected intertemporal utility takes the form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{1}{1+\nu} \int_0^1 N_{st}^{1+\nu} ds \right) \quad (2.1)$$

where N_s represents only the supply of labor (FFP) for service s with $s \in [0,1]$, while L_s will be the labor demand of the firms for this labor service s .

Furthermore, households must satisfy their budget constraint, which prevents the present value of the expenditure exceeding the stream of income and the value of their initial assets. However, unlike Chapter 1, the budget constraints must consider the effect of unemployment, the unemployment subsidy and the way this subsidy is financed. The exact expressions for the two models are the following:

Schumpeterian model

$$C_t + \frac{B_t}{P_t} + R\&D_t = \frac{B_{t-1}}{P_t} R_t^{st} + D_t + (1 - d_t) \int_0^1 (1 - \tau_t) \frac{W_{st}}{P_t} N_{st} ds + \int_0^1 z_t d_t N_{st} ds \quad (2.2)$$

Human capital model

$$C_t + \frac{B_t}{P_t} + K_{t+1} = \frac{B_{t-1}}{P_t} R_t^{st} + D_t + (1 - d_t) \int_0^1 (1 - \tau_t) \frac{W_{st}}{P_t} u_{st} N_{st} h_{st} ds + \int_0^1 z_t d_t N_{st} h_{st} ds + (1 + R_{t+\tau} - \delta) K_t \quad (2.3)$$

where d_t represents unemployment rate, τ_t the tax on the wages, and z_t the subsidy paid to the unemployed. The rest of the variables are the same as in Chapter 1: C_t is consumption, B_t nominal value of the stock of one-period life bonds that households hold in their portfolios, P_t the price of the final good, $R\&D_t$ investment in research and development, R_t real gross interest rate, R_t^{st} nominal gross interest rate, D_t firms' dividends, u_s proportion of time a service s employed devotes to production, h_{st} the human capital of the labor service s and W_{st} nominal wage for labor service s .

We assume the existence of government's budgetary equilibrium, which implies that the unemployment subsidy is completely funded with the tax on wages:

Schumpeterian model

$$\int_0^1 \tau_t \frac{W_{st}}{P_t} (1 - d_t) N_{st} ds = \int_0^1 z_t d_t N_{st} ds \quad (2.4)$$

Human capital mode

$$\int_0^1 \tau_t \frac{W_{st}}{P_t} (1 - d_t) u_{st} N_{st} h_{st} ds = \int_0^1 z_t d_t N_{st} h_{st} ds$$

considering the following expression as the labor demand in the human capital model:

$$L_{st} = (1 - d_t) u_{st} N_{st} h_{st} \quad (2.5)$$

Consequently, the previous budget constraints can be simplified as follows.

Schumpeterian model

$$C_t + \frac{B_t}{P_t} + R \& D_t = \frac{B_{t-1}}{P_t} R_t^{st} + D_t + (1 - d_t) \int_0^1 \frac{W_{st}}{P_t} N_{st} ds \quad (2.6)$$

Human capital model

$$C_t + \frac{B_t}{P_t} + K_{t+1} = \frac{B_{t-1}}{P_t} (R_t^{st} - 1) + D_t + \int_0^1 \frac{W_{st}}{P_t} L_{st} ds + (R_{t+\tau} - \delta) K_t \quad (2.7)$$

In such a way that, neither tax rate nor the unemployment subsidy appear in the expressions.

The labor supply expressions can be obtained from the solution to the decision problem of the individuals in each of the models.

Schumpeterian model

$$N_{st} = \left(\frac{1}{C_t} (1 - d_{st}) w_{st} \right)^{1/\nu} \quad (2.8)$$

where $N_t = \int_0^1 N_{st} ds$.

Human capital model

For flexibility, according to the optimal control problem presented in Appendix B.1:

$$N = \frac{1}{\xi(1-d)} \left(1 - \frac{\beta}{1+g(C)} \right) \quad (2.9)$$

for all services s .

For sticky wages (Appendix B.1):

$$N^1 = \frac{1}{\xi(1-d)} \left(1 - \frac{\beta/\Pi}{1+g(C)} \right) \quad (2.10)$$

$$N^0 = \frac{1}{\xi(1-d)} \left(1 - \frac{\beta\Pi^{J-1}}{1+g(C)} \right) \quad (2.11)$$

where N^1 corresponds to services maintaining the wage and N^0 to those who revise it.

Intermediate good firms

The behavior of intermediate good firms is different, depending on the model.

Schumpeterian model

As in the first chapter, monopolistically competitive firms obtain intermediate goods through a simple technology that generates one unit of a given intermediate good from one unit of final output. The profit for the firm producing intermediate good i will be:

$$F_{it} = P_{it}x_{it} - P_t x_{it} \quad (2.12)$$

where P_i and x_i are price and output of producer i .

They sell their output to final goods firms and set the prices according to Taylor contracts for I periods.

Human capital model

Intermediate goods producers are indexed by $j \in [0, 1]$ and have a Cobb–Douglas production function:

$$Y_{jt}^i = AK_{jt}^\alpha L_{jt}^{1-\alpha} \quad (2.13)$$

where Y_{jt}^i is the output of the intermediate good j , A is total factor productivity, K_{jt} is physical capital stock, and L_{jt} a composite index of differentiated labor services

$$\left(L_{jt} = \left(\int_0^1 L_{s jt}^{\frac{\sigma-1}{\sigma}} ds \right)^{\sigma/\sigma-1} \right).$$

The intermediate goods producer's optimal conditions are the following:

$$L_{jt} = L_t = \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{(1 - \alpha)A}{\Delta_t^w} \right]^{\frac{1}{\alpha}} K_t \quad (2.14)$$

$$R_t = \alpha \left[A \left(\frac{\varepsilon - 1}{\varepsilon} \right) \right]^{\frac{1}{\alpha}} \left[\frac{1 - \alpha}{\Delta_t^w} \right]^{\frac{1-\alpha}{\alpha}} \quad (2.15)$$

where Δ_t^w represents average real wage:

$$\Delta_t^w = \left[\int_0^1 \left(\frac{w_{st}}{P_t} \right)^{1-\sigma} ds \right]^{\frac{1}{1-\sigma}} \quad (2.16)$$

Considering expression 2.5 and those of N (2.10 and 2.11) and u (Appendix B.1), we can obtain the employment rate for each value of N^s and u^s with sticky wages:

$$\begin{aligned} L^0 &= (1 - d)u^0 N^0 \\ L^1 &= (1 - d)u^1 N^1 \end{aligned} \quad (2.17)$$

$$L^{01} = (1 - d)u^{01}N^1$$

Retail firms or final goods producers

Schumpeterian model

According to Aghion and Howitt (1992), final goods production function is the following:

$$Y_t = L_t^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^\alpha d_i \quad (2.18)$$

where x_{it} is the amount of intermediate good i used at t , $0 < \alpha < 1$, L_t is the composite demand of labor services and A_{it} is the productivity of intermediate good i (quality level).

The demand function for labor service s is obtained from profit maximization:

$$L_{st} = \left[(1 - \alpha) Y_t L_t^{\frac{1-\sigma}{\sigma}} \left(\frac{W_{st}}{P_t} \right)^{-1} \right]^\sigma \quad (2.19)$$

Considering that labor supply is not equal to labor demand, integrating L_{st} we obtain the aggregate labor demand function:

$$\left(\int_{j=0}^1 L_s^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} = L_t = ((1 - \alpha) Y_t)^\sigma L_t^{1-\sigma} \left[\int_{s=0}^1 \left(\frac{W_{st}}{P_t} \right)^{1-\sigma} ds \right]^{\frac{\sigma}{\sigma-1}}$$

$$L_t = \frac{(1 - \alpha) Y_t}{\Delta_t^W} \quad (2.20)$$

where Δ_t^W again represents average real wage:

$$\Delta_t^W = \left[\int_{s=0}^1 \left(\frac{W_{st}}{P_t} \right)^{1-\sigma} ds \right]^{\frac{1}{1-\sigma}} \quad (2.21)$$

Additionally, we obtain the expression of the partial labor demand for each value of sticky wage:

$$L_{st} = \left(\frac{(1 - \alpha)L_t^{1-\sigma/\sigma}}{w_{st}} \right)^\sigma \quad (2.22)$$

The expression of unemployment rate is obtained from the difference of labor supply and demand for every labor service s (2.8 and 2.22):

$$d_{st} = \frac{N_{st} - L_{st}}{N_{st}} \quad (2.23)$$

where $d_t = \frac{\int_0^1 (N_{st} - L_{st}) ds}{\int_0^1 N_{st} ds}$.

Human capital model

There are an infinite number of retail firms over the continuum $[0,1]$, which repackage the homogeneous intermediate goods and sell them to households according to the demand function:

$$Y_t = \left[\int_{s=0}^1 Y_{rt}^{\frac{1-\varepsilon}{\varepsilon}} dr \right]^{\frac{\varepsilon}{1-\varepsilon}} \quad r \in [0,1]$$

They sell their goods to households and set the price according to Taylor contracts for each interval of I periods.

2.2.2 Growth, innovation and human capital accumulation

Schumpeterian model

This model displays Schumpeterian growth because it occurs by increasing the quality of intermediate goods, and by “quality” we must understand the technological (or productivity) level of these capital goods.

As it has been explained in the previous chapter, on the basis of the good producers' profit function and the expected profits, if innovation is successful, we can obtain (Chapter 1) the gross growth rate in steady state:

$$g = \left[\chi \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1 \right) \right]^{\frac{\chi}{1-\chi}} (\gamma - 1) + 1 \quad (2.24)$$

Human capital model

The growth process in human capital model is obtained from the solution of the dynamic optimization problem recorded in Appendix B.1, which establishes these conditions in steady state:

$$g(Y) = g(Y^i) = g(K) = g(L) = g(h) \quad (2.25)$$

$$g(h) = \begin{cases} [1 + \xi(1 - u_{ss}(1 - d))N_{ss}] & \text{Wage Flexibility} \\ [1 + \xi(1 - u^1(1 - d))N^1](1 + g(h^1))\left(\frac{J-2}{J}\right) + \\ + [1 + \xi(1 - u^{01}(1 - d))N^1](1 + g(h^{01}))\left(\frac{1}{J}\right) + \\ + [1 + \xi(1 - u^0(1 - d))N^0](1 + g(h^0))\left(\frac{1}{J}\right) & \text{Wage rigidity} \end{cases} \quad (2.26)$$

where u_{ss} y N_{ss} are steady-state values with wage flexibility, while u^1 , h^1 and N^1 are the decisions for labor services with constant nominal wage for $s \in [0, J - 2)$, u^{01} , h^{01} and N^1 for $s \in [J - 2, J - 2]$, and u^0 , h^0 and N^0 for $s \in [J - 1, J - 1]$ the labor services that will reset nominal wage in the following period.

2.2.3 Unemployment and wage stickiness: efficiency wages and staggered contracts

The labor market friction introduced by the existence of efficiency wages involves incentive problems: asymmetric information, moral hazard and adverse selection. The main implication of this theory is a lower labor demand than labor supply and, consequently, the existence of unemployment, which works as a discipline mechanism for the workers and generates inefficiencies in resource allocation, according to Shapiro and Stiglitz (1984).

Employees choose between two effort levels (0,1), with 0 being the real cost of not making an effort and e the cost of doing so. However, if an employee shirks work tasks (effort 0), there is a probability q of being caught and being dismissed.

If we consider wage flexibility, workers' employment discounted present value (DPV) depends on the strategy about shirking or complying. The following expressions for the arbitrage equations must be considered:

$$\begin{aligned} rV_E^S &= w + (b+q)(V_U - V_E^S) \\ rV_E^N &= w - e + b(V_U - V_E^N) \\ rV_U &= z + a(V_E - V_U) \end{aligned} \tag{2.27}$$

Where V_E^S represents the DPV of employment for the worker that shirks, V_E^N the DPV for the worker that does not shirk and, finally, V_U that of the unemployed. In addition, r is interest rate, b the probability rate of employment loss, q the probability of being caught and being fired, a the rate of job-finding and z the utility of leisure time and unemployment benefits.

Consequently, the employers will set a wage consistent with workers incentives, that is, a wage ensuring workers will make the effort instead of shirking. This wage must fulfill this condition:

$$V_E^N = V_E^S \quad (2.28)$$

$$w = z + e + \left(r + \frac{bN}{N-L}\right) \frac{e}{q} = z + e + \left(r + \frac{b}{d}\right) \frac{e}{q}$$

where N is labor force, L employment and d unemployment rate. From this expression, we can deduce that a higher $\frac{bN}{N-L}$, that is, a lower level of unemployment, requires a higher wage in order to satisfy the no shirking condition. Therefore, unemployment acts as a discipline mechanism.

If we consider quarterly data and wage stickiness during four periods according to Taylor staggering wage-setting process, the average values of workers employment and unemployment satisfy the following arbitrage conditions:

$$rV_E^S = \frac{1}{4} [w\Delta_w^{bq} + (b\Delta_b + q\Delta_q)(V_U - V_E^S)]$$

$$rV_E^N = \frac{1}{4} [w\Delta_w^b - e\Delta_b + b\Delta_b(V_U - V_E^N)] \quad (2.29)$$

$$rV_U = \frac{1}{4} [z\Delta_a + a\Delta_a(V_E - V_U)]$$

The variable w is the steady wage value set in every revision. In order to obtain this wage value, we need the expressions of the different parameters Δ that represent the cumulative probabilities of being employed or unemployed:

$$\Delta_b = 1 + (1-b) + (1-b)^2 + (1-b)^3$$

$$\Delta_q = 1 + (1-q) + (1-q)^2 + (1-q)^3 \quad (2.30)$$

$$\Delta_a = 1 + (1-a) + (1-a)^2 + (1-a)^3$$

The parameter Δ_b contains the four probabilities of being employed due to structural reasons, the parameter Δ_q the four probabilities of being employed despite shirking, and the parameter Δ_a the four probabilities of being unemployed also for structural reasons.

The parameter Δ_w^{bq} contains the four coefficients differencing the steady wage w after each revision from the other three possible values of steady wages that coincide simultaneously each quarter in the case of workers that shirk. This parameter is also different in the two models.

Schumpeterian model

$$\Delta_w^{bq} = 1 + \frac{(1-b)(1-q)}{\Pi g} + \left(\frac{(1-b)(1-q)}{\Pi g} \right)^2 + \left(\frac{(1-b)(1-q)}{\Pi g} \right)^3 \quad (2.31)$$

Human capital model

$$\Delta_w^{bq} = 1 + \frac{(1-b)(1-q)}{\Pi} + \left(\frac{(1-b)(1-q)}{\Pi} \right)^2 + \left(\frac{(1-b)(1-q)}{\Pi} \right)^3 \quad (2.32)$$

The parameter Δ_w^b contains the four coefficients differencing the four possible values of steady net wages that coincide simultaneously each quarter in the case of workers that do not shirk. This parameter is also different in the two models.

Schumpeterian model

$$\Delta_w^b = 1 + \frac{(1-b)}{\Pi g} + \left(\frac{(1-b)}{\Pi g} \right)^2 + \left(\frac{(1-b)}{\Pi g} \right)^3 \quad (2.33)$$

Human capital model

$$\Delta_w^b = 1 + \frac{(1-b)}{\Pi} + \left(\frac{(1-b)}{\Pi}\right)^2 + \left(\frac{(1-b)}{\Pi}\right)^3 \quad (2.34)$$

The three arbitrage conditions for every DPV can be rewritten as follows:

$$(4r + b\Delta_b + q\Delta_q)V_E^S = w\Delta_w^{bq} + (b\Delta_b + q\Delta_q)V_U$$

$$(4r + b\Delta_b)V_E^N = w\Delta_w^b - e\Delta_b + b\Delta_b V_U$$

$$(4r + a\Delta_a)V_U = z\Delta_a + a\Delta_a V_E$$

Taking into account as the point of departure the condition of consistency with workers incentives:

$$V_E^S = \frac{w\Delta_w^{bq} + (b\Delta_b + q\Delta_q)V_U}{(4r + b\Delta_b + q\Delta_q)} = \frac{w\Delta_w^b - e\Delta_b + b\Delta_b V_U}{(4r + b\Delta_b)} = V_E^N$$

Afterwards, we can obtain some algebra:

$$w\Delta_w^{bq} = (w\Delta_w^b - e\Delta_b) \frac{(4r + b\Delta_b + q\Delta_q)}{(4r + b\Delta_b)} - \frac{4rq\Delta_q}{(4r + b\Delta_b)} V_U$$

$$V_U = \frac{4r + b\Delta_b}{4r4r + 4ra\Delta_a + 4rb\Delta_b} z\Delta_a + \frac{(w\Delta_w^b - e\Delta_b)a\Delta_a}{4r4r + 4ra\Delta_a + 4rb\Delta_b}$$

And from here, the expression of the steady wage after each wage revision:

$$w \left[\Delta_w^{bq} - \Delta_w^b \frac{(4r + b\Delta_b + q\Delta_q)}{(4r + b\Delta_b)} \right] = -e\Delta_b \frac{(4r + b\Delta_b + q\Delta_q)}{(4r + b\Delta_b)} - \frac{4rq\Delta_q}{(4r + b\Delta_b)} V_U$$

$$w \left[\frac{\Delta_w^{bq}(4r + b\Delta_b) - \Delta_w^b(4r + b\Delta_b + q\Delta_q)}{(4r + b\Delta_b)} \right]$$

$$= -e\Delta_b \frac{(4r + b\Delta_b + q\Delta_q)}{(4r + b\Delta_b)} - \frac{4rq\Delta_q}{(4r + b\Delta_b)} V_U$$

$$w[\Delta_w^{bq}(4r + b\Delta_b) - \Delta_w^b(4r + b\Delta_b + q\Delta_q)] = -e\Delta_b(4r + b\Delta_b + q\Delta_q) - 4rq\Delta_q V_U$$

$$\begin{aligned}
& w[\Delta_w^{bq}(4r + b\Delta_b) - \Delta_w^b(4r + b\Delta_b + q\Delta_q)] \\
& = -e\Delta_b(4r + b\Delta_b + q\Delta_q) - \frac{q\Delta_q(4r + b\Delta_b)}{(4r + a\Delta_a + b\Delta_b)}z\Delta_a \\
& \quad - \frac{(w\Delta_w^b - e\Delta_b)q\Delta_qa\Delta_a}{(4r + a\Delta_a + b\Delta_b)} \\
w & = \frac{e\Delta_b \left[(4r + b\Delta_b + q\Delta_q) - \frac{q\Delta_qa\Delta_a}{(4r + a\Delta_a + b\Delta_b)} \right] + \frac{q\Delta_q(4r + b\Delta_b)}{(4r + a\Delta_a + b\Delta_b)}z\Delta_a}{\Delta_w^b \left[(4r + b\Delta_b + q\Delta_q) - \frac{q\Delta_qa\Delta_a}{(4r + a\Delta_a + b\Delta_b)} \right] - \Delta_w^{bq}(4r + b\Delta_b)} \quad (2.35)
\end{aligned}$$

In the case of wage flexibility, we have $\Delta_w^b = \Delta_w^{bq} = \Delta_b = \Delta_q = \Delta_a = 1$, and the expression of w is simplified. The flexible steady wage would take the following expression:

$$w = \frac{e \left[(r + b + q) - \frac{rqa}{r(r + a + b)} \right] + \frac{q(r + b)}{(r + a + b)}z}{\left[(r + b + q) - (r + b) - \frac{qa}{(r + a + b)} \right]} \quad (2.36)$$

From where, after some algebra, we obtain the well-known expression of the efficiency wage introduced initially (2.28):

$$\begin{aligned}
w & = \frac{e \left[(r + b + q) - \frac{qa}{(r + a + b)} \right] + \frac{q(r + b)}{(r + a + b)}z}{\left[(r + b + q) - (r + b) - \frac{qa}{(r + a + b)} \right]} \\
& = \frac{e[(r + b)(r + a + b) + q(r + a + b) - qa] + q(r + b)z}{[q(r + a + b) - qa]} \\
& = \frac{e[(r + b)(r + a + b) + q(r + b)] + q(r + b)z}{q(r + b)} \\
& = z + e + \frac{e}{q}(r + a + b) = z + e + \frac{e}{q} \left(r + \frac{b}{d} \right)
\end{aligned}$$

2.2.4 Price setting

Schumpeterian model

As explained in Chapter 1, it is intermediate goods producers who set every I period (quarters) the price P^* that maximizes their expected profits, which will be the following value (relative to the final good price) in steady state for all periods:

$$\frac{P^*}{P} = \frac{1 \sum_{\tau=0}^{I-1} \left(\beta (\Pi)^{1/1-\alpha} \right)^\tau}{\alpha \sum_{\tau=0}^{I-1} \left(\beta (\Pi)^{\alpha/1-\alpha} \right)^\tau} \quad (2.37)$$

Human capital model

It is the retail firms who set, for the I periods, the price that maximizes their expected profits in that time interval. The optimal relative price in steady state will be:

$$\frac{P^*}{P} = \frac{\sum_{\tau=0}^{I-1} (\beta \Pi^\varepsilon)^\tau}{\sum_{\tau=0}^{I-1} (\beta \Pi^{\varepsilon-1})^\tau} \quad (2.38)$$

2.2.5 Equilibrium conditions

Schumpeterian model

As described in Chapter 1, the aggregate equilibrium of the good markets is the equality between final output and the sum of consumption and gross investment. We again consider that there are neither public expenditures nor an external sector and the demand for final goods is composed of consumption, investment in R&D, and intermediate goods production. As a result, the ratio consumption/output satisfies the following expression in steady state:

$$\frac{C}{Y} = 1 - \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \frac{A}{Y} - \left[\chi \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1 \right) \right]^{\frac{1}{1-\chi}} \frac{A}{Y} \quad (2.39)$$

where additionally:

$$\frac{A}{Y} = \frac{1}{\left(\alpha^{\frac{1}{1-\alpha}} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \right)^{\alpha} L} \quad (2.40)$$

Human capital model

We again assume, also as in Chapter 1, that there are neither public expenditures nor an external sector. Therefore, final good output is composed of consumption and investment, and the steady-state consumption to physical capital ratio in steady state, C/K , will be as follows:

$$\frac{C}{K} = A^{\frac{1}{\alpha}} \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{1 - \alpha}{\Delta_t^W} \right]^{\frac{1-\alpha}{\alpha}} - g(C) - \delta \quad (2.41)$$

$$g(Y) = g(Y^i) = g(K) = g(L) = g(C) \quad (2.42)$$

2.2.6 Steady state

We must characterize the steady state and the system of equations that determine the values of the endogenous variables in this situation, considering that our models incorporate economic growth and, therefore, the growing steady-state variables must be normalized.

Schumpeterian model

The normalization of all the growing variables of the Schumpeterian model is carried out by dividing them by the production level of the final good Y . The

system of equations is presented in Appendix B2.1 for the endogenous variables:

$$\frac{P^*}{P}, \frac{P_{-s}^*}{P}, g, L, L_{-s}, LL, \Delta_W^Y, w, \left(\frac{w}{Y}\right)_{-s}, N_{-s}, N, d, \frac{C}{Y}, \frac{A}{Y} \text{ and } R.$$

Human capital model

Considering the representative household's optimal control problem of human capital model developed in Appendix B1, the steady-state system of equations for flexible wages is defined for the endogenous variables: $w, w_{-s}, \Delta_W, R, C/K, g, N_{ss}, u_{ss}, P^*/P, \Delta_P, R, LL_{ss}$ and d_{ss} . If there is wage stickiness, the equations system contains the following unknowns: $w, w_{-s}, \Delta_W, C/K, g, N^0, N^1, N, u^1, u^0, u^{01}, P^*/P, P_{-s}^*/P, \Delta_P, R, L^0, L^1, L^{01}, LL$ and d . We present the system of equations in Appendix B2.2.

2.3. Trend inflation influence on unemployment, employment and labor force participation rates

As we have done in the first chapter, the two models have been simulated through Dynare in order to obtain the values of the main endogenous variables in steady state and their responses to changes in trend inflation. Given the changes introduced in the setup of the models, we will pay special attention to the role played by unemployment, employment and labor force participation rates in the relationship between trend inflation and long-run growth, and the differences with the conclusions of the first chapter in this relationship.

The values of the parameters used in the simulations are presented in Table 2.1. They are appropriate for quarterly data and, excluding the specific parameters of efficiency wage (z, q, e and b), commonly used in New-Keynesian models. Related

to efficiency wage parameters, in this section we use a combination of values ($z=0.2$, $e=0.05$, $q=0.9$ and $b=0.1$) for the Schumpeterian model and ($z=0.49$, $e=0.0756$, $q=0.9$ and $b=0.6$) for the Lucas human capital model. These combinations lead to a set of consequences, which, although they are not unique, allow us the presentation of all the possibilities that will be completely explained in Section 5, where we present the sensibility analysis for every one of these parameters.

2.3.1 Schumpeterian model

The long-term inflation–growth relationship for the parameters of Table 2.1 is very similar to that obtained in Chapter 1. Figure 2.1 shows how the growth rate remains constant at a value near 0.514% whatever the value of the trend inflation rate in the case of flexibility or only price stickiness. Similarly, under wage stickiness, growth is maximized for a negative trend inflation rate near -0.512%, reaching a value somewhat lower than under flexibility (0.512%). Consequently, we can maintain all the conclusions of Chapter 1 regarding the long-term inflation–growth relationship, except the coincidence of the value of the maximum growth rate achievable with wage stickiness and wage flexibility.

Table 2.1: Parameter values chosen

Parameter	Description	Schumpeterian model	Human capital model
δ	Capital depreciation rate		0.0275
α	Output elasticity with respect to capital	0.332	0.332
β	Utility discount factor	0.97	0.97
ε	Elasticity of substitution among retail or intermediate goods		5
ϕ_π	Coefficient of inflation reaction in the Taylor rule	2.05	2.05

σ	Elasticity of substitution among labor services	12	12
ν	Relative utility weight of labor	1	1
I	Periods it takes to reset prices	(1, 2)	(1, 2)
J	Periods it takes to reset wages	(1, 4)	(1, 4)
γ	Productivity upgrade after every innovation	1.009	
χ	Elasticity of the probability of success in the innovation with respect to relative investment	0.1	
ξ	Productivity parameter of human capital accumulation		0.07
A	Constant total factor productivity		1
z	Utility of leisure time and unemployment benefits	0.2	0.49
e	Cost of labor effort	0.05	0.075
q	Probability of being fired after shirking	0.9	0.9
b	Exogenous probability of losing the job	0.1	0.6

This difference implies that we verify for this combination of parameters (z, e, q, b) the existence of a “growth loss” when wages are sticky as a consequence of the presence of the unemployment originated by the friction introduced by efficiency wages. It will be interesting below to discover the mechanism at work in the origin of this loss; at the same time we show that the growth loss is not the only possible result because the possibilities of “no growth loss” or a “growth premium” are also open.

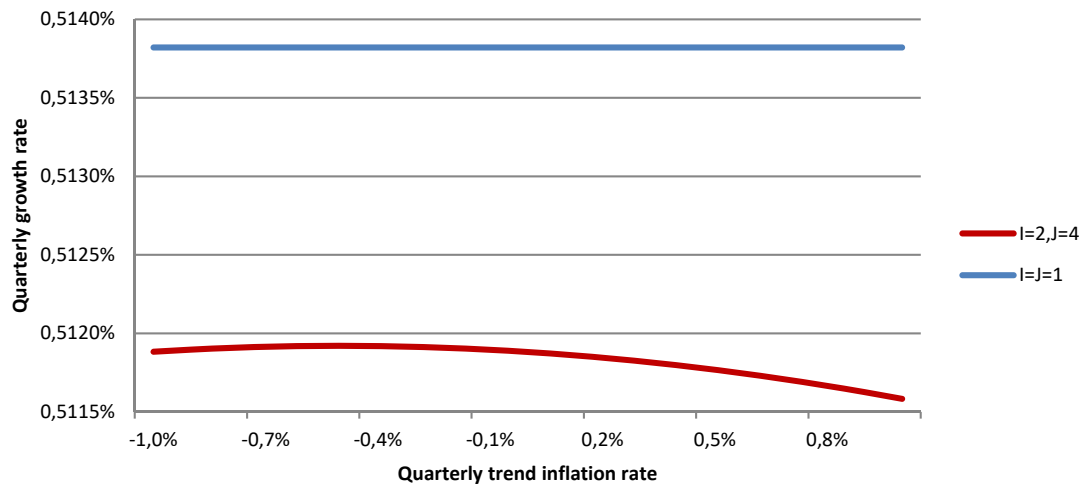


Figure 2.1: Long-term inflation-growth relationship – Schumpeterian growth model.

The relationship between inflation and unemployment, employment and labor force participation rates are shown in Figures 2.2, 2.3 and 2.4, respectively. As in the previous chapter, rates with flexibility remain constant whatever the value of trend inflation and these rates slightly vary when only price stickiness exists (flexibility relationships are represented as the blue line while the red line corresponds to stickiness).

Regarding wage stickiness, we must consider the three rates separately since they show a different behavior. Concerning unemployment rate (Figure 2.2), this does not change with trend inflation even in the case of wage stickiness. Consequently, we can confirm that unemployment rate is not a relevant variable in the long-term relationship between inflation and growth, since long-term unemployment is independent from trend inflation. This is a common feature of wage and price flexibility and wage stickiness, confirming Friedman’s revision of the Phillips curve. The specific mechanism derived from the role played by the unemployment rate is that, as a consequence of the greater rate of stickiness, the “growth loss”

related to flexibility occurs In Section 5 we show that, when unemployment rate is lower in the case of stickiness, a “growth premium” occurs or, when it is the same in stickiness and flexibility, the “no growth loss” possibility appears (as in Chapter 1).

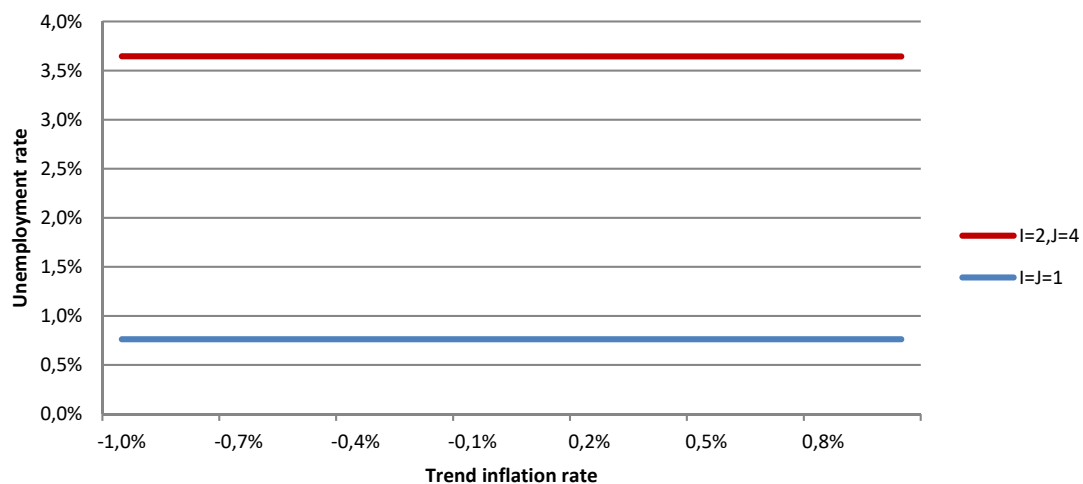


Figure 2.2: Long-term inflation–unemployment relationship – Schumpeterian growth model.

If we pay attention to the long-term relationships inflation–labor force participation and inflation–employment, displayed in Figures 2.3 and 2.4, respectively, we can observe again independence in the case of price and wage flexibility and an inverted U-shape in the case of wage stickiness with the maximum value for these two relevant rates of the labor market at exactly the same trend inflation rate value that maximizes long-run growth (-0.512%). This result means, as a consequence, that the dynamics of these two rates contribute, and are closely related, to the long-run growth maximization with wage stickiness. The two rates reach their highest value for this trend inflation rate value due to the effect that the lowest average real wage at that point has on the employment rate

(expression 2.24) and to the combination of the four different wages, the consumption and the unemployment rate at that point that provide the maximum average labor force participation rate.

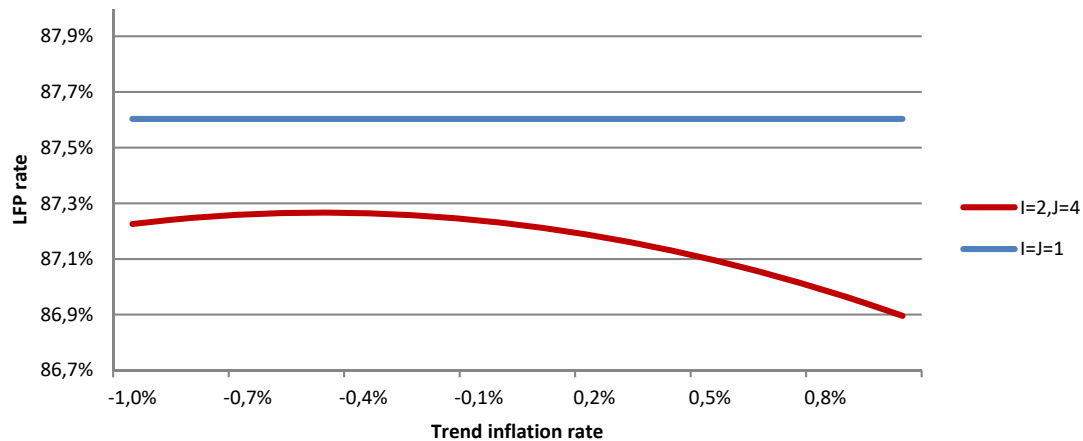


Figure 2.3: Long-term inflation-LFP relationship – Schumpeterian growth model.

Moreover, for the efficiency wage parameters chosen, both employment and LFP rates reach—at most—a lower value when wages are sticky than when they are flexible at the point where both rates are maximized. However, higher or the same values cannot be discarded because they appear when there is “growth premium” or “no growth loss.”

We can see, comparing Figures 2.3 and 2.4, that the difference between flexibility and wage stickiness is greater in the employment rate than in the labor force participation rate, this difference being the reason for the greater unemployment rate in the case of stickiness.

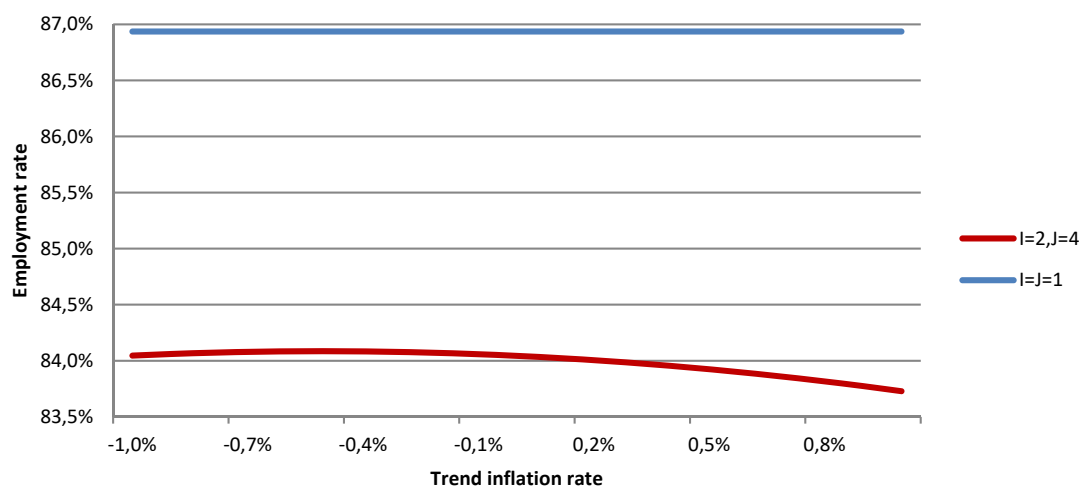


Figure 2.4: Long-term inflation–employment relationship – Schumpeterian growth model.

2.3.2 Human capital model

If we consider wages-per-unit of human capital, Figure 2.5 shows the relationship between economic growth and trend inflation in the long term. The blue line represents the case of wage and price flexibility, where the growth rate remains constant at a value of 0.83% whatever the inflation rate. The red line represents the case of wage stickiness, where the growth rate is maximized for null trend inflation reaching a value lower than flexibility (0.5466%). Consequently, we can also maintain the conclusions of Chapter 1 regarding the human capital model except for the coincidence of the maximum growth rate value achievable with wage stickiness and wage flexibility. The “growth loss” occurring in the Schumpeterian model with unemployment (when sticky wages are per hour or per worker) is also confirmed in the case of wages-per-unit of human capital. The mechanism is the same, depending on the relative unemployment rate values corresponding to wage stickiness and flexibility and, consequently, it is one of the possibilities along with other two alternatives: “growth premium” and “no growth

loss.” The difference in the case of the human capital model is that the position of the unemployment rate values is the opposite to those in the Schumpeterian model because the “growth loss” corresponds to a greater unemployment rate for the wage flexibility situation, while the “growth premium” corresponds to a lower one.

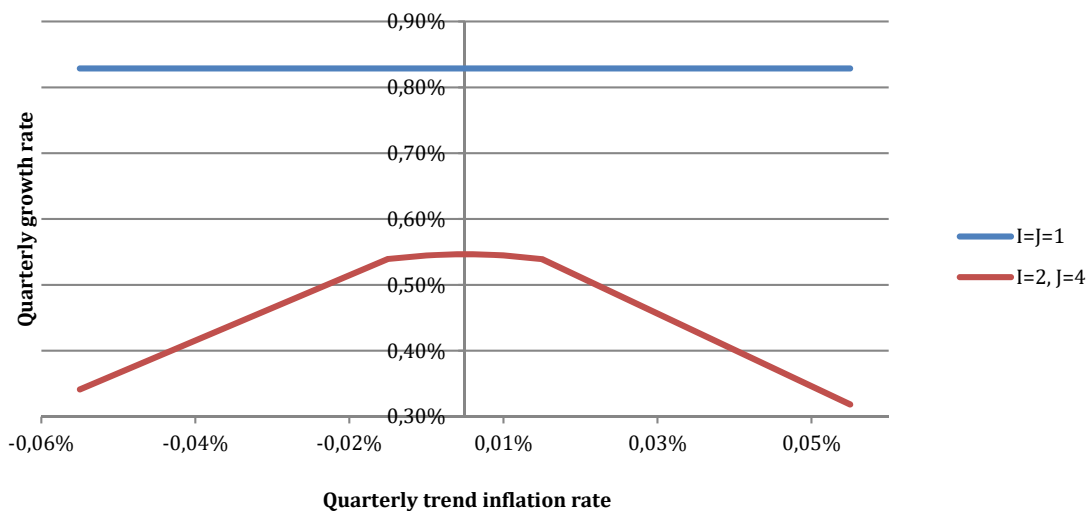


Figure 2.5: Long-term inflation-growth relationship – Human capital growth model.

The following charts, Figures 2.6, 2.7 and 2.8, show the relationships between inflation and unemployment, employment and labor force participation rates. As expected, these three labor market rates remain constant whatever the value of trend inflation with wage and price flexibility (blue line in the charts).

In the case of wage stickiness, each variable must also be considered separately. Regarding the unemployment rate (Figure 2.6), this remains unchanged after changes in trend inflation. Consequently, we can again confirm that unemployment rate is not a relevant variable in the maximization of long-term growth and also the validity of the revision made by Friedman to the Phillips curve in the case of wages-per-unit of human capital.

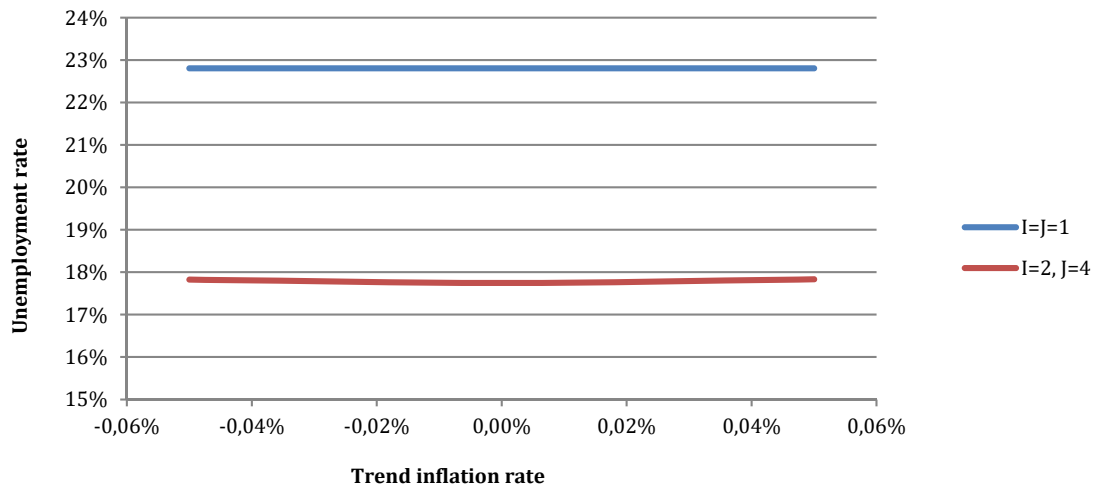


Figure 2.6: Long-term inflation–unemployment relationship – Human capital growth model.

With regard to employment and labor force participation rates, we can observe their relationships with trend inflation in Figures 2.7 and 2.8, respectively. The two rates reach their maximum value for null trend inflation, the same value for which economic growth is maximized. Consequently, as in the Schumpeterian model, these two variables contribute, and are closely related, to the maximization of long-run economic growth, unlike unemployment rate.

The reason why these two important labor market rates are maximized for the same trend inflation rate value that maximizes the growth rate is much clearer than in the Schumpeterian model. For that trend inflation rate value, the real average wage reaches its minimum and, hence, the employment rate its maximum. In the case of the LFP rate, as it depends positively on the growth rate, the maximum for both rates are coincident.

Moreover, both rates reach—at most—a lower value than with flexibility because of the negative effect of the lower growth rate on the labor force participation rate,

according to expressions (2.10) and (2.11). These results have significant consequences because the lower labor force participation rate with wage stickiness results in a lower unemployment rate, unlike in the Schumpeterian model. The underlying cause of the higher unemployment rate is a relatively higher LFP encouraged by the human capital accumulation. In any case we can conclude that a lower unemployment rate does not necessarily involve a higher long-term economic growth.

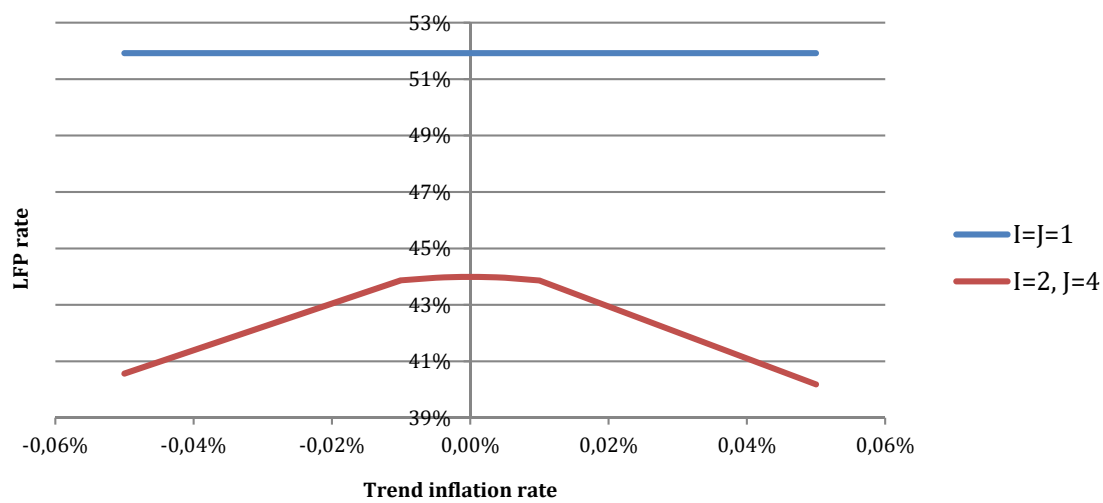


Figure 2.7: Long-term inflation-LFP relationship – Human capital growth model.

We can see, comparing Figures 2.7 and 2.8, that the difference between flexibility and wage stickiness is greater in the labor force participation rate than in the employment rate, this difference being the reason for the lower unemployment rate in the case of stickiness.

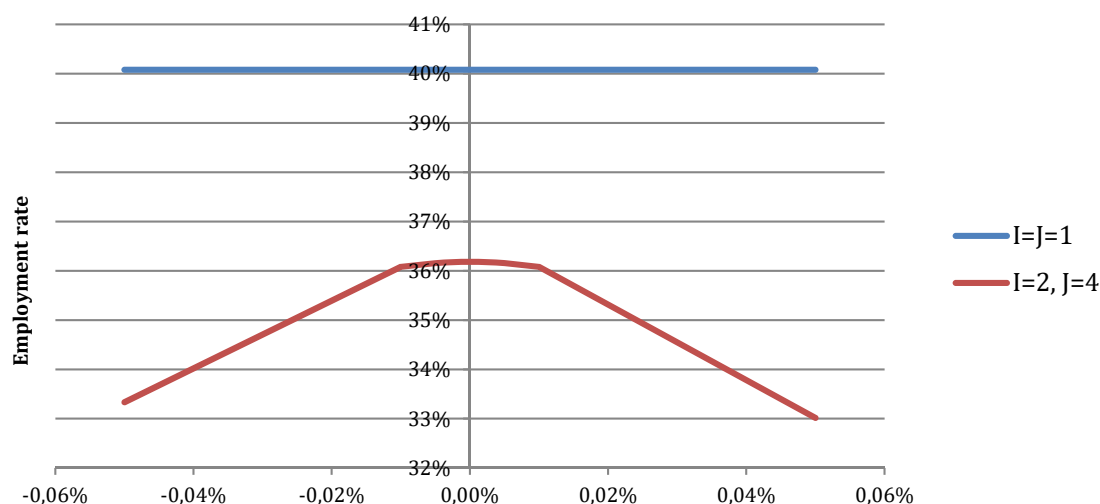


Figure 2.8: Long-term inflation–employment relationship – Human capital growth model.

Finally, as in the previous chapter, we also find a sharp difference in the magnitude of the effect of trend inflation on long-run growth and labor market variables between the two models. We can continue maintaining a higher influence of monetary policy on long-run growth and the labor market variables when wages are set per unit of human capital.

2.4. Effects of considering labor market stickiness on maximum growth rate: transmission mechanisms

Having evaluated and compared through simulations the consequences of a labor market friction (efficiency wages) on the long-run relationships between inflation, labor market variables and growth, it is necessary to identify from the steady-state equations (Appendix B.2) the main mechanisms that explain these results. Table 2.2 summarizes the main results we have obtained.

One of the main results is the value of trend inflation that maximizes the long-term growth rate in each model. This was the focus of the previous chapter and adequately explained there. In spite of the different labor market context, the same conclusions can be maintained in this chapter. According to this new context of efficiency wages, real average wage is minimum (and growth rate maximum) when (negative) trend inflation has exactly the same absolute value as the growth rate in the Schumpeterian model and for null trend inflation in the Lucas human capital model due to the effect of the average real wage on labor demand and growth. Consequently, the previous chapter results and conclusions regarding this point can be maintained.

Table 2.2: Long-run maximum (quarterly) rates

Growth engine	Wage-setting process	Growth rate	Labor force participation	Employment rate	Unemployment rate
Schumpeterian Model	Wage Flexibility	0.514%	87.60%	86.94%	0.76%
	Wage Stickiness (*)	0.512%	87.27%	84.08%	3.64%
Human Capital Model	Wage Flexibility	0.8288%	51.92%	40.08%	22.8%
	Wage Stickiness (**)	0.5466%	43.99%	36.18%	17.75%

(*) Trend inflation rate = -0.512%

(**) Trend inflation rate = 0%

But not all the results coincide because the unemployment originated by the friction introduced is the cause of a growth loss when wages are sticky. A second significant result is that the economic growth rate with sticky wages does not reach the value of wage flexibility. The transmission mechanisms of the two models were explained in section 5 of the previous chapter and, accordingly, both growth rates are maximized for the minimum value of the average real wage (Δ_w). According to B2.1.3, the effect of the average real wage on growth rate takes place through the aggregate employment in the Schumpeterian model, while this effect

is direct in the Lucas human capital model according to B.2.2.5. Consequently, if the maximum growth rate achievable with sticky wages does not reach the value of wage flexibility this is due to a higher average real wage value for the trend inflation that maximizes growth, unlike Chapter 1. The introduction of the labor market friction involves a different labor market context (efficiency wages), which results in a higher average real wage than in flexibility, due to the effect of the different cumulative crossed quarterly probabilities.

The third relevant result, which is different in the two growth engines, is that the value of unemployment rate when wages are sticky is greater than in flexibility in the Schumpeterian model, while in the human capital model it is lower. The different transmission mechanisms were advanced in the previous section.

Considering the Schumpeterian model, the higher average real wage with wage stickiness affects the employment rate both directly and negatively (B2.1.4), and the labor force participation rate positively, since the slightly higher consumption is offset by the higher average real wage (B2.2.10), increasing, as a consequence, the unemployment rate (B2.2.12).

Regarding the human capital model, the higher average real wage directly involves a lower growth rate and, then, a lower labor force participation rate (B2.2.6 and B2.2.7). The lower labor force participation rate decreases the unemployment rate (B.2.20) because the negative effect on the employment rate is offset (B2.2.19). Consequently, while the labor force participation rate decreases, the employment rate decreases to a lesser extent and, therefore, unemployment rate sinks.

As a result of these transmission mechanisms, while unemployment rate with wage stickiness is higher than with flexibility in the Schumpeterian model, it is lower in the human capital model. Here again, the ultimate reason is the different ways of setting wages and the mechanism to determine the employment rate.

2.5. A sensitivity analysis for efficiency wage parameters

A sensitivity analysis of the main macroeconomic variables to changes in efficiency wage parameters is the way to display the possibilities opened for the two growth models in the two wage-setting alternatives considered. Figures 2.9 and 2.10, which show the results for this analysis in the two models, summarize these possibilities and will be referred to throughout this section.

As a conclusion of this analysis, we seek to be able to determine the values of the efficiency wage parameters consistent with the charts presented in the previous section and the reasons to discard other alternatives.

It is important to say that, while in the case of flexibility the simulations are independent of the trend inflation rate, with wage stickiness the simulations are made for the value -0.512, for which the growth rate is maximized with the combination ($z=0.2$, $e=0.05$, $q=0.9$, $b=0.1$) in the Schumpeterian model and 0 with the combination ($z=0.49$, $e=0.075$, $q=0.9$, $b=0.6$) in the Lucas human capital model. Moreover, when the sensibility for one of the four parameters is studied the other three are maintained in the values of the combination of Table 2.1.

There are two features of the results that are present in almost all the cases considered below. The first is that the sensibility of the real wage to all the parameters is much more immediate and important with wage flexibility than with

wage stickiness, except in the case of the probability of employment loss (parameter b). In fact, the response of the main variables in the case of stickiness is almost negligible. The reason for this is the absence of wage revision for some of the workers and the crossed influence on this variable originated by the probabilities of losing employment (b and q).

The second feature is that, fortunately, the real wage response to changes in the parameters is the same in the two models, even though in the first case LFP and the employment rate depend on the real wage while in the case of the Lucas model they depend on the growth rate.

2.5.1 Schumpeterian model

Figure 2.9 shows the response of growth, employment, LFP and unemployment rates in the Schumpeterian model to changes in the parameters z (Figure 2.9a), e (Figure 2.9b), q (Figure 2.9c) and in b (Figure 2.9d). In order to perform this analysis, we must bear in mind expressions (2.8) for LFP rate, (2.22) for employment rate, (2.35) and (2.36) for efficiency wages and (2.24) for economic growth.

Variations in z

The growth rate with flexibility is higher than that of stickiness until the value 0.6 for z , and lower for z greater than 0.6. The same can be said for LFP and the employment rate and the opposite for the unemployment rate.

- Under wage flexibility, for values lower than 0.6 efficiency wages increase very slightly, reducing the employment rate slowly and, thus, economic

growth. The value of the average real wage is lower than that corresponding to wage stickiness. The labor force participation rate is not affected since the increase of efficiency wages is compensated by higher consumption. As a consequence, unemployment rate slightly increases with the decrease of employment rate. For values greater than 0.6 the impact is higher.

- Under wage stickiness, the effect of z in efficiency wages is negligible due to the fixed value of the trend inflation rate, its positive influence is compensated by the lack of wage revision for some workers and the crossed influence of the probabilities of losing employment.

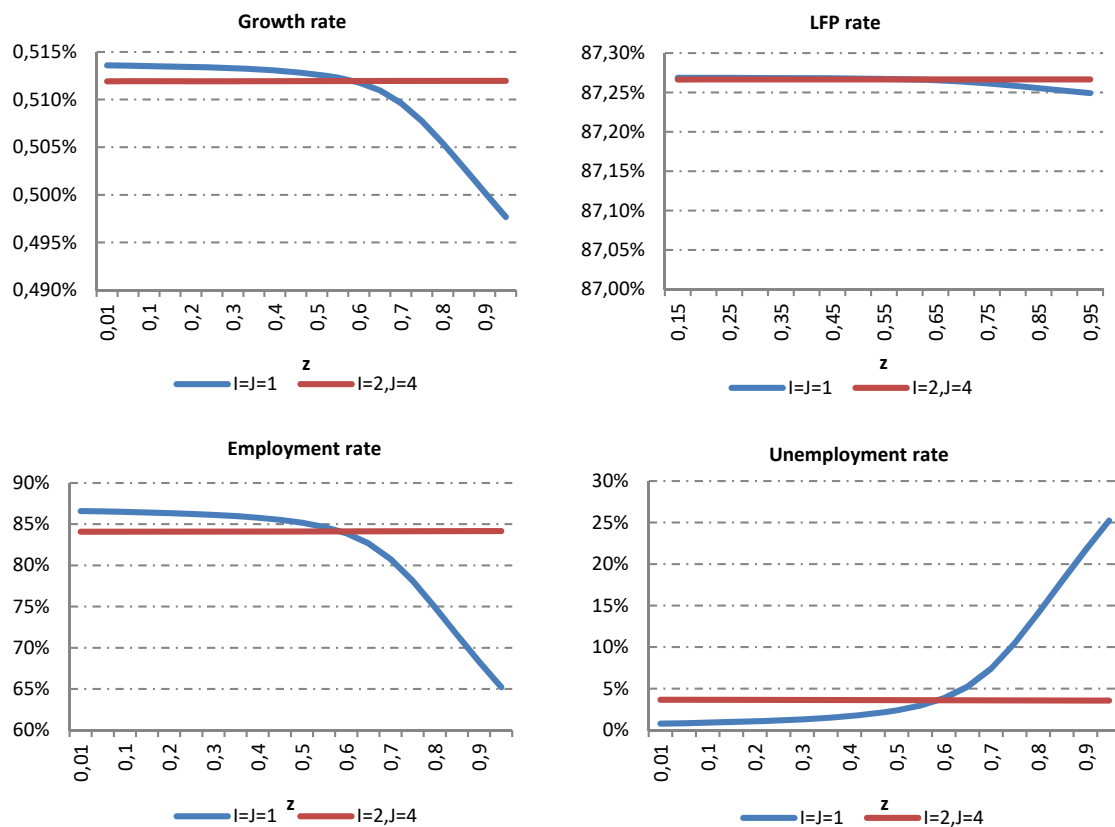


Figure 2.9a: Schumpeterian model: Sensitivity of the main economic rates to changes in parameter z .

- The reason why the real wage is greater with stickiness than with flexibility is the greater unemployment rate and the crossed effects of the probabilities of staying employed.
- What is the meaning of the threshold? The meaning is that, in this model, the utility of leisure time and unemployment benefits has an intense effect on the flexible average real wage, especially from 0.6. This wage grows with z , for $z=0.6$ is equal to the sticky wage and from 0.6 becomes greater and greater.

The charts previously presented (Figures 2.1 to 2.4) are correct for values of z lower than 0.6. For 0.6 the maximum values of growth, employment and LFP rates of stickiness will be the same as in flexibility, like the constant unemployment rate. For values greater than 0.6 the maximum values of growth, employment and LFP rates of stickiness will be greater than those corresponding to flexibility and the constant unemployment rate will be lower. Values lower than 0.6 are the most likely because this value corresponds to 80% of the wage. As the proportion grows with z , values greater than 0.6 are not plausible.

Variations in e

As for parameter z , there is an upper limit at the value 0.15 for parameter e from where stickiness growth turns higher than that of flexibility. For lower values of e , the wage rate is lower with flexibility than with stickiness, which is also like that of parameter z , and the opposite happens for values greater than 0.15.

- Under wage flexibility, efficiency wages increase with e , decreasing employment and economic growth rates, and a rising unemployment rate.

LFP rate is not affected as in the case of z because the rise in consumption compensates the increase of efficiency wages.

- Contrary to flexibility, sticky efficiency wages slightly change due to the constant value of the trend inflation rate, the lack of revision with growth rate in no updated wages, and the crossed effects of the probabilities of staying employed. Consequently, both employment and growth rates slightly grow with e .

As a conclusion, the charts previously presented (Figures 2.1 to 2.4) are correct for values of e lower than 0.15. For 0.15 the maximum values of growth, employment and LFP rates of stickiness will be the same as in flexibility, like the constant unemployment rate. For values greater than 0.15, the maximum values of growth, employment and LFP rates of stickiness will be greater than those corresponding to flexibility, and the constant unemployment rate will be lower.

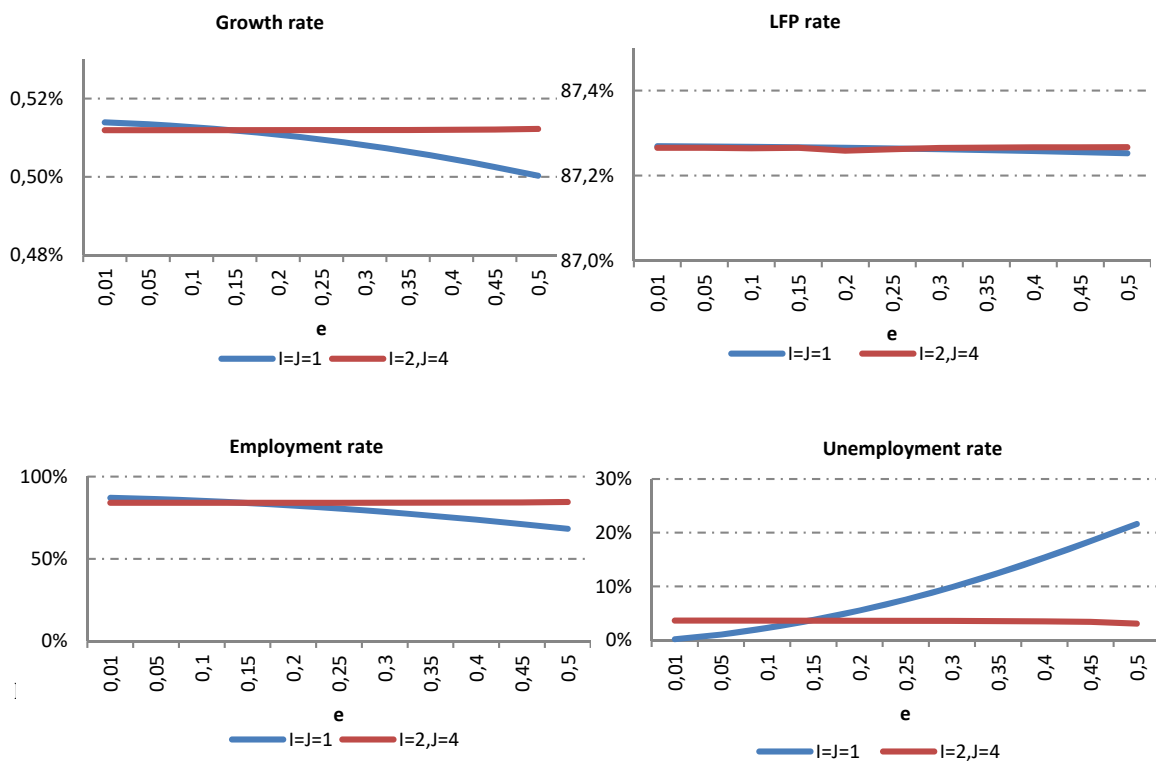


Figure 2.9b: Schumpeterian model: Sensitivity of the main economic rates to changes in parameter e .

Once again, the most likely values are lower than 0.15 because e is 20% of the wage and the proportion is increasing with e .

Variations in q

Unlike parameters z and e , there is a minimum threshold of q from which growth of flexibility exceeds the stickiness value. This value is exactly 0.25.

- Flexible efficiency wages decrease with q increasing employment and growth rates and decreasing unemployment rate. LFP rate is minimally affected by an increase of q due to the effect of consumption.

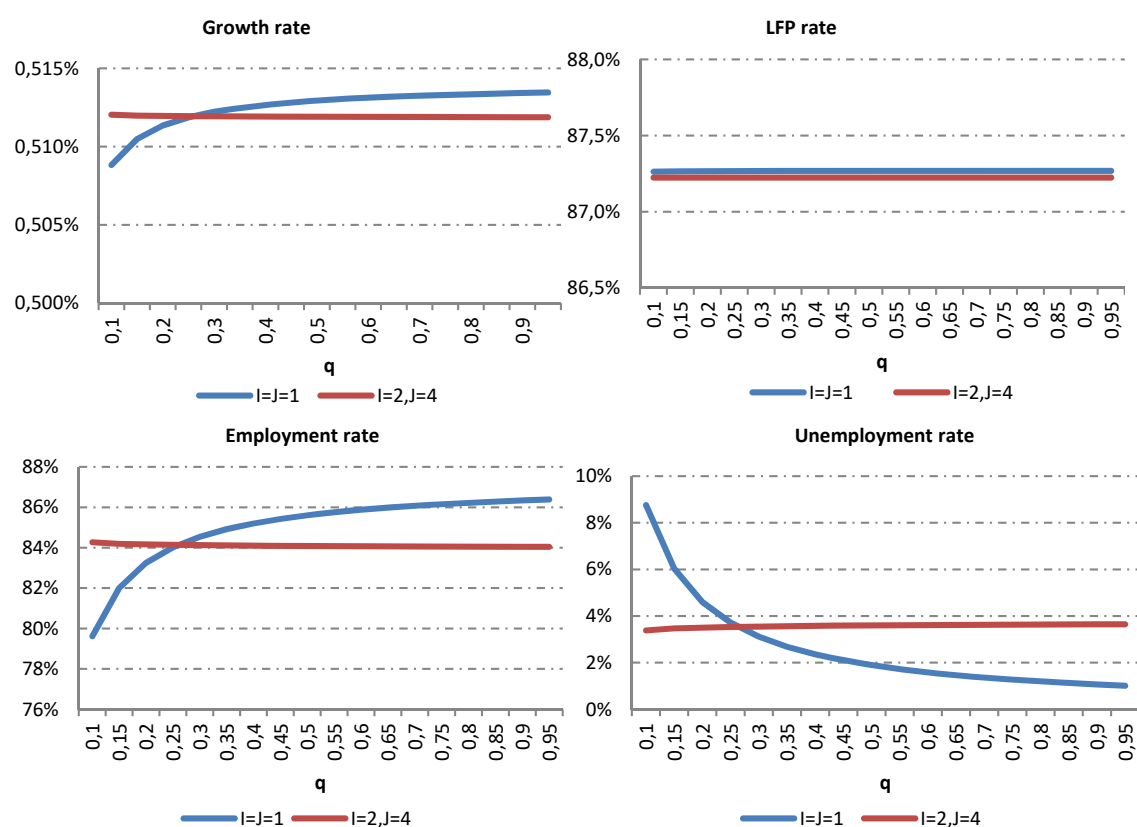


Figure 2.9c: Schumpeterian model: Sensitivity of the main economic rates to changes in parameter q .

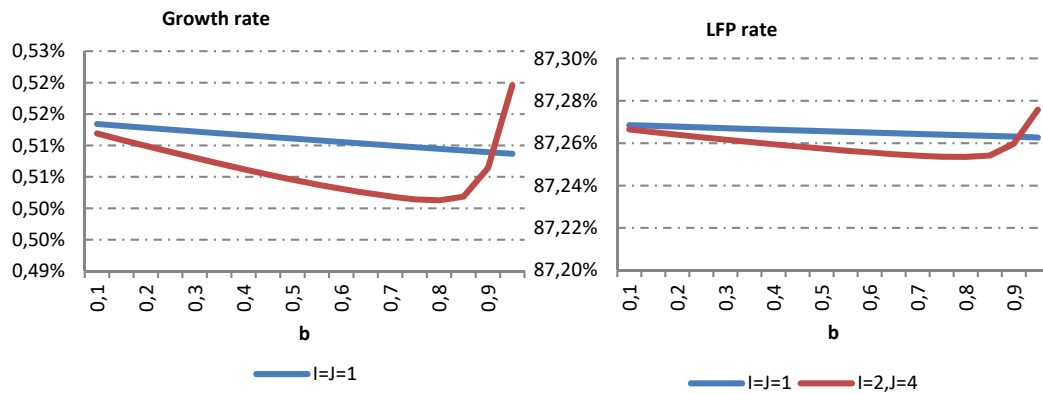
- The effect on sticky efficiency wages is very low because of the effect of the fixed trend inflation rate and because the increase in q is offset by the

decline in the probability of being employed (Δ_q and Δ_w^{bq}). Consequently, employment rate slightly increases with the minimum drop in efficiency wages but the effect on economic growth is negligible.

In conclusion, for values of q smaller than 0.25, the flexible wage is higher than that of stickiness and then flexibility growth is smaller than the growth of stickiness. The charts previously presented (Figures 2.1 to 2.4) are correct for values of z greater than 0.25. For 0.25 the maximum values of growth, employment and LFP rates of stickiness will be the same as in flexibility, like the constant unemployment rate. For values greater than 0.25, the maximum values of growth, employment and LFP rates of stickiness will be lower than those corresponding to flexibility and the constant unemployment rate will be higher. In this case the most plausible values are greater than 0.25.

Variations in b

Flexibility growth rate is higher than that of stickiness until the value 0.9 for b . The same can be said for employment and LFP rates and the opposite for the unemployment rate. The opposite happens for values greater than 0.9.



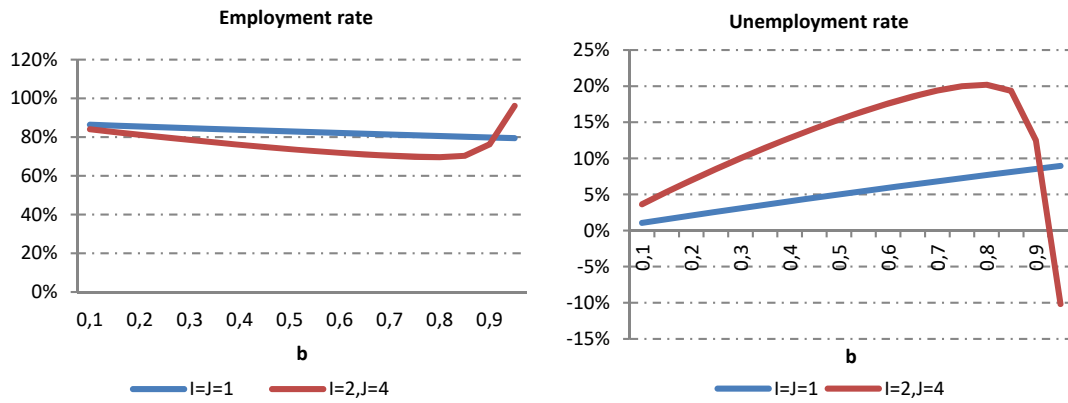


Figure 2.9d: Schumpeterian model: Sensitivity of the main economic rates to changes in parameter b .

- Flexible real wages grow with b , which decreases employment and growth rates and increases unemployment rate. The effect on LFP is low, since the increase in efficiency wages is partly compensated by higher consumption.
- Under stickiness, efficiency wages grow with b to a greater extent than flexible wages due to the negative effect of Δ_w^b on efficiency wages until 0.9. The effect on LFP is also negligible.

As a result, flexibility growth is higher than that of stickiness and this difference becomes greater as b increases until 0.9. Regarding unemployment rate, the flexible rate is lower and the difference becomes greater as b grows until 0.9.

The charts previously presented (Figures 2.1 to 2.4) are correct for values of b lower than 0.9. For 0.9 the maximum values of growth, employment and LFP rates of stickiness will be the same as in flexibility, like the constant unemployment rate. For values greater than 0.9, the maximum values of growth, employment and LFP rates of stickiness will be greater than those corresponding to flexibility and the constant unemployment rate will be lower. Obviously, the most plausible values are below 0.9.

2.5.2 Human capital model

Figure 2.10 shows the sensitivity of growth, employment, labor force participation and unemployment rates in the human capital model to variations in z , e , q and b . In order to perform this analysis, we must bear in mind expressions (2.9) to (2.11) for the LFP rate, (2.14) and (2.17) for the employment rate, (2.35) and (2.36) for efficiency wages and (B2.2.5) for the growth rate.

Variations in z

The growth rate with flexibility is higher than that of stickiness until the value 0.6 for z , and lower for z when it is greater than 0.6. The same can be said for LFP, employment and unemployment rates.

- When $z=0$, the difference of the growth rate is highest between flexibility and stickiness. However, under flexibility, economic growth rate decreases with z and does the LFP rate. In addition, it boosts real wage and, thus, employment rate decreases. The effect on unemployment rate (function 2.17) is the same because of the higher effect on the LFP rate than in employment rate.
- Under stickiness, the effect on the uploaded wage is offset by the lack of revision with the growth rate and the crossed effects of the probabilities of keeping employed. Consequently, the effects on stickiness rates are negligible.



Figure 2.10a: Lucas model: Sensitivity of the main economic rates to changes in parameter z .

The charts previously presented (Figures 2.5 to 2.8) are correct for values of z lower than 0.6. For 0.6 the maximum values of growth, employment and LFP rates of stickiness will be the same as in flexibility, like the constant unemployment rate. For values greater than 0.6 the maximum values of growth, employment, LFP and unemployment rates of stickiness will be greater than those corresponding to flexibility. The first situation is the most plausible one.

Variations in e

There is no minimum threshold of e from which flexibility growth exceeds the growth of stickiness, at least initially. It is greater whatever the value of e , like the rest of rates considered.

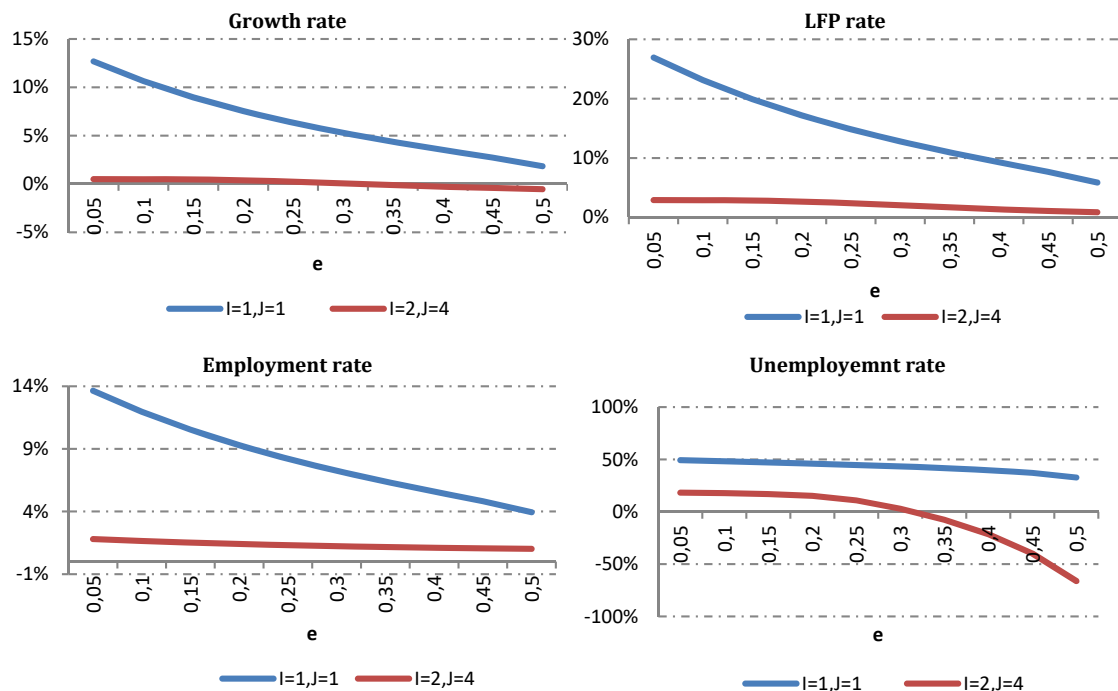


Figure 2.10b: Lucas model: Sensitivity of main economic rates to changes in parameter e .

- Under wage flexibility, economic growth decreases with e , because LFP, employment and unemployment rates decrease as a response to the subsequent increase in the real wage.
- Similar to parameter z , under stickiness the effect on efficiency wages is offset by the lack of revision for three quarters of the workers and the crossed effects of the probabilities of staying employed, making negligible the net variation.
- A singular result in this case is that for e greater than 0.30 the unemployment rate is negative, implying the impossibility of feasible equilibria for higher values of e .

The charts previously presented (Figures 2.5 to 2.8) are correct for any feasible value of e , as long as they must be lower than 0.30.

Variations in q

There is not a lower limit of q where flexibility growth turns higher than that of stickiness. The growth with wage flexibility exceeds the growth of wage stickiness whatever the value of q .

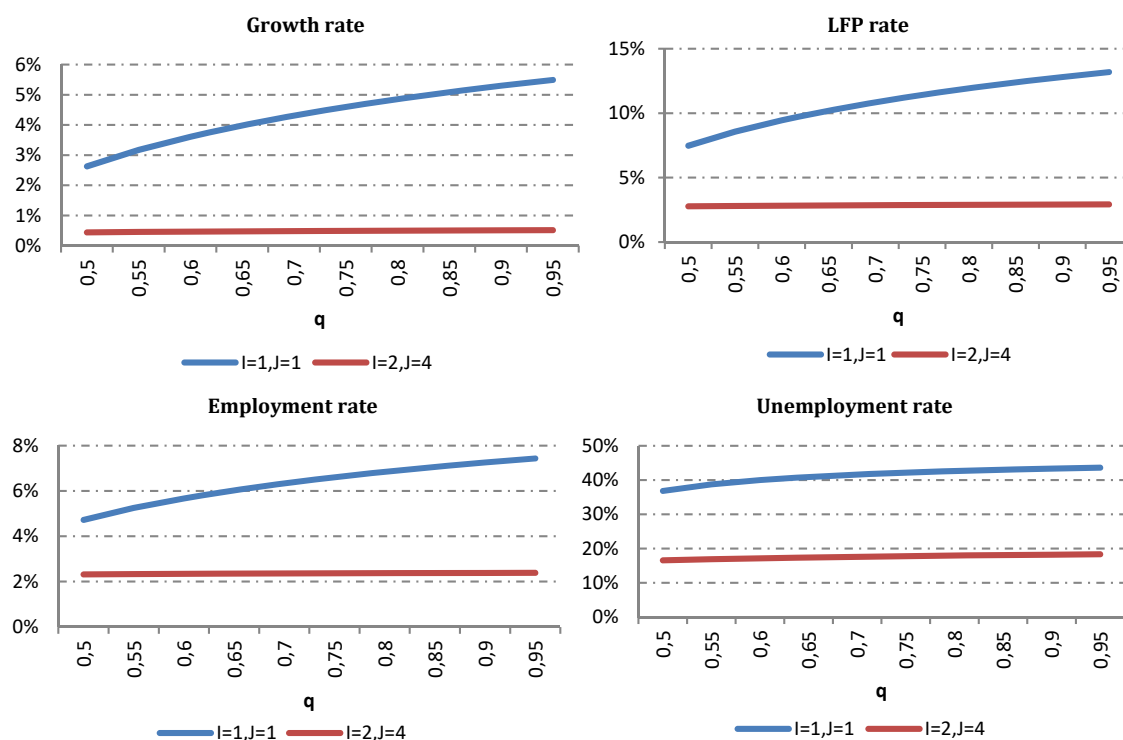


Figure 2.10c: Lucas model: Sensitivity of main economic rates to changes in parameter q .

Variations in b

Similar to parameter q in the Schumpeterian model, there is a minimum threshold of b from which the growth rate of flexibility exceeds that of stickiness. This value is exactly 0.54. Feasible equilibria require $b < 0.9$ (for $b > 0.9$ unemployment rate is negative).

- Wage flexibility erodes economic growth, LFP, employment and unemployment rates as b grows, because the real wage increases.
- The same happens with wage stickiness but the drop in the quoted rates is much more intensive until 0.55.

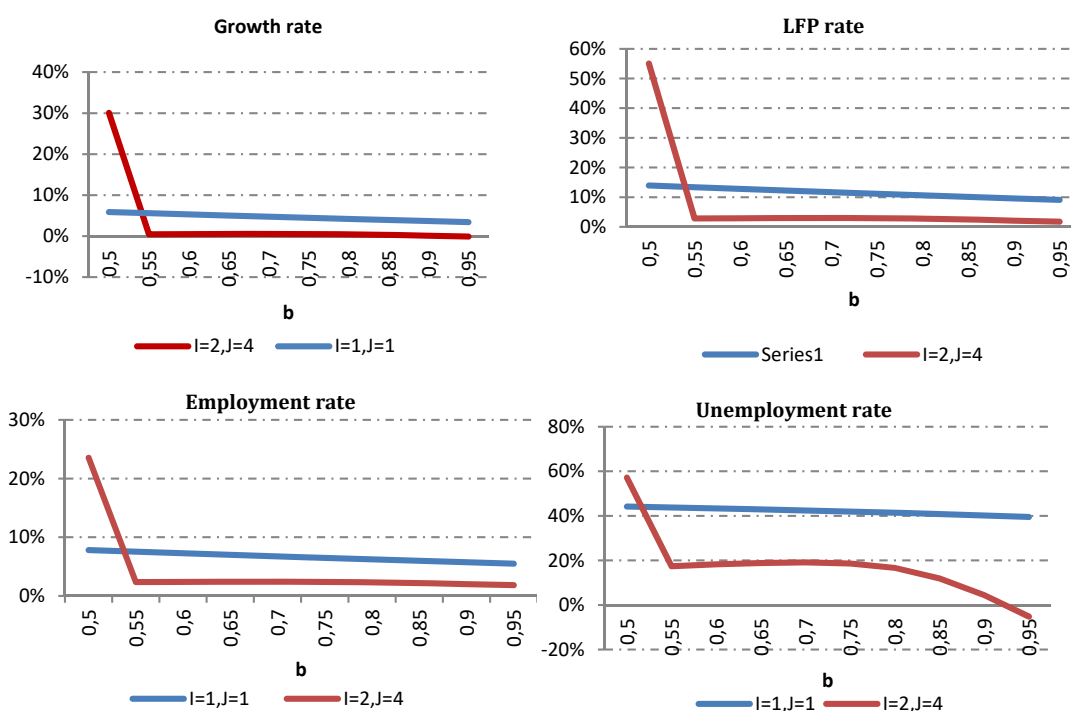


Figure 2.10d: Lucas model: Sensitivity of main economic rates to changes in parameter b .

The charts previously presented (Figures 2.5 to 2.8) are correct for values of b greater than 0.54. For 0.54 the maximum values of growth, employment and LFP rates of stickiness will be the same as in flexibility, like the constant unemployment rate. For values lower than 0.54, the maximum values of growth, employment, LFP and unemployment rates of stickiness will be greater than those corresponding to flexibility.

We can clearly observe in Table 2.3 the same conditions in the two models for parameters z , e and b , and the addition of a lower bound for parameter b (0.54). In

spite of the different conditions in the last case, the limits of the two models are fully compatible. Related to parameter q , there are no conditions in the human capital model, while the condition in the Schumpeterian model is very plausible.

Table 2.3: Threshold values for growth rate with wage flexibility higher than in wage stickiness

Model	Parameter z	Parameter e	Parameter q	Parameter b
Schumpeterian	< 0.60	< 0.15	> 0.25	< 0.9
Human Capital	< 0.60	< 0.3		> 0.54 < 0.9

From all the possibilities of parameter combinations contained in Table 2.3, the more likely combinations of efficiency wage parameters in the two models are those leading to a “growth loss.”

2.6. Conclusions

Two DSGE models with endogenous growth and nominal stickiness have been extended to include the presence of unemployment in order to know its effects on the relationship “trend inflation/long-run growth” and how it affects the conclusions of the first chapter. The main results on these effects, as well as the implications on other very relevant macroeconomic variables, have been obtained from simulations using Dynare.

We have concluded firstly that the introduction of unemployment does not affect the relationship between trend inflation and long-run growth in its basic features, since their general structure is similar to that shown in Chapter 1. However, those basic characteristics are substantially extended. The analysis of the relationships

between trend inflation and the new endogenous variables of the labor market introduced allow us, secondly, to find some results assigning a key role for two of those variables in the long-term transmission mechanisms of the monetary policy. They are employment and labor force participation rates.

Our results confirm that, as in Friedman's critique of the Phillips curve, trend inflation and the unemployment rate are independent in the long run but, at the same time, employment and labor force participation rates are maximized for the value of the trend inflation, which maximizes the long-run growth rate.

Consequently, maintaining the Friedman criticism after having introduced additional endogenous variables and a distortion in the labor market, we simultaneously find additional results that provide a more general perspective for the long-run macroeconomic dynamics. In particular, these results mean that the trend inflation rate value for which the natural rate of unemployment takes place is not indifferent because it is associated to different growth, employment, and labor force participation rates, the same trend inflation rate maximizing the three rates.

Therefore, the point here is the new macroeconomic relevance of employment and labor force participation rates, which play a decisive role in the mechanisms that make it possible for monetary policy to reach the maximum achievable economic growth rate. As these two variables do not play any theoretical role in mainstream macroeconomic models, this result could mean that the preconceived notion maintaining that unemployment rate is the labor market variable playing the key role in the long-run macroeconomic dynamics should be overturned.

Moreover, the new labor market context (efficiency wages) has relevant additional consequences. While unemployment rate with wage stickiness is higher than that of flexibility in the Schumpeterian model, we find the opposite in the human capital model. But the more remarkable result is that, unlike in Chapter 1, sticky average real wage can be higher, equal or lower than a flexible wage, and the value of the achievable growth rate will be respectively lower, equal or higher with wage stickiness than with wage flexibility. Consequently, the unemployment caused by the labor market distortion introduced can cause a “growth loss” or a “growth premium” in the case of wage stickiness, as well as a loss or a gain in employment and labor force participation rates. From all these possibilities, the more likely combinations of efficiency wage parameters in the two models are those leading to a “growth loss.”

Chapter 3

Financial frictions, unemployment, and long-run inflation–growth relationship: empirical implications

Abstract

Taking into consideration both unemployment and a financial sector, we present an extended long-run inflation–growth relationship for two wage-setting processes and some of its empirical implications. The financial extension includes new variables, from which one of them—leverage ratio—plays a key role in the maximization of long-run growth. However, the negative impact of the financial friction (and leverage ratio) on the achievable economic growth cannot be generalized since it depends on the kind of friction considered. Additionally, the empirical application explores the implications of the models for six countries in order to identify to what extent they could improve their observed long-run growth, employment and labor force participation rates according to the obtained inflation–growth relationship.

3.1. Introduction

The results of the two previous chapters have allowed us to know the relationships between trend inflation, long-run growth and labor market variables in greater detail, in the presence of endogenous growth, nominal price and wage stickiness and unemployment. They have confirmed the non-neutrality of monetary policy with wage stickiness, the non-generality of the results of Amano et al. (2009) and Amano, Carter and Moran (2012), since the optimal trend inflation depends on the type of wage-setting process. They have also confirmed Friedman’s critique of Phillips’ curve, which has been extended with the relevant role played by employment and labor force participation rates in the maximization of the achievable long-run growth.

Throughout this chapter, we complete our analysis with the introduction of a financial sector. We can find different precedents in the existing literature on the link between the financial system and long-run economic growth, where we are interested in the long-run relationship leverage ratio/growth. Up to now, the results in the literature appear conclusive in that there is not a relationship that is generally applicable to all the possible situations.

Goldsmith (1969) was the first to consider whether financial structure influences the pace of economic growth. He thought that one of the most important problems in the field of finance, if not the single most important one, was the effect that financial structure and development have on economic growth. But although he was largely successful in documenting the evolution of national financial systems, he was unable to provide much cross-country evidence on the relationship between economic development and the mixture of financial markets, due to data limitations.

Recent research has not substantially completed Goldsmith's goal of assessing the relationship between financial structure and economic growth in a broad cross-section of countries. Researchers have developed rigorous theories of the evolution of financial structures and how the mixture of markets and banks influences economic development but the empirical results are mixed.

Auerbach (1985) stated that, according to agency models of financial activity, risky firms should borrow less, while fast-growing firms should borrow less because of their higher ratio of growth opportunities to existing capital. But he found a positive relation between the firm growth (profit growth) and long-term leverage ratio, concluding that the effect of firm growth rates on the level of borrowing is

inconsistent with the predictions of "agency" models of leverage.

Lang, Ofek and Stulz (1996) showed that leverage does not reduce growth for firms known to have good investment opportunities (high-q firms), but is negatively related to growth for firms whose growth opportunities are either not recognized by the capital markets or are not sufficiently valuable to overcome the effects of their debt overhang (low Tobin's q ratio).

Demirgüç-Kunt and Levine (2001) conclude in the introduction of their book that financial structure is not an analytically very useful way to distinguish among national financial systems. Countries do not grow faster, new firms are not created more easily, firms' access to external finance is not easier, and firms do not grow faster in either market- or bank-based financial systems.

Beck et al. (2001) conduct a comprehensive assessment of the relationship between economic performance and financial structure. Using different data and econometric methodologies, the authors find astonishingly consistent results. First, no evidence exists that distinguishing countries by financial structure helps explain differences in economic performance. More precisely, countries do not grow faster, financially dependent industries do not expand at higher rates, new firms are not created more easily, firms' access to external finance is not easier, and firms do not grow faster in either market-based or bank-based financial systems. Second, they find that distinguishing countries by overall financial development does help explain cross-country differences in economic performance.

Arestis, Luintel and Luintel (2008) take as their point of departure the previous results and they contribute to the empirical literature surrounding financial

structure and economic growth, revealing first that time series results show that, in sharp contrast to existing results, for the majority of countries financial structure significantly explains economic growth. Second, they find significant heterogeneity in cross-country parameters and adjustment dynamics, concluding that data cannot be pooled for the countries considered because panel regressions mask important cross-country differences.

They conclude that a robust co-integrating relationship between output per capita, capital stock per capita and the financial structure exists, in sharp contrast to Levine (2002) and Beck and Levine (2002), amongst others. The main policy message of their findings is that financial structure matters for economic growth but with the effect of the leverage ratio being different depending on the countries.

Finally, Gambacorta, Yang, Tsatsaronis (2014) maintain that, up to a point, banks and markets both foster economic growth. Beyond that limit, expanded bank lending or market-based financing no longer adds to real growth. But when it comes to moderating business cycle fluctuations, banks and markets differ considerably in their effects. In normal downturns, healthy banks help to cushion the shock but, when recessions have coincided with financial crises, the impact on GDP has been three times as severe for bank-oriented economies as it has for market-oriented ones.

In short, there is no a unique relation between the leverage ratio and the growth rate, with any direction of the causality and even the absence of causality being possible when the sample pools cross-country and time series data.

The first objective of this chapter is to know how a distortion in the financial market impacts on the quoted conclusions of the two previous chapters, taking

into consideration that monetary policy is closely related to interest rates and, hence, to financial activity. The results obtained from this analysis allow us to confirm that the incorporation of financial frictions has not any impact on the main results from Chapters 1 and 2. On the other hand, the consequences of introducing financial frictions cannot be generalized regardless of the friction type, since we have found that a *costly verification* model has no any impact on the long-run inflation–growth relationship if we consider nominal stickiness in wages, unlike flexibility. Moreover, we confirm the previously quoted results regarding the non-conclusive influence of the leverage ratio on the growth rate, given that the influence in the two models of financial friction considered the relationship between the two variables is contrary.

The second objective of this third chapter is to explore the empirical implications of the models for six developed countries in order to conclude to what extent they could improve their long-run growth, employment and labor force participation rates. The conclusions from the Schumpeterian model are that the two countries with more potential increase in the long-run growth are Japan and Germany. The USA and France are situated at an intermediate level of improvement, while Australia and Spain are the two countries with the lowest level of growth gain. In the Lucas human capital model, France is added to the first group, Australia and Spain would be in the intermediate group and the USA would have the lowest improvement.

The way to achieve these gains would be a change in trend inflation (inflation target). The one country with a positive change is Japan (+0.21%), while the rest of the countries should at least decrease the target: -0.27% Germany, -0.36% France,

-0.76% Spain and -0.88% the USA and Australia. For these last two countries the gain in the employment and LFP rates would be at most one percentage point, three quarters of a percentage point in Spain and near zero in Japan, Germany and France. So, the growth gain in the first three countries would come from the improvement in the LFP rate, while in the case of the last three the growth gain would come from a change in the allocation of resources leading to an increase in the TFP growth. In other words, the growth gain would come from the increase in the TFP growth in the second group while in the first one the origin would come from the LFP rate.

To address our analysis, we will continue using the two growth models analyzed in the previous chapter, that is, the Schumpeterian growth model of Aghion and Howitt and the Lucas human capital model. We start from these models as defined in Chapter 2 with unemployment, completing them with the specific features of the corresponding financial sector.

This approach involves intermediate goods producers or retailers needing external resources to fund their investment in R&D or their working capital, respectively, because their internal funds are no longer enough. But in addition to that, we will consider the existence of asymmetric information in the financial market. As mentioned, we will consider two different frictions to obtain a broader analysis, one in the Schumpeterian model and the second in the Lucas human capital model. Both frictions are based on the existence of asymmetric information. Both frictions are based on the existence of asymmetric information: namely, in Gertler and Karadi's (2011) *financial intermediation model* the financial entities have an information advantage in the Schumpeterian model, and in the *costly verification*

model of Bernanke, Gertler and Gilchrist (1999) and Gertler (2009) the information advantage in on the part of the borrowers in the Lucas human capital mode. All their features are explained in Section 2.

Section 3 presents the relationships between inflation, growth, labor market variables and leverage ratio, which confirm all the conclusions from previous chapters and the ambiguous results on the relationship leverage ratio/trend inflation, and the impact of the financial friction on long-run growth. While the *financial intermediation model* shrinks economic growth, the *costly verification model* barely has an impact on the level of economic growth under wage stickiness. Section 4 summarizes the results of section 3 and contains the assessment of the main effect of considering wage stickiness, unemployment and financial friction on the long-run growth rate. Section 5 contains the sensitivity analysis of growth rate, labor market variables, and leverage ratio to changes in efficiency wage parameters. Section 6 includes the empirical application of two models to six economies governed by different central banks: United States, Australia, Japan, France, Spain and Germany. The estimations have been made using Dynare in order to obtain all the parameters, and thus compare the observed with the best values of the main macroeconomic variables. Finally, section 7 summarizes the main findings.

3.2. Two DSGE models with endogenous growth, staggered wage and price setting, efficiency wages and financial friction

In line with the two previous chapters, we will enrich the Schumpeterian and Lucas human capital models with the introduction of financial frictions. The credit

market features will be introduced throughout this section and we will analyze how the new activity affects agents and expressions.

In order to obtain broader conclusions, we will consider a different financial friction for each one of the growth models. The *financial intermediation model* of Gertler and Karadi (2011) has been selected to introduce the financial distortion in the Schumpeterian model and, the *costly verification model* of Bernanke and Gertler (1989) in the Lucas human capital model.

As in the previous chapters, price stickiness has been considered for two periods, and wage stickiness for four. Similarly, efficiency wages introduce the distortion in the labor market causing unemployment and the expression for sticky wages has been obtained, as in Chapter 2, through cumulative probabilities of the efficiency wage-setting process.

3.2.1 Agents

Financial intermediaries

Schumpeterian model

The financial friction model considered is based on asymmetric information in favor of financial entities that have more information than depositors. As a consequence, the latter are those who must take precautions against or impose restrictions on the deposits placement. The main elements used by Gertler and Karadi (2011) to define the financial intermediaries' behavior are set out below.

The balance of a financial intermediary j will be the following:

$$S_{jt} = T_{jt} + B_{jt} \quad (3.1)$$

where S_{jt} represents its total credit in t , B_{jt} deposits of households and T_{jt} its net wealth or equity. The credit is demanded by the intermediate producers to found their investment in R&D.

The net wealth or equity of the financial intermediary T_{jt} evolves as follows:

$$T_{jt+1} = R_{t+1}^k S_{jt} - R_{t+1} B_{jt} = (R_{t+1}^k - R_{t+1}) S_{jt} + R_{t+1} T_{jt} \quad (3.2)$$

where R^k is credit return and R deposit cost. This agent maximizes the present value of their wealth V :

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} \beta^i T_{jt+i} = \max E_t \sum_{i=0}^{\infty} \beta^i [(R_{t+1+i}^k - R_{t+1+i}) S_{jt+i} + R_{t+1+i} T_{jt+i}]$$

β being the intertemporal discount factor and taking into consideration the condition of the presence of financial frictions:

$$E_t \beta^i (R_{t+1+i}^k - R_{t+1+i}) \geq 0$$

The agency problem entails that any financial intermediary could divert a proportion λ of their assets, in which case depositors would obtain $(1-\lambda)$ of theirs. Consequently, the compatibility of incentives would lead to the constraint:

$$V_{jt} \geq \lambda S_{jt} \quad (3.3)$$

The wealth of the financial intermediary can be simplified and rewritten as follows:

$$V_{jt} = E_t \sum_{i=0}^{\infty} \beta^i (R_{t+1+i}^k - R_{t+1+i}) S_{jt+i} + E_t \sum_{i=0}^{\infty} \beta^i R_{t+1+i} T_{jt+i} = v_t S_{jt} + \eta_t T_{jt} \quad (3.4)$$

where v_t is the marginal return of an additional unit of investment and η_t the marginal return of an additional unit of wealth (equity). Both variables can be expressed as follows:

$$v_t = E_t(R_{t+1}^k - R_{t+1}) + E_t(\beta G(S)_{t+1} v_{t+1}) \quad (3.5)$$

$$\eta_t = E_t R_{t+1} + \beta E_t(G(T)_{t+1} \eta_{t+1}) \quad (3.6)$$

$$G(T)_{t+1} = \frac{T_{jt+1}}{T_{jt}} \quad (3.7)$$

$$G(S)_{t+1} = \frac{S_{jt+1}}{S_{jt}} \quad (3.8)$$

The compatibility of incentives (with equality) leads to the constraint:

$$v_t S_{jt} + \eta_t T_{jt} = \lambda S_{jt} \quad (3.9)$$

obtaining as a consequence the leverage ratio ϕ_t :

$$S_{jt} = \frac{\eta_t}{\lambda - v_t} T_{jt} = \phi_t T_{jt} \text{ where } \phi_t = \frac{\eta_t}{\lambda - v_t} = \frac{S_{jt}}{T_{jt}} \quad (3.10)$$

Considering that ϕ_t does not depend on specific elements of each intermediary, we can rewrite it as follows: $S_t = \phi_t T_t$ ($S_t = \sum_j S_{jt}$, $T_t = \sum_j T_{jt}$). Once ϕ_t is known, we can obtain the growth rates of T and S :

$$G(T)_{t+1} = \frac{T_{jt+1}}{T_{jt}} = (R_{t+1}^k - R_{t+1}) \phi_t + R_{t+1} \quad (3.11)$$

$$G(S)_{t+1} = \frac{S_{jt+1}}{S_{jt}} = \frac{\phi_{t+1} T_{jt+1}}{\phi_t T_{jt}} = \frac{\phi_{t+1}}{\phi_t} G(T)_{t+1} \quad (3.12)$$

Consequently, the total wealth of the financial intermediaries can be stated as:

$$T_t = \gamma[(R_t^k - R_t)\phi_{t-1} + R_t]T_{t-1} + \psi R_t^k S_{t-1} \quad (3.13)$$

where ψ represents the wealth proportion of the new bankers. This expression can be rewritten as follows:

$$T_t = \gamma[(R_t^k - R_t)\phi_{t-1} + R_t]T_{t-1} + \psi R_t^k \phi_{t-1} T_{t-1} \quad (3.14)$$

$$G(T)_t = \gamma[(R_t^k - R_t)\phi_{t-1} + R_t] + \psi R_t^k \phi_{t-1} \quad (3.15)$$

Human capital model

This model is also based on asymmetric information but, in this case, in favor of borrowers. This financial friction is derived from the *costly verification* of contracts, due to the existence of asymmetric information (Bernanke and Gertler, 1989). As a result, an external finance premium appears and corporate balance sheets are of essential importance. We consider that the retailers are the agents who need to fund their working capital. They allocate the benefits of each period to fund the productive activity, but in addition to that, they have to resort to external funding to cover all the production costs.

The minimum return on total investment of an entrepreneur i (Q_{it}) required by financial intermediaries to lend is $\omega^* R_k Q_{it}$, where R_k is the average gross return and ω is an idiosyncratic stochastic shock for each borrower in the interval $[\underline{\omega}, \bar{\omega}]$ with cumulative probability function $H(\omega)$ and probability density function $h(\omega)$. Consequently, if $\omega \geq \omega^*$ the lender obtains $D = \omega^* R_k Q_{it}$ but if $\omega \leq \omega^*$, the entrepreneur announces the bankruptcy and the lender will have to monitor, in

such a way that the financial intermediary's benefit will be $(1 - \mu)\omega R_k Q_{it}$. The total cost of bankruptcy is $\mu\omega R_k Q_{it}$.

As a consequence, the expected return of the bank over the yield of the entrepreneur is:

$$\Gamma(\omega^*) = \int_{\omega^*}^{\bar{\omega}} \omega^* dH + \int_{\underline{\omega}}^{\omega^*} \omega dH \quad (3.16)$$

where we denote the second term $G(\omega^*) (= \int_{\underline{\omega}}^{\omega^*} \omega dH)$.

The relation that synthesizes the equilibrium of lending activity for each producer is:

$$\frac{Q_{it}}{F_{yit}} = \frac{1}{1 - [\Gamma(\omega^*) - \mu G(\omega^*)] R_k/R} = \phi(R_k/R) \quad (3.17)$$

where F_{yit} is the retail firms' own financing (their period benefits) and R the real interest rate on bonds. $\phi(R_k/R)$ is the term denoting the leverage ratio.

Due to all retailers having the same behavior and $\phi(R_k/R)$ not depending on idiosyncratic aspects, we can rewrite for all the economy:

$$\frac{Q_t}{F_{yt}} = \phi(R_k/R) \quad (3.18)$$

where

$$Q_t = \int_0^1 Q_{it} di$$

$$F_{yt} = \int_0^1 F_{yit} di$$

As a result, the gross rate of a loan in period t , R_t^k , is:

$$R_t^k = R_t \left(1 + \frac{\mu G(\omega_t^*)}{[\Gamma(\omega_t^*) - G(\omega_t^*)]} \right) \quad (3.19)$$

where the term $\frac{\mu G(\omega_t^*)}{[\Gamma(\omega_t^*) - G(\omega_t^*)]} R_t$ represents the external finance premium.

Households

Household members offer labor to intermediate or final goods producers, depending on the model, consume the final good and hold bonds. Supply and demand for labor are no longer equal and the expected utility must take the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{1}{1+\nu} \int_0^1 N_{st}^{1+\nu} ds \right) \quad (3.20)$$

where N_s represents only the supply of labor (or labor force participation) for labor service s with $s \in [0,1]$, while L_s will be the labor demand of the firms for labor service s .

Furthermore, households must satisfy the same budget constraint as in Chapter 2 because household members do not need funding and, then, the financial friction does not impact on their decision problem.

Schumpeterian model

$$C_t + \frac{B_t}{P_t} + R_t D_t = \frac{B_{t-1}}{P_t} R_t^{st} + D_t + (1 - d_t) \int_0^1 \frac{W_{st}}{P_t} L_{st} ds \quad (3.21)$$

We can maintain the labour supply expression of the previous chapter:

$$N_{st} = \left(\frac{1}{C_t} (1 - d_{st}) w_{st} \right)^{1/\nu} \quad (3.22)$$

where $N_t = \int_0^1 N_{st} ds$.

Human capital model

$$C_t + \frac{B_t}{P_t} + K_{t+1} = \frac{B_{t-1}}{P_t} R_t^{st} + D_t + (1 - d_t) \int_0^1 \frac{W_{st}}{P_t} L_{st} ds + (1 + R_{t+\tau} - \delta) K_t \quad (3.23)$$

Similarly to the Schumpeterian model, financial friction does not impact on the consumer decision problem and we can maintain the same expression of the previous chapter.

Labor supply for flexibility will be:

$$N_{ss} = \frac{1}{\xi} \left(1 - \frac{\beta}{1 + g(C)} \right) \quad (3.24)$$

For sticky wages:

$$N^1 = \frac{1}{\xi} \left(1 - \frac{\beta/\Pi}{1 + g(C)} \right) \quad (3.25)$$

$$N^0 = \frac{1}{\xi} \left(1 - \frac{\beta\Pi^{J-1}}{1 + g(C)} \right) \quad (3.26)$$

where N^0 represents the labor supply for individuals who will reset the nominal wage in the following period and N^1 represents those who maintain the nominal wage.

Intermediate goods firms

Schumpeterian model

As explained in the first and second chapters, monopolistically competitive firms obtain intermediate goods through a simple technology that generates one unit of a given intermediate good from one unit of final output. The profit for the firm producing intermediate goods i will be:

$$F_{it} = P_{it}X_{it} - P_tX_{it}$$

They sell their output to final goods firms and, under price stickiness, set the prices according to Taylor contracts for I periods:

$$\frac{P^*}{P} = \frac{1}{\alpha} \frac{\sum_{\tau=0}^{I-1} (\beta(\Pi)^{1/1-\alpha})^\tau}{\sum_{\tau=0}^{I-1} (\beta(\Pi)^{\alpha/1-\alpha})^\tau} \quad (3.27)$$

Human capital model

Intermediate goods producers are indexed by $j \in [0, 1]$ and have a Cobb–Douglas production function:

$$Y_{jt}^i = AK_{jt}^\alpha L_{jt}^{1-\alpha} \quad (3.28)$$

Consequently, the demand function for labor service s is obtained from profit maximization, but now considering the opportunity cost of financing their productive activity with their own funds (perfect competition market):

$$F_{Yt} = P_{jt} AK_{jt}^\alpha L_{jt}^{1-\alpha} - \int_{s=0}^1 W_{st} L_{st} ds - R_t P_t K_t - R_t^k D_t \quad (3.29)$$

$D_t (= \int_{s=0}^1 W_{st} L_{st} ds + R_t P_t K_t)$ would represent the amount needed to fund the working capital, and R_t^k the opportunity cost.

From profit maximization, we obtain the demand function for labor service and the cost of capital:

$$L_t = \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{A}{(1 + R_t^k)} \frac{(1 - \alpha)}{\Delta_t^w} \right]^{\frac{1}{\alpha}} K_t \quad (3.30)$$

$$R_t = \alpha \left[\frac{A}{(1 + R_t^k)} \left(\frac{\varepsilon - 1}{\varepsilon} \right) \right]^{\frac{1}{\alpha}} \left[\frac{(1 - \alpha)}{\Delta_t^w} \right]^{\frac{1-\alpha}{\alpha}} \quad (3.31)$$

where Δ_t^w represents average real wage.

$$\Delta_t^w = \left[\int_0^1 \left(\frac{w_{st}}{P_t} \right)^{1-\sigma} ds \right]^{1/1-\sigma} \quad (3.32)$$

Under wage stickiness, we must obtain the partial and aggregate labor demand functions:

$$L_0 = (1 - d)u^0 N^0 \quad (3.33)$$

$$L_1 = (1 - d)u^1 N^1 \quad (3.34)$$

$$L_{01} = (1 - d)u^{01} N^1 \quad (3.35)$$

$$LL = \frac{1}{J} \sum_{s=0}^{J-1} L_{-s} \quad (3.36)$$

The expression of unemployment is obtained from the difference of labor supply and demand:

$$d_t = \frac{N_t - LL_t}{N_t} \quad (3.37)$$

Final goods producers and retail firms

Schumpeterian model

According to Aghion and Howitt (1992), final goods production function is the following:

$$Y_t = L_t^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^\alpha di \quad (3.38)$$

The demand functions for labor service s and intermediate good i are also obtained from profit maximization, but now the profit F_{Yt} considers the opportunity cost of financing the working capital $D_t (= \int_{s=0}^1 W_{st} L_{st} ds + \int_{i=0}^1 P_{it} x_{it} di)$:

$$F_{Yt} = P_t \int_0^1 (A_{it} L_t)^{1-\alpha} x_{it}^\alpha di - \int_{s=0}^1 W_{st} L_{st} ds - \int_{i=0}^1 P_{it} x_{it} di - R_t^k D_t$$

We obtain from profit maximization the demand functions for intermediate goods and labor services:

$$x_{it} = \left(\frac{P_t}{(1 + R_t^k) P_t^i} \right)^{1/1-\alpha} \alpha^{1/1-\alpha} A_{it} L_{st} \quad (3.39)$$

$$L_{st} = \left(\left(\frac{1 - \alpha}{1 + R_t^k} \right) Y_t L_t^{\frac{1-\sigma}{\sigma}} \left(\frac{W_{st}}{P_t} \right)^{-1} \right)^{\sigma} \quad (3.40)$$

Considering that labor supply is not equal to labor demand and integrating L_{st} to obtain the aggregate function for labor demand:

$$\left(\int_{s=0}^1 L_{st}^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} = L_t = \frac{(1 - \alpha)}{(1 + R_t^k)} \frac{Y_t}{\Delta_t^w} \quad (3.41)$$

Under wage stickiness, we must obtain the partial and aggregate labor demand functions:

$$L_{st} = \frac{(1 - \alpha)}{(1 + R_t^k)} \frac{L_t^{\frac{1-\sigma}{\sigma}}}{W_{st}} \quad (3.42)$$

$$LL = \frac{1}{J} \sum_{s=0}^{J-1} L_s \quad (3.43)$$

The expression of unemployment rate is obtained from the ratio between the difference of labor supply and demand and labor supply:

$$d_t = \frac{N_t - LL_t}{N_t} \quad (3.44)$$

Human capital model

There are an infinite number of retail firms over the continuum $i \in [0,1]$, which repackage the homogeneous intermediate goods and sell them to households. They are the borrowers in order to fund the acquisition of intermediate goods. The leverage ratio will be:

$$\phi(R_k/R) = \frac{Q_{it}}{F_{it}} = \frac{P_{jt}Y_{jt}}{P_t Y_t - (1 + R_t^k)P_{jt}Y_{jt}} = \frac{1}{\frac{\varepsilon}{\varepsilon - 1} \frac{1}{\Delta_p} - (1 + R_t^k)}$$

The equilibrium condition for the credit market will be:

$$\phi(R_k/R) = \frac{1}{\frac{\varepsilon}{\varepsilon - 1} \frac{1}{\Delta_p} - (1 + R_t^k)} = \frac{1}{1 - [\Gamma(\omega^*) - \mu G(\omega^*)] R_k/R}$$

Retailers sell their goods to households and set the price according to Taylor contracts each interval of l periods:

$$\frac{P^*}{P} = \frac{\sum_{\tau=0}^{l-1} (\beta \Pi^\varepsilon)^\tau}{\sum_{\tau=0}^{l-1} (\beta \Pi^{\varepsilon-1})^\tau} \quad (3.45)$$

3.2.2 Growth, innovation and human capital accumulation

Schumpeterian model

This model displays growth by increasing the quality (productivity) of intermediate goods. The intermediate producers found their activity of R&D by means of credit.

The average expected profit of producer i in a period t , VF_{it} , will be:

$$VF_{it} = \alpha^{\frac{1}{1-\alpha}} A_{it} L_t \frac{1}{l} \sum_{s=0}^{l-1} \left(\frac{P_{t-s}^*}{P_t} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{t-s}^*}{P_t} - 1 \right) \quad (3.46)$$

We assume the following probability function for the success of the innovation:

$$\phi(n_{it}) = n_{it}^\chi \quad 0 < \chi < 1 \quad (3.47)$$

with $\phi'(n_{it}) = \chi n_{it}^{\chi-1} > 0$ and $\phi''(n_{it}) = \chi(\chi - 1)n_{it}^{\chi-2} < 0$

If innovation is successful, expected profits will be:

$$\phi(n_{it})VF_{it}^* \quad (3.48)$$

where $n_{it} = S_{it}/A_{it}^*$, S_{it} being the received credit, equal to the quantity of final goods devoted to innovation, and A_{it}^* the intermediate goods productivity achieved if innovation is successful. Consequently, the expected profit of the R&D activity that can provide an innovation is:

$$\phi\left(\frac{S_{it}}{A_{it}^*}\right)VF_{it}^* - (1 + R_t^k)S_{it} \quad (3.49)$$

The optimal value of n_{it} will be common for all entrepreneurs, due to the fact that n only depends on market elements:

$$n_{it} = n = \left[\frac{\chi}{(1 + R_t^k)} \alpha^{\frac{1}{1-\alpha}} L_t \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{t-s}^*}{P_t} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{t-s}^*}{P_t} - 1 \right) \right]^{\frac{1}{1-\chi}} \quad (3.50)$$

Consequently the gross growth rate in the steady state is

$$g = \left[\frac{\chi}{(1 + R_t^k)} \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_s^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_s^*}{P} - 1 \right) \right]^{\frac{\chi}{1-\chi}} (\gamma - 1) + 1 \quad (3.51)$$

Human capital model

The growth process in the human capital model is obtained from the solution to the dynamic optimization problem, where the condition 1.81 in Chapter 1 is affected by the financial friction and expressions (2.25) y (2.26) are maintained.

We have, then, the following expressions for economic growth:

$$1 + g(C) = \frac{\beta}{(1 + \delta) - \left[\frac{A}{(1 + R_t^k)} \left(\frac{\varepsilon - 1}{\varepsilon} \right) \right]^{\frac{1}{\alpha}} \left[\frac{(1 - \alpha)}{\Delta_t^w} \right]^{\frac{1-\alpha}{\alpha}}} \quad (3.52)$$

$$g(Y) = g(Y^i) = g(K) = g(L) = g(h)$$

$$g(h) = \begin{cases} [1 + \xi(1 - u_{ss}(1 - d))N_{ss}] & \text{Wage Flexibility} \\ [1 + \xi(1 - u^1(1 - d))N^1](1 + g(h^1))\left(\frac{J-2}{J}\right) + \\ + [1 + \xi(1 - u^{01}(1 - d))N^1](1 + g(h^{01}))\left(\frac{1}{J}\right) + \\ + [1 + \xi(1 - u^0(1 - d))N^0](1 + g(h^0))\left(\frac{1}{J}\right) & \text{Wage rigidity} \end{cases}$$

where u_{ss} y N_{ss} are steady-state values with wage flexibility, while u^1 , h^1 and N^1 are the decisions for labor services with constant nominal wage for $s \in [0, J - 2)$, u^{01} , h^{01} and N^1 for $s \in [J - 2, J - 2]$, and u^0 , h^0 and N^0 for $s \in [J - 1, J - 1]$ those corresponding to labor services that will reset the nominal wage in the following period.

Regarding the time devoted to the production activity, we can maintain the expressions for u^{ss} , u^1 , u^0 and u^{01} of the two previous chapters.

$$u_{ss} = \frac{1}{1 - d_t} \left(1 - \frac{g(C)}{\xi N_{ss}} \right) \quad (3.53)$$

$$u^1 = \frac{[2 - (1 + g(C))\Pi] - \frac{\beta/\Pi}{1 + g(C)}}{1 - \frac{\beta/\Pi}{1 + g(C)}} \quad s = 2, 3, \dots, J - 1 \quad (3.54)$$

$$u^0 = \frac{\left[2 - \frac{(1 + g(C))N^0}{\Pi^{J-1}} \right] - \frac{\beta\Pi^{J-1}}{1 + g(C)}}{1 - \frac{\beta\Pi^{J-1}}{1 + g(C)}} \quad s = 0 \quad (3.55)$$

$$u^{01} = \frac{\left[2 - (1 + g(C))\Pi \frac{N^1}{N^0} \right] - \frac{\beta/\Pi}{1 + g(C)}}{1 - \frac{\beta/\Pi}{1 + g(C)}} \quad s = 1 \quad (3.56)$$

3.2.3 Unemployment and wage stickiness: efficiency wages and staggered contracts

Just as in Chapter 2, the labor market friction is introduced by the existence of efficiency wages and we can maintain the same expressions. Sticky efficiency wage, regardless of the endogenous growth model, will be:

$$w = \frac{e\Delta_b \left[(4r + b\Delta_b + q\Delta_q) - \frac{q\Delta_q a\Delta_a}{(4r + a\Delta_a + b\Delta_b)} \right] + \frac{q\Delta_q (4r + b\Delta_b)}{(4r + a\Delta_a + b\Delta_b)} z\Delta_a}{\Delta_w^b \left[(4r + b\Delta_b + q\Delta_q) - \frac{q\Delta_q a\Delta_a}{(4r + a\Delta_a + b\Delta_b)} \right] - \Delta_w^{bq} (4r + b\Delta_b)} \quad (3.57)$$

With wage flexibility we have $\Delta_w^b = \Delta_w^{bq} = \Delta_b = \Delta_q = \Delta_a = 1$, and the flexible steady wage is:

$$w = z + e + \frac{e}{q} \left(r + \frac{b}{d} \right) \quad (3.58)$$

3.2.4 Equilibrium conditions and steady state

Schumpeterian model

We again consider that there are neither public expenditures nor an external sector and the demand for final goods is composed of consumption, investment in R&D and intermediate goods production. As a result, the ratio consumption/output satisfies the following expression in steady state:

$$\begin{aligned} \frac{C}{Y} = 1 - \left(\frac{\alpha}{1 + R^k} \right)^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \frac{A}{Y} \\ - \left[\frac{\chi}{(1 + R_t^k)} \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1 \right) \right]^{\frac{1}{1-\chi}} \frac{A}{Y} \end{aligned} \quad (3.59)$$

where additionally:

$$\frac{A}{Y} = \frac{1}{\left(\left(\frac{\alpha}{1 + R^k} \right)^{\frac{1}{1-\alpha}} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \right)^\alpha L} \quad (3.60)$$

Taking into account the leverage ratio \emptyset_t from expression (3.10), we can obtain the following relation for the proportion of the debt on the final production:

$$\emptyset \frac{T}{Y} = \frac{S}{Y} = \frac{A}{Y} \left[\frac{\chi}{(1 + R_t^k)} \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1 \right) \right]^{\frac{1}{1-\chi}} \quad (3.61)$$

The normalization of all the growing variables of the Schumpeterian model is carried out dividing them by the production level of the final good, Y . The system of equations is presented in section C.1 of the Appendix C for the endogenous variables: $\frac{P^*}{P}, \frac{P_{-s}^*}{P}, g, L, L_{-s}, LL, \Delta_W^Y, w, \left(\frac{w}{Y}\right)_{-s}, N_{-s}, N, u, \frac{C}{Y}, \frac{A}{Y}, R, R^k, v, \eta, G(T), G(S), \frac{T}{Y}$ and \emptyset .

Related to the model without financial sector, six endogenous variables are added, and correspondingly six equations.

Human capital model

We again assume that there are neither public expenditures nor an external sector. Therefore, the steady-state consumption to physical capital ratio in steady state, C/K , will be as follows:

$$\frac{C}{K} = A^{\frac{1}{\alpha}} \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{1 - \alpha}{(1 + R_t^k)} \frac{1}{\Delta_t^W} \right]^{\frac{1-\alpha}{\alpha}} - g(C) - \delta \quad (3.62)$$

$$g(Y) = g(Y^i) = g(K) = g(L) = g(C) \quad (3.63)$$

Finally, as R_t^k represents the opportunity cost of resorting to external funds, we can obtain the following expression for the relation of credit market:

$$(1 + R_t^k) = [1 + R_t(1 + \text{premium})] \quad (3.64)$$

As in the previous chapters, the normalization of all the growing variables is carried out by dividing them by the physical capital level, K . The system of equations is presented in section C.2 for the endogenous variables: $\frac{P^*}{P}, \frac{P_{-s}^*}{P}, \Delta_p, g(C), d, L^0, L^1, L^{01}, LL, \Delta_W^Y, w, N^0, N^1, N, u^1, u^0, u^{01}, \frac{C}{Y}, R, R^k$ and *premium*. In this case only two are the endogenous variables added.

3.3. Trend inflation influence on growth, labor market variables and leverage ratio

As in the previous chapters, the two models have been simulated through Dynare in order to obtain the endogenous variable values in steady state and their responses to changes in trend inflation. In this chapter, we will pay special attention to the role played by the financial friction.

The values of the parameters are presented in Table 3.1. They are appropriate for quarterly data and, excluding the specific parameters of efficiency wage (z , q , e and b), commonly used in New-Keynesian models.

Table 3.1: The choice of parameter values

Parameter	Description	Schumpeterian model	Human capital model
δ	Capital depreciation rate		0.0115
α	Output elasticity with respect to capital	0.332	0.3256
β	Utility discount factor	0.99	0.98
ε	Elasticity of substitution among retail or intermediate goods		4.8
σ	Elasticity of substitution among labor services	12	10
ν	Relative utility weight of labor	1	1
I	Periods it takes to reset prices	1, 2	1, 2
J	Periods it takes to reset wages	1, 4	1, 4
γ	Productivity upgrade after every innovation	1.0125	
χ	Elasticity of the probability of success in the innovation with respect to relative investment	0.16	
ξ	Productivity parameter of human capital accumulation		0.07
A	Constant total factor productivity		0.706
z	Utility of leisure time and unemployment benefits	0.5	0.4
e	Cost of effort	0.03	0.075
q	Rate of job-finding	0.9	0.9
b	Rate of loss of employment	0.5	0.6
γ_2	Bankers' survival rate	0.97	
λ	Proportion of diverted assets	0.33	
ψ	Wealth proportion of the new bankers	0.005	
μ	Supervision cost of bankruptcy		0.04

3.3.1. Schumpeterian model

The behavior of the long-term inflation–growth relationship is very similar to that obtained in Chapters 1 and 2. Figure 3.1 shows, as in flexibility or only price stickiness, growth rate remains constant whatever the value of the trend inflation rate. Similarly, under wage stickiness, we can observe a relationship with an

inverted-U shape being maximized for a negative trend inflation of -0.5%. Although the chart does not allow the value of trend inflation to be differentiated where growth rate is exactly maximized, we can obtain it by observing the value of the rest of variables, especially the value of labor demand, since growth rate is maximized when labor demand is maximized, at a deflation rate of -0.5%.

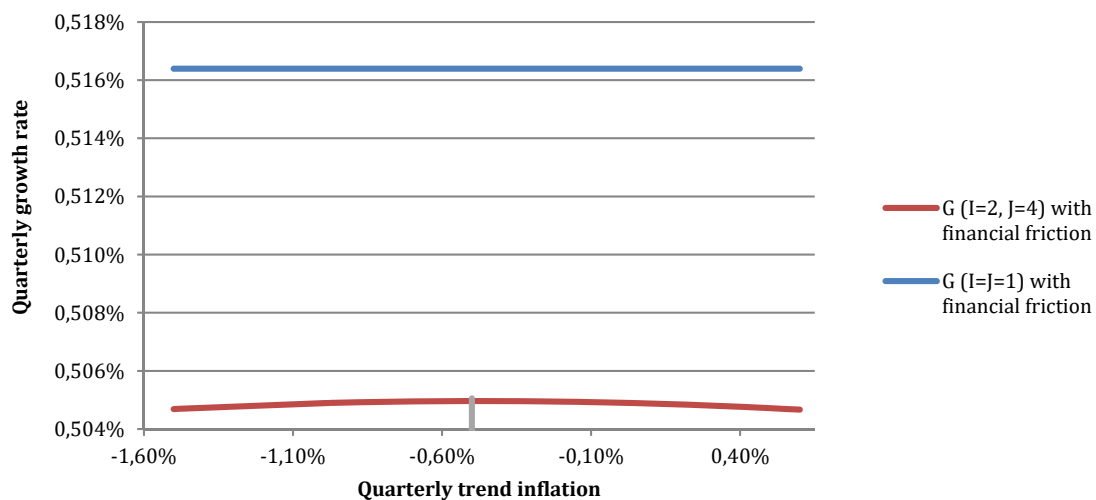


Figure 3.1: Long-term inflation-growth relationship under financial friction – Schumpeterian growth model.

As might be expected, a “growth loss” takes place as a consequence of the existence of financial frictions, since financial intermediaries represent an additional cost in the production process for retail firms, which signifies a shrinking of labor demand, L , and hence, of the production, Y , productivity level, A , and economic growth, G . Figure 3.2 shows this “growth loss” as a consequence of the financial friction under flexibility and wages stickiness.

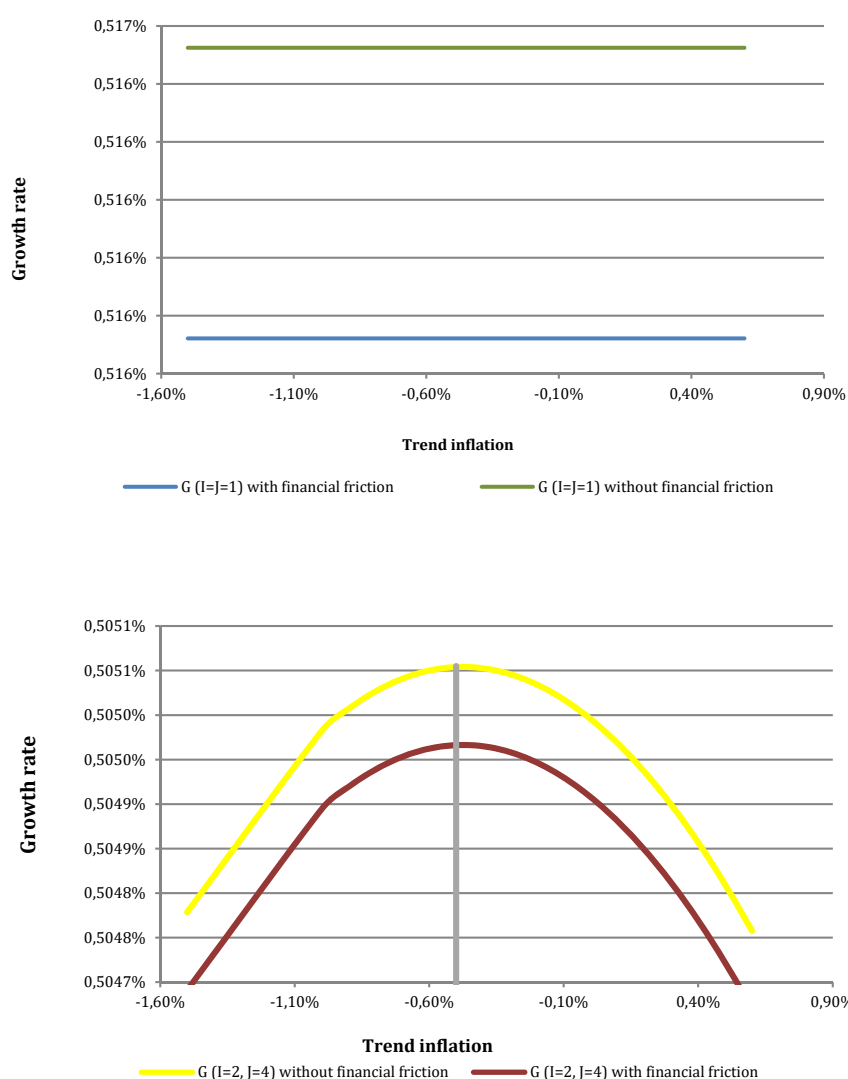


Figure 3.2: Growth loss due to financial friction under flexibility and wage stickiness – Schumpeterian growth model.

The relationships between trend inflation and unemployment, employment and labor force participation rates are shown in the following charts (Figures 3.3, 3.4 and 3.5, respectively). As in the previous chapter, flexibility rates remain constant whatever the value of trend inflation and these rates hardly vary when only price stickiness exists (flexibility relationships are represented through blue lines).

Regarding wage stickiness, we also find similar results to those of the previous

chapter. Concerning unemployment rate (Figure 3.3), this does not change with trend inflation, so we can also confirm Friedman's revision of the Phillips curve under financial frictions. Regarding employment and labor force participation rates, these rates do influence the maximization of long-run growth rate with wage stickiness since both rates are maximized at exactly the same trend inflation rate value that maximizes the long-run growth rate. Consequently, the presence of financial frictions does not affect the behavior of the main labor market variables.

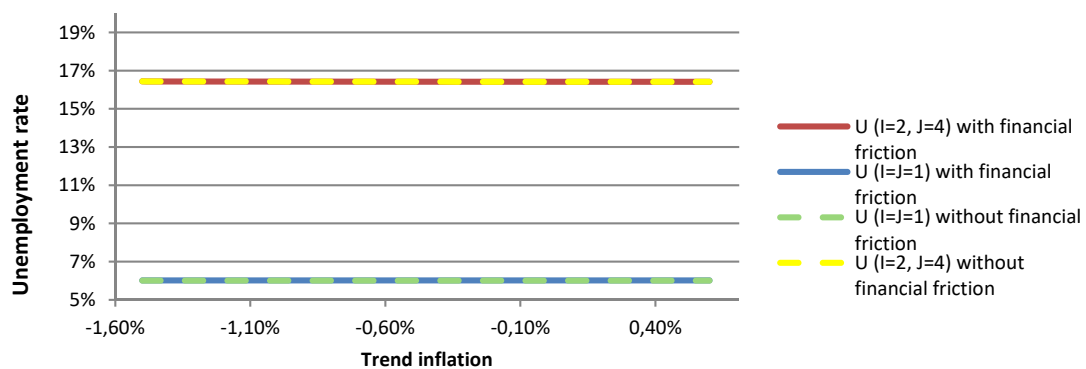


Figure 3.3: Long-term inflation-unemployment rate relationship – Schumpeterian growth model.

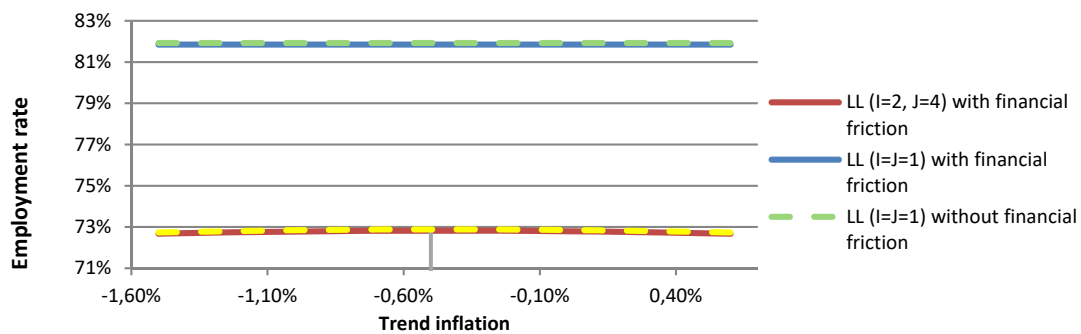


Figure 3.4: Long-term inflation-employment rate relationship – Schumpeterian growth model.

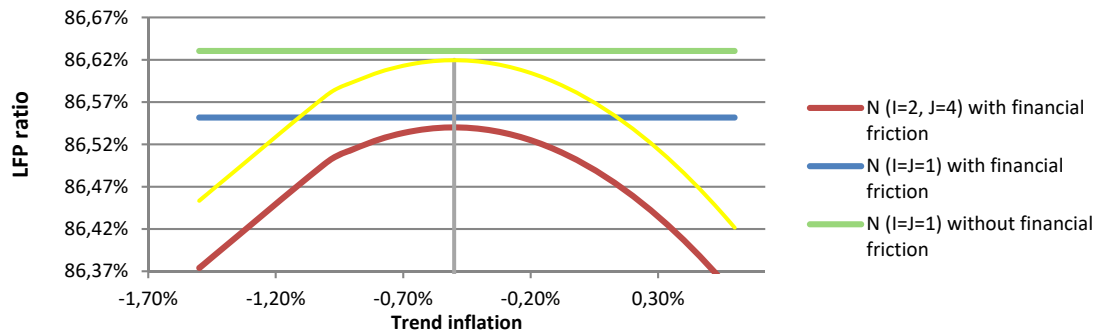


Figure 3.5: Long-term inflation-labor force participation rate relationship – Schumpeterian growth model.

Additionally, the relationship between trend inflation and leverage ratio is outstanding because the latter summarizes the activity of the credit market. Parameter \emptyset represents the leverage ratio of the economy, that is, the relation between total funding necessary to fund product activity of final goods producers and the financial intermediaries' net wealth. Figure 3.6 shows how, under wage stickiness, leverage ratio is minimized at exactly the same trend inflation value that maximizes economic growth, or in other words, growth rate is maximized for the minimum value of \emptyset (6.05982145). However, it is independent of trend inflation when wages are flexible.

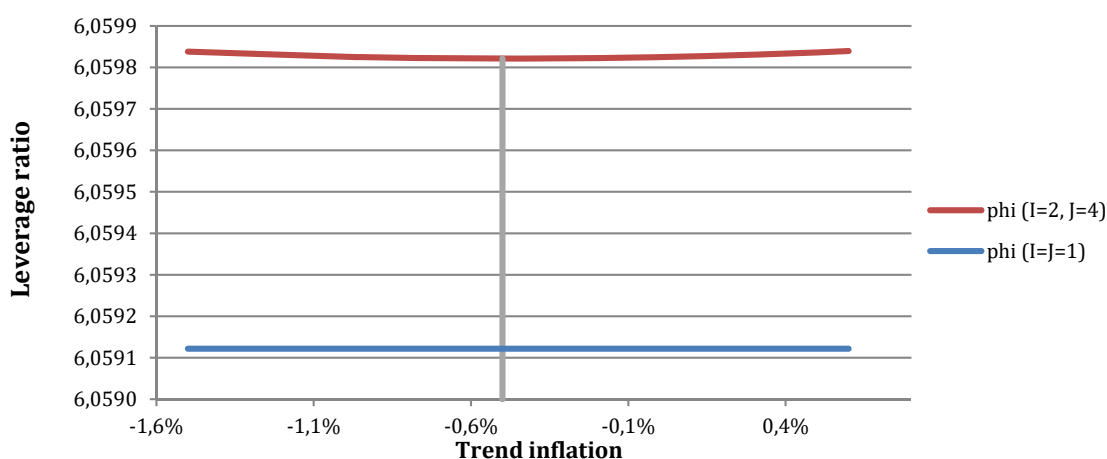


Figure 3.6: Long-term inflation-leverage ratio relationship under financial frictions – Schumpeterian growth model.

Finally, we must analyze the relationship between the measure of the financial friction and long-term economic growth. As we mentioned previously, λ represents the proportion of possible diverted assets by financial intermediaries causing depositors to redeem $(1 - \lambda)$ of their initial assets. Figures 3.7 and 3.8 show the negative effect of the financial distortion on the long-term growth rate whatever the type of wage (flexible or sticky). That is, the more asymmetric the information is, the higher the contraction in the economy. This growth shrinking is caused because the more asymmetric the information is, the higher the possibility of diverting assets, the higher the financial costs to achieve the compatibility of incentives and, then, the lower the employment and labor force participation rates and the higher the unemployment rate.

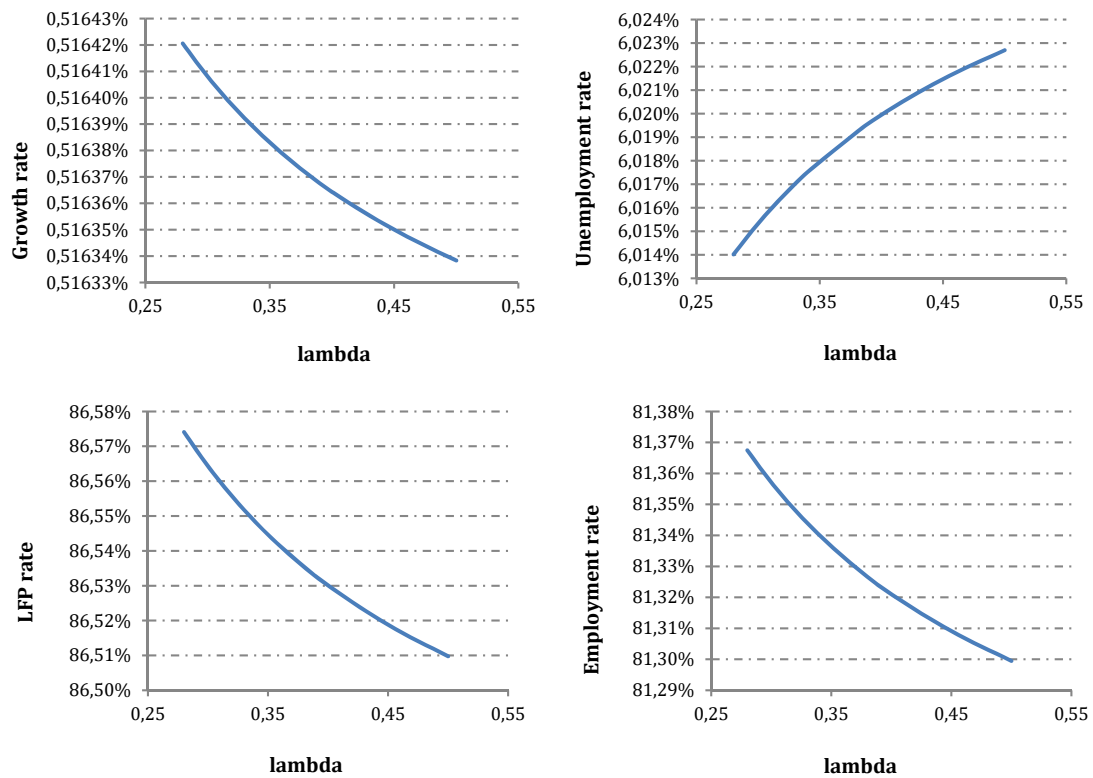


Figure 3.7: Variations of the main macroeconomic variables to changes in λ under flexibility – Schumpeterian growth model.

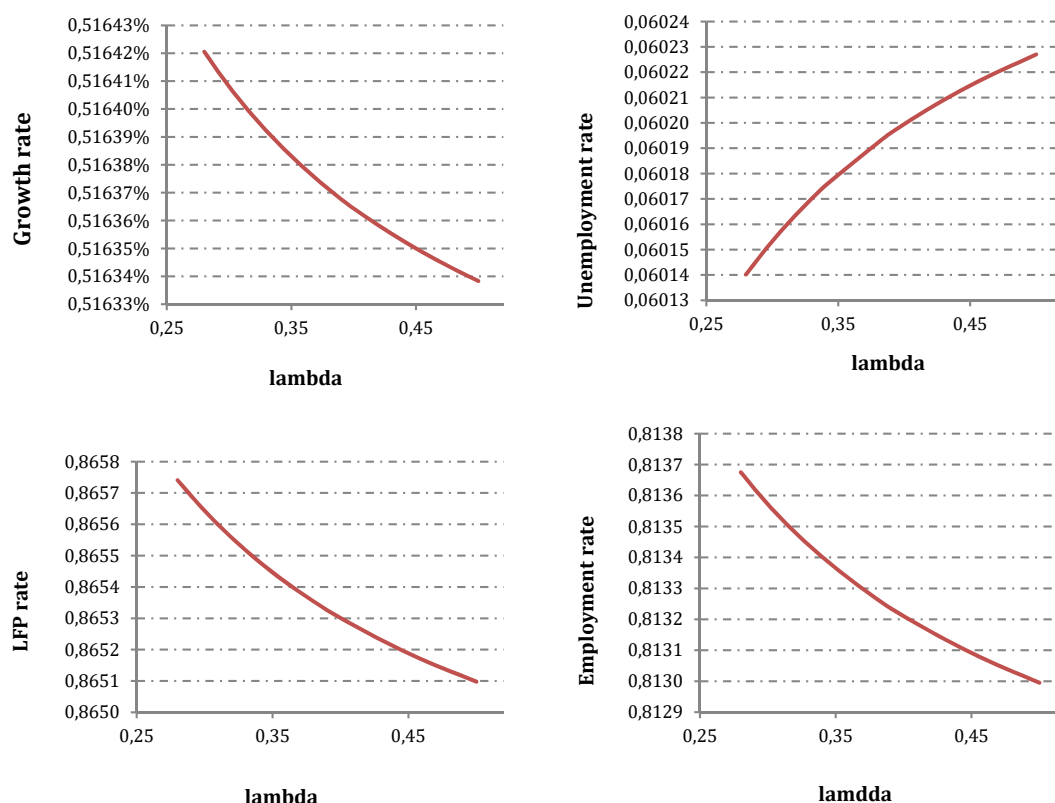


Figure 3.8: Variations of the main macroeconomic variables to changes in λ under wage stickiness – Schumpeterian growth model.

3.3.2. Human capital model

Regarding the Lucas human capital model, we also observe a similar relationship between inflation and economic growth to those obtained in Chapters 1 and 2. Figure 3.9 shows, on the one hand, how growth rate remains constant, whatever the value of trend inflation rate, in flexibility or only price stickiness and, on the other hand, a relationship with an inverted U-shape being maximized for null trend inflation under wage stickiness. Again, we can obtain the value of trend inflation where growth rate is maximized by observing the value of the other variables, especially the employment rate.

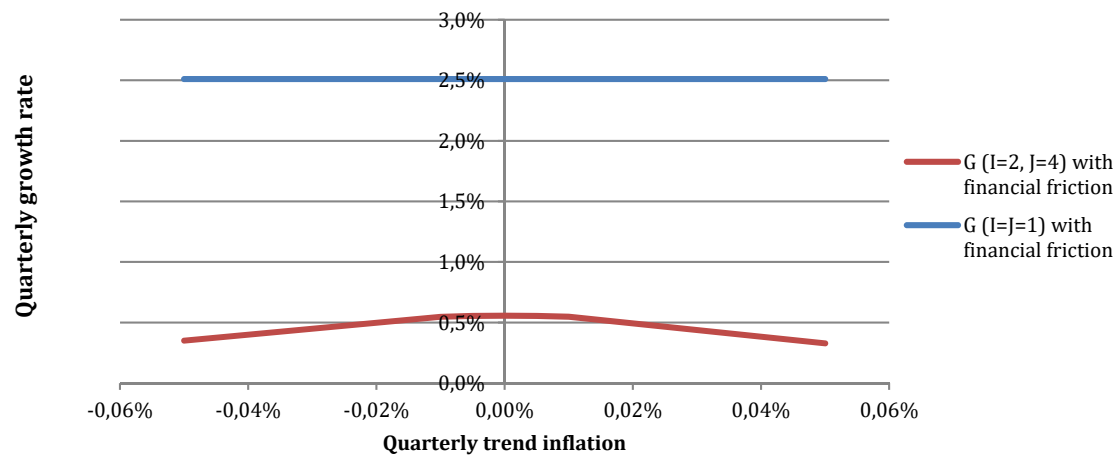


Figure 3.9: Long-term inflation-growth relationship under financial friction – Human capital model.

We can expect a “growth loss” as a consequence of the financial friction, since financial intermediaries represent an additional cost in the production process for retail and intermediate goods firms (opportunity cost in the last case).

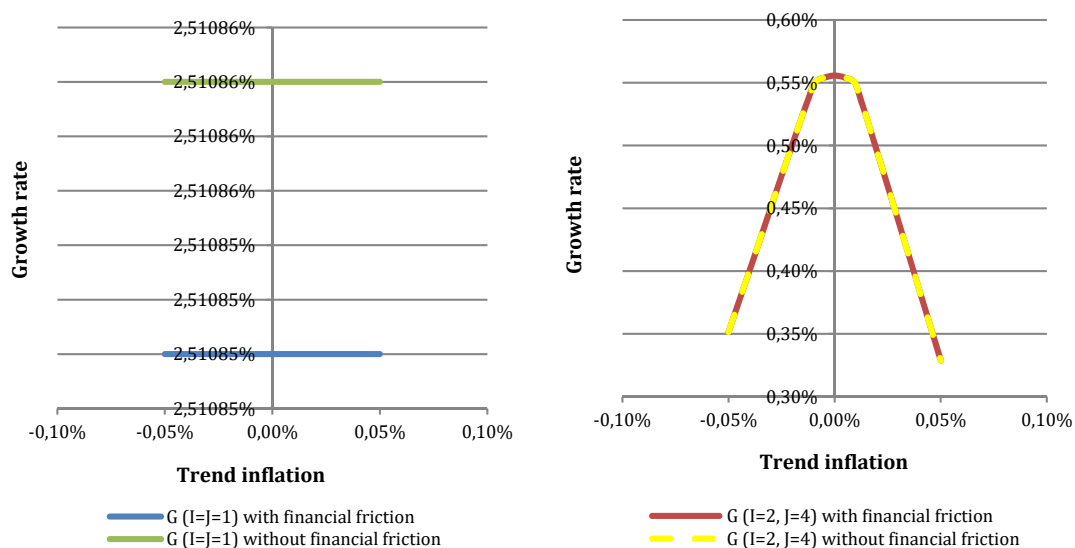


Figure 3.10: Growth loss due to financial friction under flexibility and wage stickiness – Human capital model.

However, although we can observe this growth loss under flexibility in Figure 3.10, financial friction has no impact on long-run growth rate under nominal rigidities. This is due to the minimum effect that this type of financial friction has on the value of the external finance premium as a consequence of the lack of wage revision for some workers. Financial friction is offset by wages stickiness resulting in such a small impact that it does not represent any variation on lending costs.

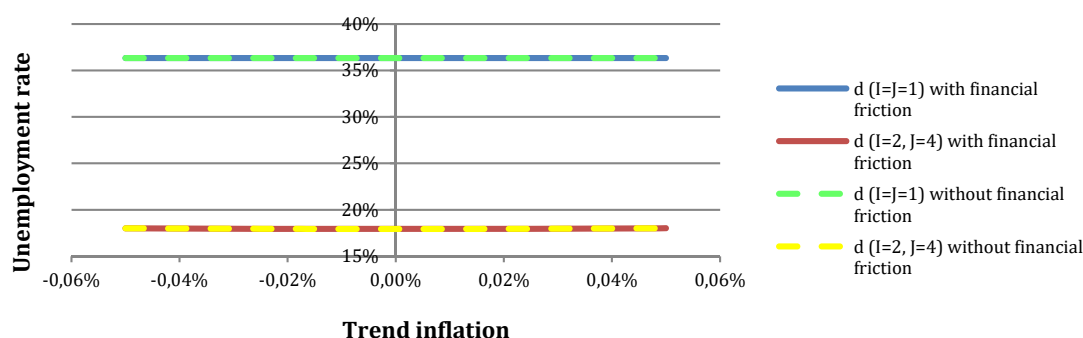


Figure 3.11: Long-term inflation–unemployment rate relationship – Human capital model.

The relationships between trend inflation and labor market rates are shown in Figures 3.11, 3.12 and 3.13. As in the previous model and chapter, flexibility rates remain constant whatever the value of trend inflation and these rates do not vary when only price stickiness exists (flexibility relationships are represented by a blue line).

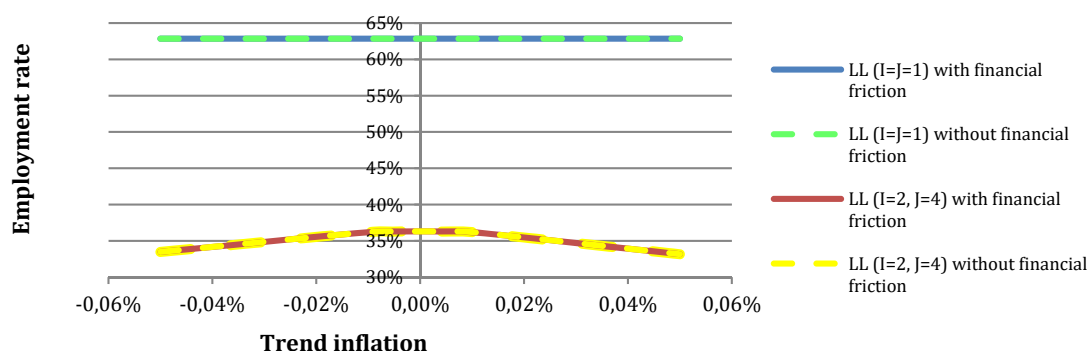


Figure 3.12: Long-term inflation–employment rate relationship – Human capital model.

Regarding wage stickiness, while unemployment rate (Figure 3.11) does not change with trend inflation, employment and labor force participation rates do change with trend inflation, employment and labor force participation rates do influence the maximization of long-run growth rate since both rates are maximized at exactly the same trend inflation rate value that maximizes long-run growth rate, that is, for null trend inflation. Therefore, as in the previous model, the presence of financial frictions does not affect the behavior of the main labor market variables.

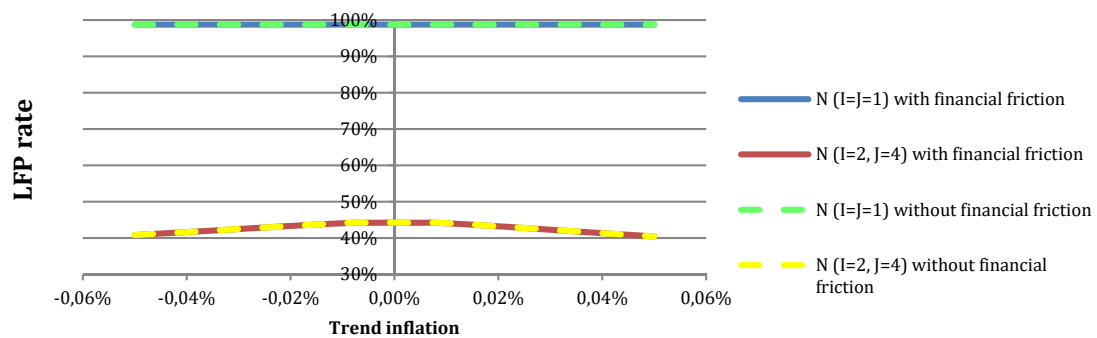


Figure 3.13: Long-term inflation–labor force participation rate relationship – Human capital model.

In contrast to the previous model, Figure 3.14 shows how, under wage stickiness, leverage ratio is maximized at exactly the same trend inflation value that maximizes economic growth, that is, economic growth maximizes for the maximum value of \emptyset (3.420103).

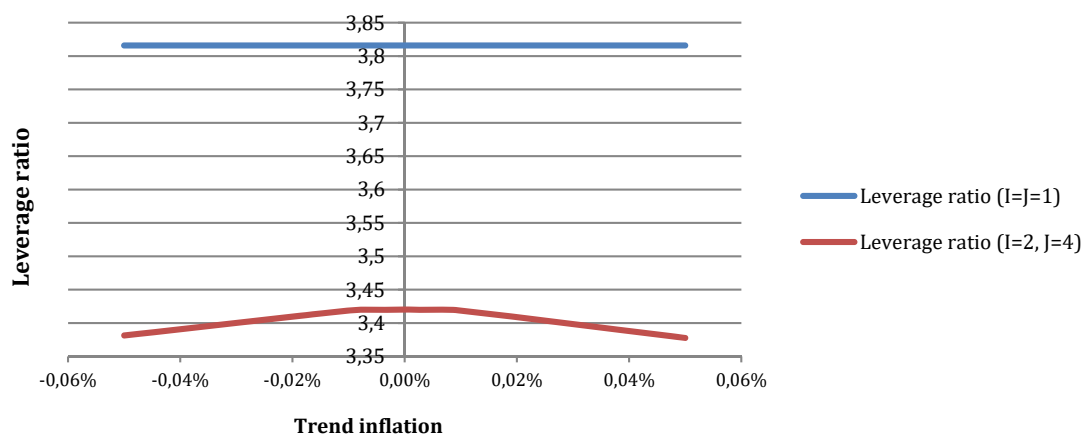


Figure 3.14: Long-term inflation-leverage ratio relationship under financial frictions – Human capital model.

Finally, we must analyze the relationship between the measure of the financial friction and long-term economic growth. As we mentioned previously, μ represents the supervision costs of bankruptcy, that is, the retailers' power and information, in such a way that the financial intermediary's benefit will be $(1 - \mu)\omega R_k Q_{it}$ if the retailer announces his/her bankruptcy and the lender has to monitor.

Figure 3.15 shows the relationship between the supervision costs and the main macroeconomic variables under nominal flexibility, where the greater the supervision costs the higher the external finance premium and the higher the production cost. Consequently, as the supervision costs are higher, the labor demand falls, producing a contraction in the economy.

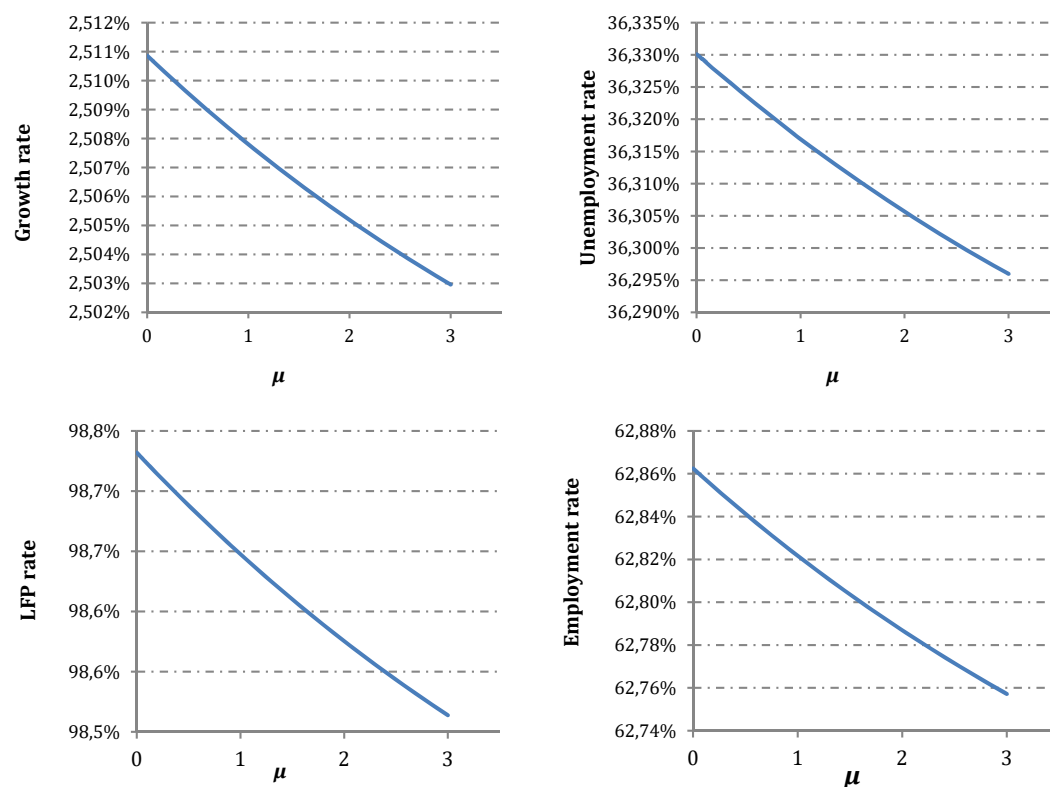


Figure 3.15: Variations of the main macroeconomic variables to changes in μ : flexible wages – Human capital model.

However, as has already been mentioned, if we consider nominal wage stickiness (Figure 3.16), supervision costs has very little impact on the external finance premium and it is compensated by the lower effect on real interest, resulting in a null impact on long-run economic growth and the rest of labor market variables.

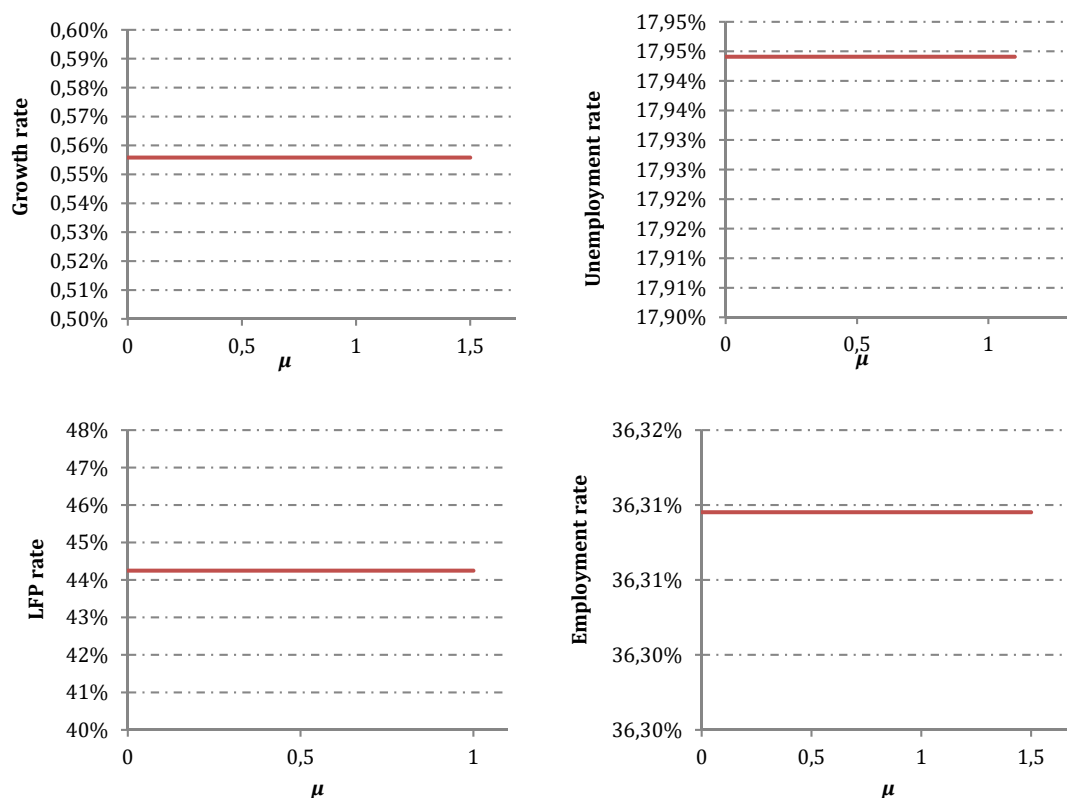


Figure 3.16: Variations of the main macroeconomic variables to changes in μ : rigid wages – Human capital model.

3.4. Effects of considering financial frictions on maximum growth rate: transmission mechanisms

After having evaluated and compared through simulations the consequences of financial market frictions on the long-run relationship between inflation and labor market variables and growth, it is necessary to identify from the steady-state equations (Appendix C2) the main mechanisms that explain these results.

Table 3.2 summarizes the main results.

Table 3.2: Maximum quarterly rates. Flexibility and wage stickiness

		Schumpeterian model					Human capital model				
		Financial intermediation model					Costly verification model				
		<i>Growth rate</i>	<i>Labor Force Participation</i>	<i>Employment</i>	<i>Unemployment</i>	<i>Leverage</i>	<i>Growth rate</i>	<i>Labor Force Participation</i>	<i>Employment</i>	<i>Unemployment</i>	<i>Leverage</i>
Flexibility	No financial friction	0.5164%	86.63%	81.4%	6.0064%	0	2.51086%	98.73%	62.86%	36.33%	0
	Financial friction	0.5163%	86.55%	81.34%	6.01703%	6.0591	2.51085%	98.73%	62.86%	36.33%	3.8159
Wage Stickiness (*)	No financial friction	0.50505%	86.62%	72.39%	16.422%	0	0.55582%	44.25%	36.31%	17.94%	0
	Financial friction	0.5049%	86.54%	72.32%	16.4216%	6.0598	0.55582%	44.25%	36.31%	17.94%	3.4201

(*) Quarterly rates for null trend inflation in human capital model and $\Pi = -0.5\%$ in Schumpeterian model.

On the one hand, under a Schumpeterian model, we can observe a growth loss as a consequence of the introduction of a financial intermediation friction regardless of the existence of nominal stickiness or flexibility.

According to the economic growth rate expression C1.3 in Appendix C.1, this decrease is due to a decline in the employment rate as a consequence of the increase in production costs (C1.4). Similarly, the LFP rate decreases as a consequence of the rise of consumption (C1.10 and C1.11). The net effect on the unemployment rate is not clear: we can observe a slight increase in flexibility, while it slightly decreases with wage stickiness. The different behavior is due to the effect on the employment rate, which is slightly greater in flexibility than

stickiness. Finally, the point where growth rate is maximum, leverage ratio is minimized since v_t , the marginal return of an additional unit of investment, is minimum (C1.18).

On the other hand, if we pay attention to the Lucas human capital model results, we observe a different behavior in nominal stickiness and flexibility. While we can note a very slight contraction under flexibility, the effect of the friction *costly state verification* has a null effect on economic variables under nominal stickiness.

Under flexible wages, supervision costs, μ , minimally increase the external finance premium (C2.17) raising production costs (C2.16) and then, shrinking economic growth (C2.5). However, as has already been mentioned, if we consider nominal wage stickiness, supervision cost has very little impact on the external finance premium, which is offset by the lack of wage revision for some workers, resulting in a null impact on long-run economic growth and the rest of the labor market variables.

Finally, regarding the value of trend inflation that maximizes the long-term growth rate, the same conclusions found in previous chapters 1 and 2 can be maintained in this chapter. According to efficiency wages, sticky nominal wage is minimum (and growth rate maximum) when trend inflation is negative and has exactly the same absolute value as the growth rate in the Schumpeterian model and is null in the Lucas human capital model. Consequently, the previous chapter results and conclusions regarding this point can be maintained. Similarly, economic growth rate with sticky wages does not reach the value of wage flexibility whatever the trend inflation and growth engine. The transmission mechanisms of the two models are explained in section 4 of the previous chapter.

3.5. A sensitivity analysis for efficiency wage parameters under financial friction

As explained in Chapter 2, the objective of this analysis is to determine the values of the efficiency wage parameters consistent with the charts presented in the two previous sections and the reasons to discard other alternatives.

The way to proceed is the same as in Chapter 2: that is, while in the case of flexibility the simulations are independent of trend inflation, with wage stickiness the simulations are made for the value -0.505%, the optimal trend inflation for the combination of parameters $z=0.5$, $e=0.03$, $q=0.9$, $b=0.5$ in the Schumpeterian model, and 0% for the combination $z=0.4$, $e=0.075$, $q=0.9$, $b=0.6$, in the Lucas human capital model. When the sensibility for one of the four parameters is studied, the other three are maintained in the values of the combinations of Table 3.1.

Similarly to Chapter 2, we can observe more immediate and significant sensibility of the flexible real wage to fluctuations in the parameters, except in the case of the probability of employment loss (parameter b). The reason is the same as in Chapter 2: that is, the crossed influence on this variable originated from the probabilities of losing employment (b and q) and from only considering the results corresponding to the inflation rate, which maximizes growth.

Even though the value of the parameters of this chapter is not the same as that in Chapter 2, and then, the threshold values are different, we can observe the same behavior with both flexible and sticky wages. Consequently, we can confirm that financial frictions do not alter the sensibility analysis previously performed and the conclusion of the plausibility of the situations shown in the Figures of section

3.3, which entail a growth loss (like employment and LFP losses) as consequence of the existence of unemployment.

As in Chapter 2, unemployment rate is acting as an endogenous variable, so this analysis cannot cover the isolated effect of unemployment rate variations in efficiency wages. However, we can observe an inverse relationship between unemployment rate and average real wage, which means that unemployment rate does act as a discipline mechanism.

Figures 3.17 and 3.18, which show the sensitivity analysis for the different parameters in the two analyzed models, will be commented on throughout this section.

3.5.1. Schumpeterian model

Figure 3.16 shows the response of growth, employment, LFP and unemployment rates in the Schumpeterian model to changes in the parameters z (Figure 3.17a), e (Figure 3.17b), q (Figure 3.17c) and in b (Figure 3.17d). In order to perform this analysis, we must bear in mind expressions (3.22) for the LFP rate, (3.42 and 3.43) for the employment rate, (3.57 and 3.58) for efficiency wages, (3.51) for economic growth and (3.10) for leverage ratio.

Considering wage flexibility, increases in z , the utility of leisure time and unemployment benefits, e , the cost of making the working effort, and b , the probability rate of employment loss, cause an increase in efficiency wages and, consequently, a decrease in employment and LFP rates and an increase in leverage ratio reducing economic growth. The impact on unemployment rate is as expected,

since increases in efficiency wages cause a decrease in employment rate and, therefore, an increase in unemployment rate.

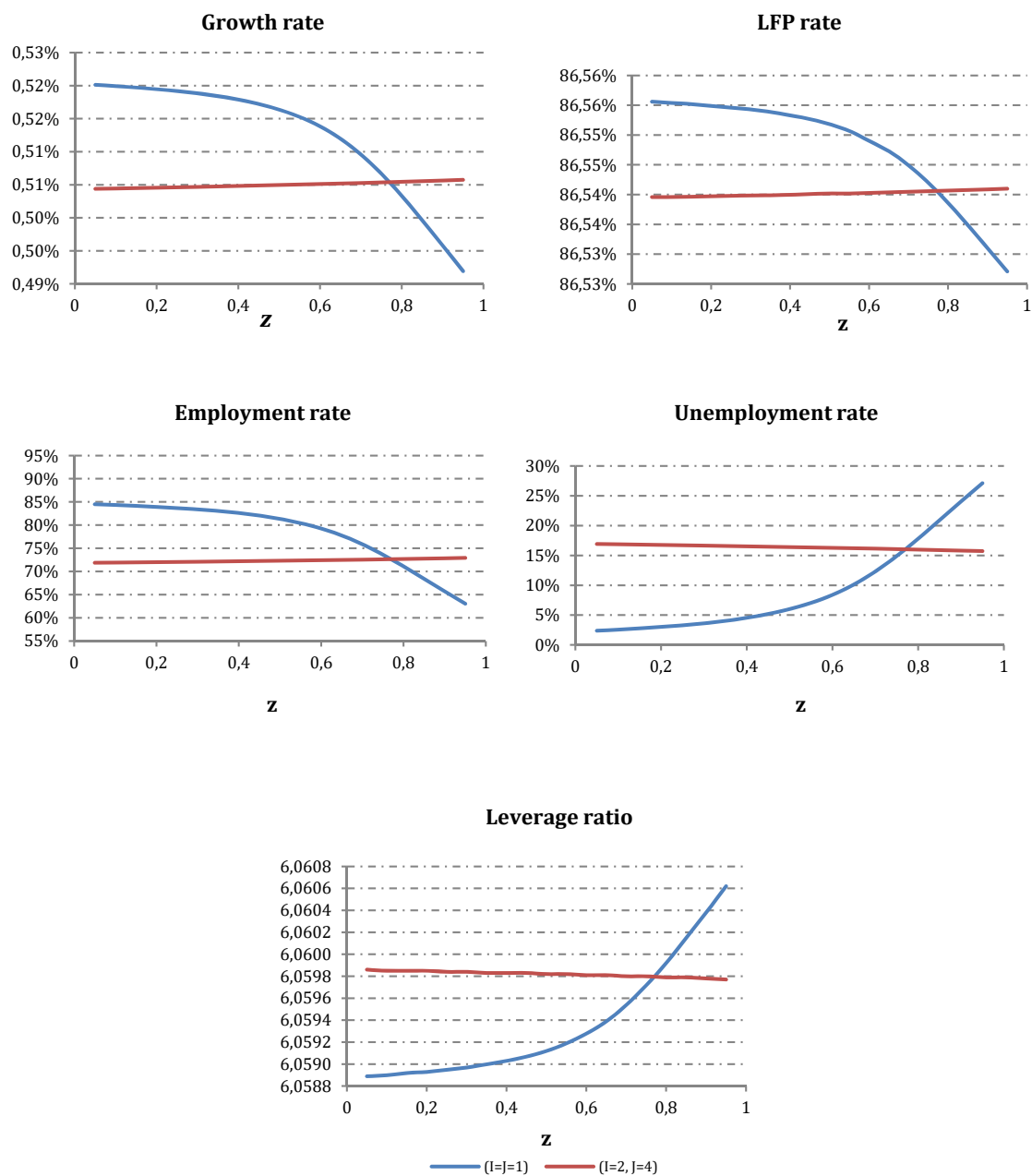


Figure 3.17a: Schumpeterian model: Sensitivity of the main economic rates to changes in parameter z .

On the other hand, changes in parameter q , the probability of being caught shirking and being fired, cause the opposite effect: a drop in efficiency wages, an increase in employment and LFP rates, a decrease in the leverage ratio and, then, a rise in the growth rate and a decrease in the unemployment rate.

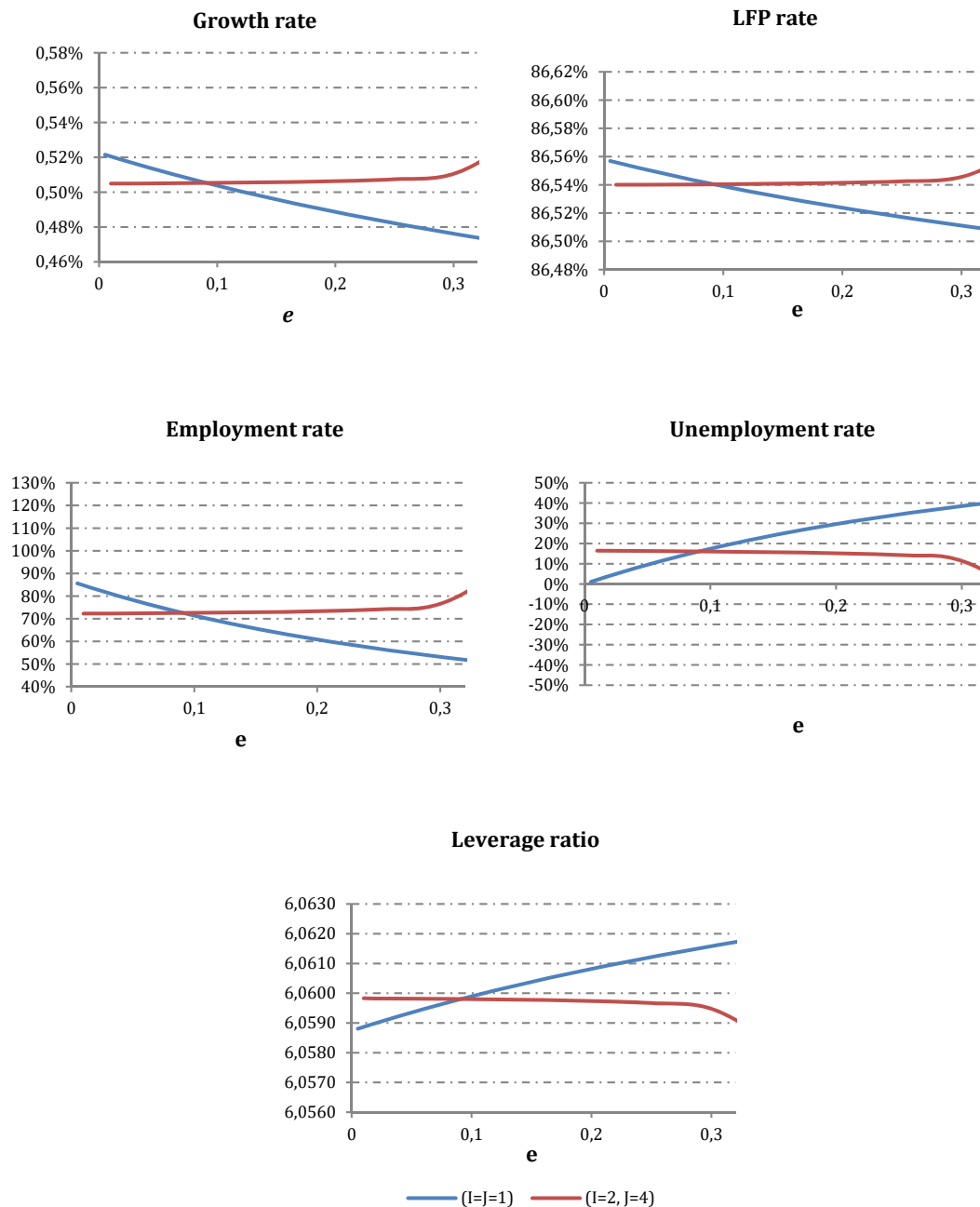


Figure 3.17b: Schumpeterian model: Sensitivity of the main economic rates to changes in parameter e .

Regarding stickiness, the lack of wage revision with the growth rate and the crossed effects of the probabilities of keeping employed offset the effect of z , q and e on the uploaded wage. Consequently, the effects on stickiness rates are barely noticeable.

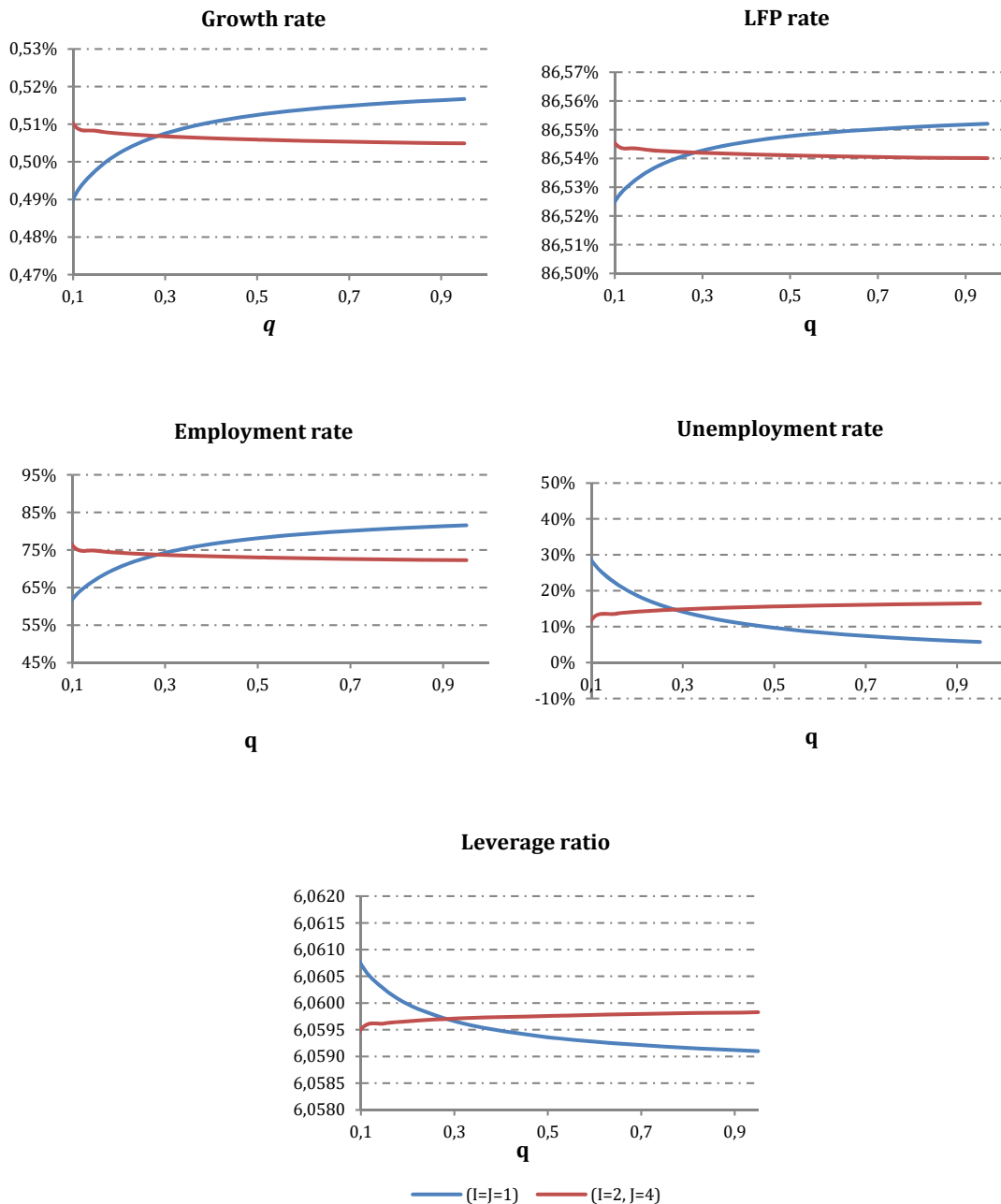


Figure 3.17c: Schumpeterian model: Sensitivity of the main economic rates to changes in parameter q .

Nevertheless, we must highlight the exception of the parameter b , the probability of employment loss, as the unemployment and growth rates of stickiness respond to fluctuations in this parameter through the effect that Δ_b , Δ_w^b and Δ_w^{bq} have on sticky wage.

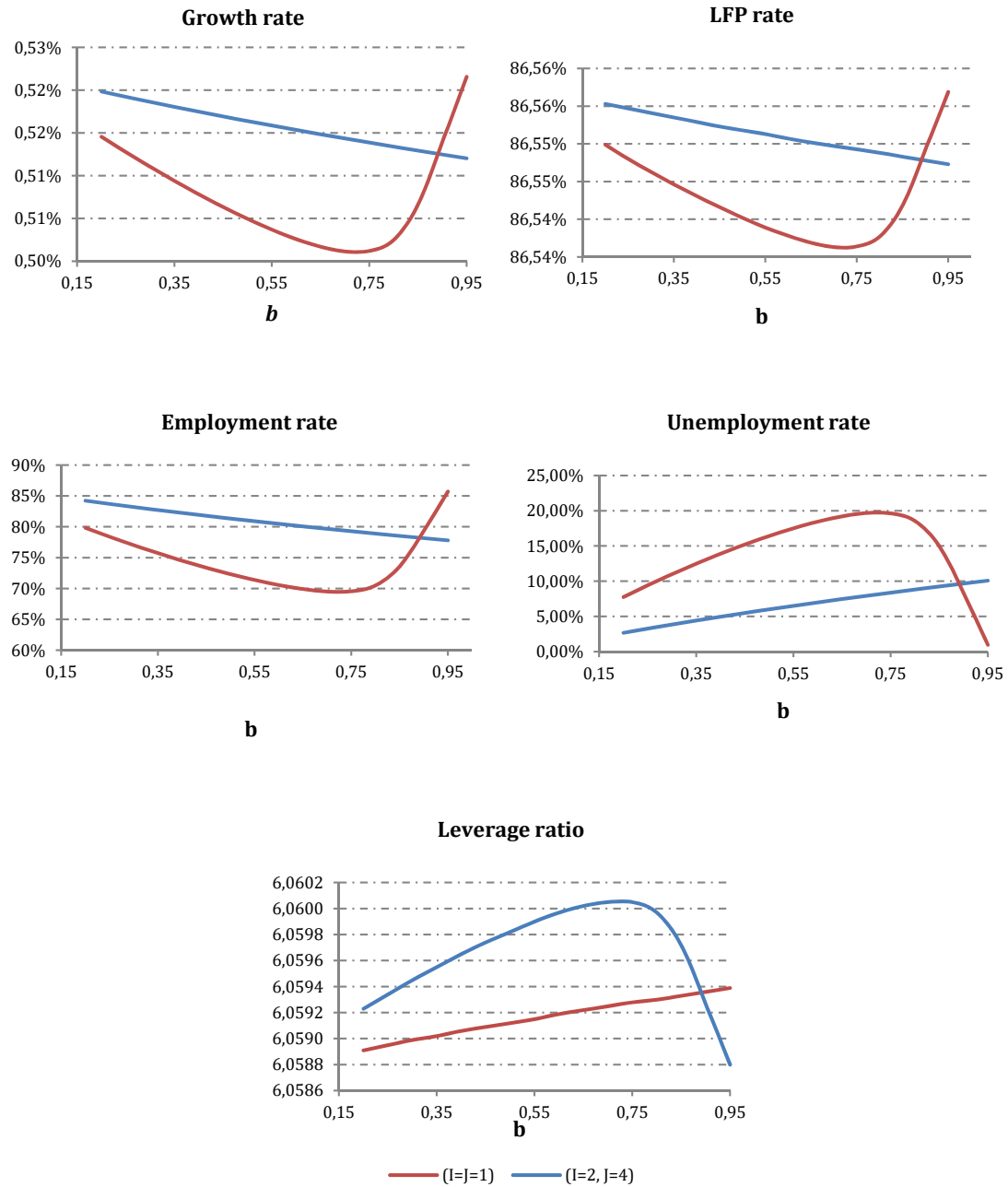


Figure 3.17d: Schumpeterian model: Sensitivity of the main economic rates to changes in parameter b .

Human capital model

Figure 3.18 shows the response of growth, employment, LFP and unemployment rates in the Lucas human capital model to changes in the parameters z (Figure 3.18a), e (Figure 3.18b), q (Figure 3.18c) and in b (Figure 3.18d). In order to perform this analysis, we must bear in mind expressions (3.24, 3.25 and 3.26) for the LFP rate, (3.33, 3.34, 3.35 and 3.36) for the employment rate, (3.57 and 3.58) for efficiency wages and (3.52) for economic growth.

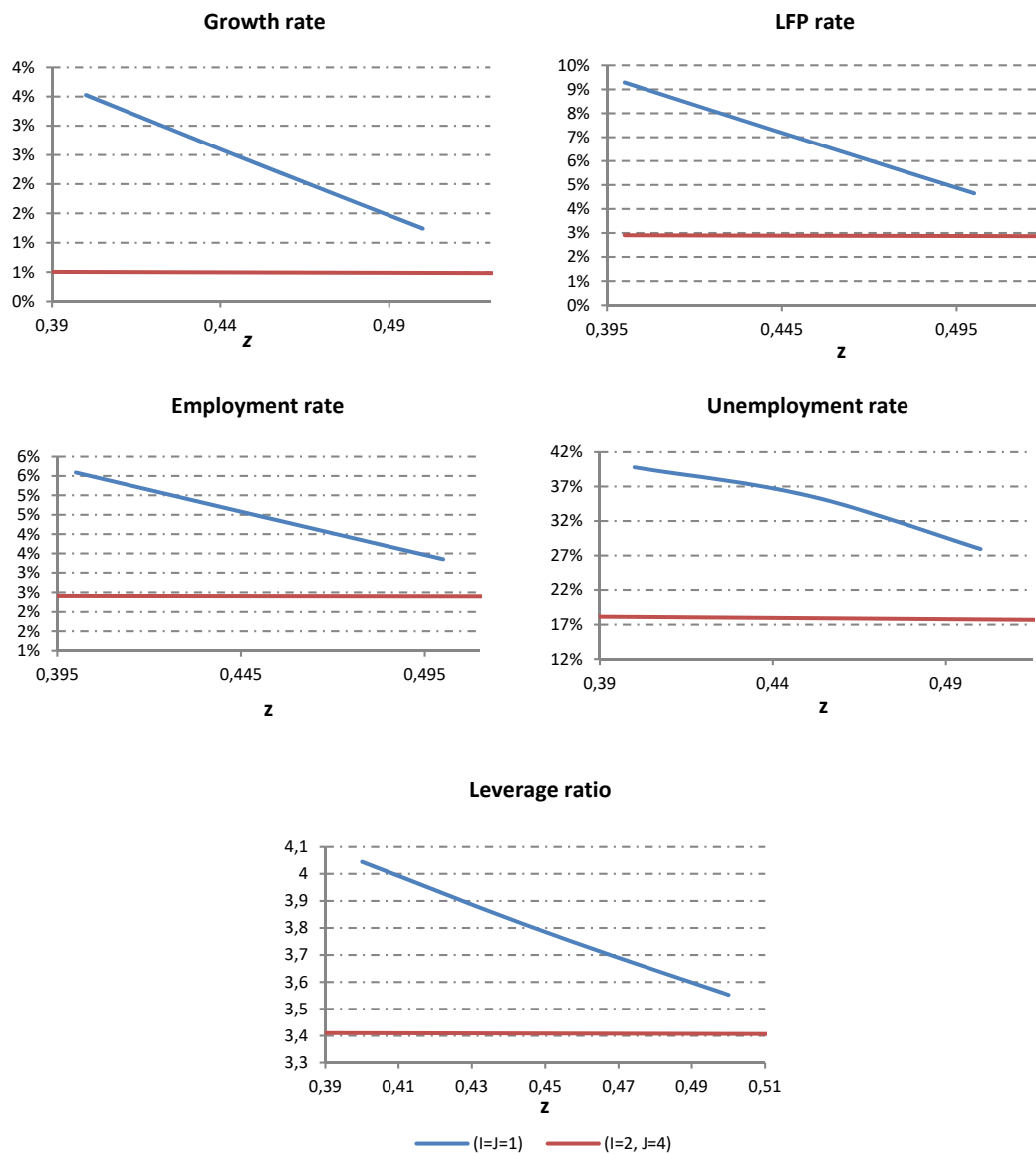


Figure 3.18a: Lucas model: Sensitivity of the main economic rates to changes in parameter z .

Similarly to the Schumpeterian model, if we consider wage flexibility, increases in z , e , and b , cause an increase in efficiency wages and, then, a decrease in employment and LFP rates, reducing economic growth. The impact on unemployment rate also is as expected, since increases in efficiency wages cause a decrease in employment and, therefore, an increase in the unemployment rate.

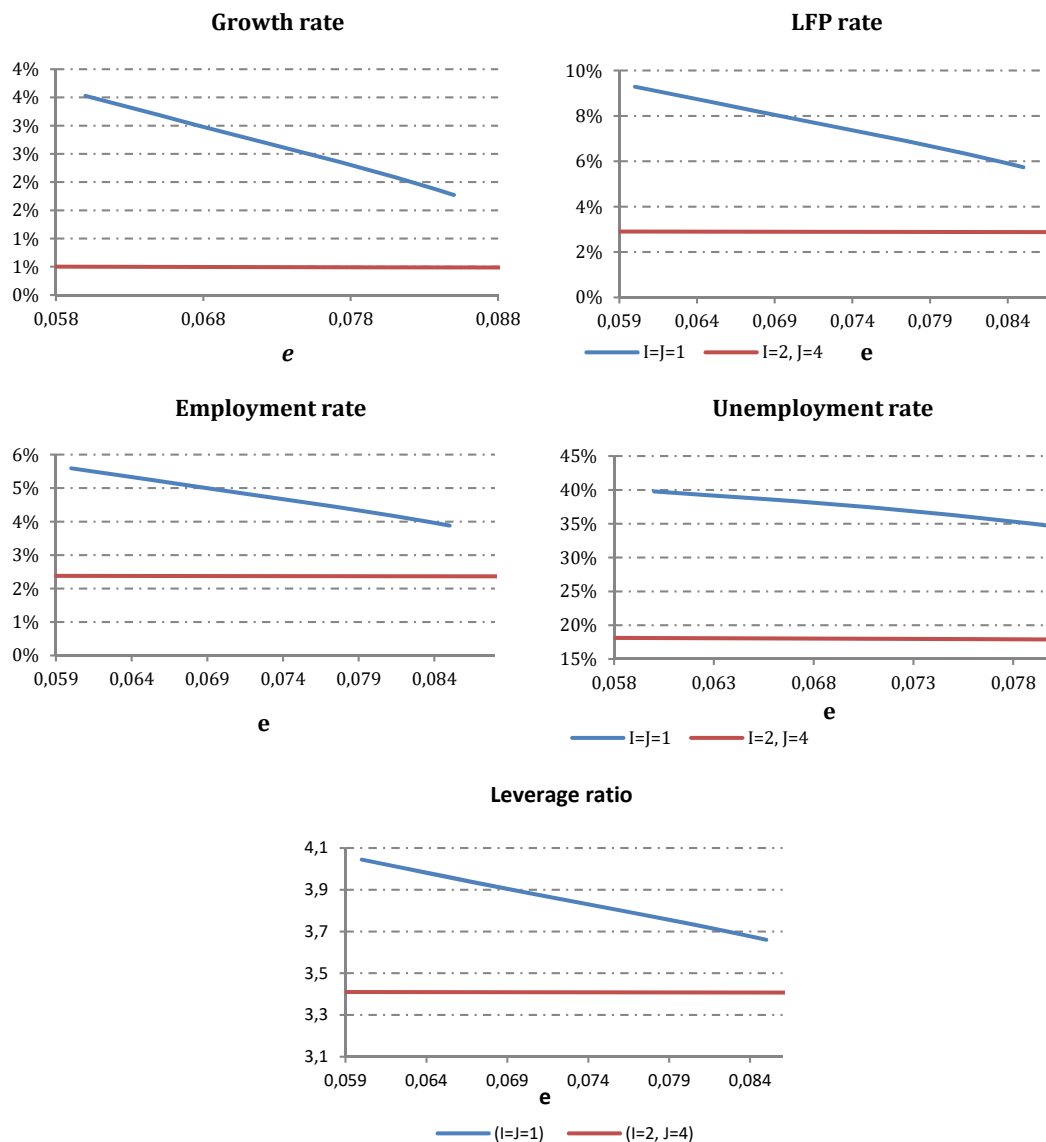


Figure 3.18b: Lucas model: Sensitivity of main economic rates to changes in parameter e .

On the other hand, an increase in parameter q causes the opposite effect: a drop in efficiency wages, an increase in employment and LFP rates and, then, a rise of the growth rate and a decrease in the unemployment rate.

Regarding stickiness, the effect of z , q and e are barely noticeable, while parameter b has a more significant effect on the growth and labor market variables due to the effect that Δ_b , Δ_w^b and Δ_w^{bq} have on sticky wages.

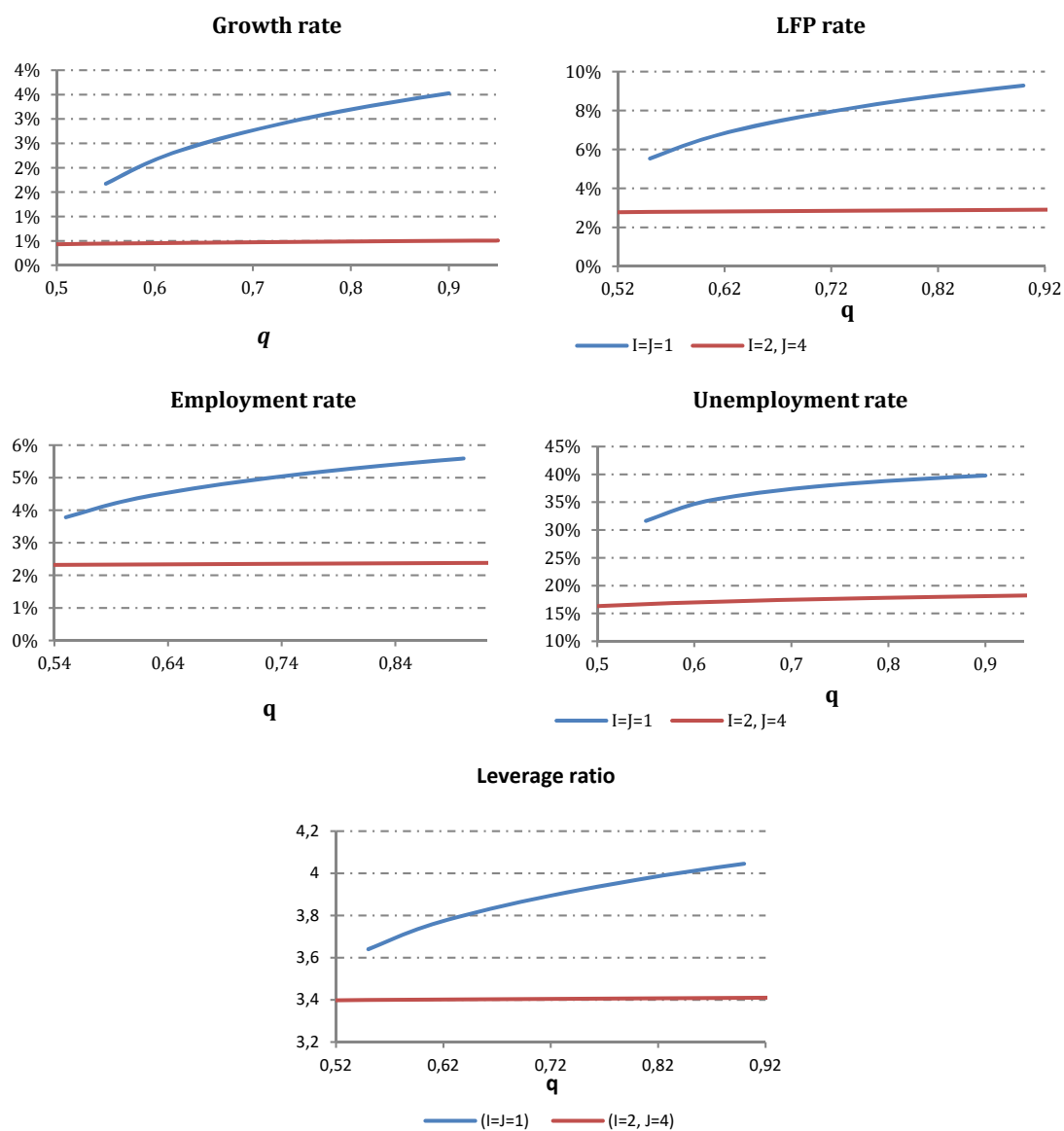


Figure 3.18c: Lucas model: Sensitivity of main economic rates to changes in parameter q .

Finally, as in the previous chapter without the financial sector, we must highlight the higher effect or sensitivity of the economic variables to fluctuations in efficiency wage parameters if we consider wages per unit of human capital instead of wage per worker. A variation in one of the parameters involves a much more significant effect on growth and labor market rates.

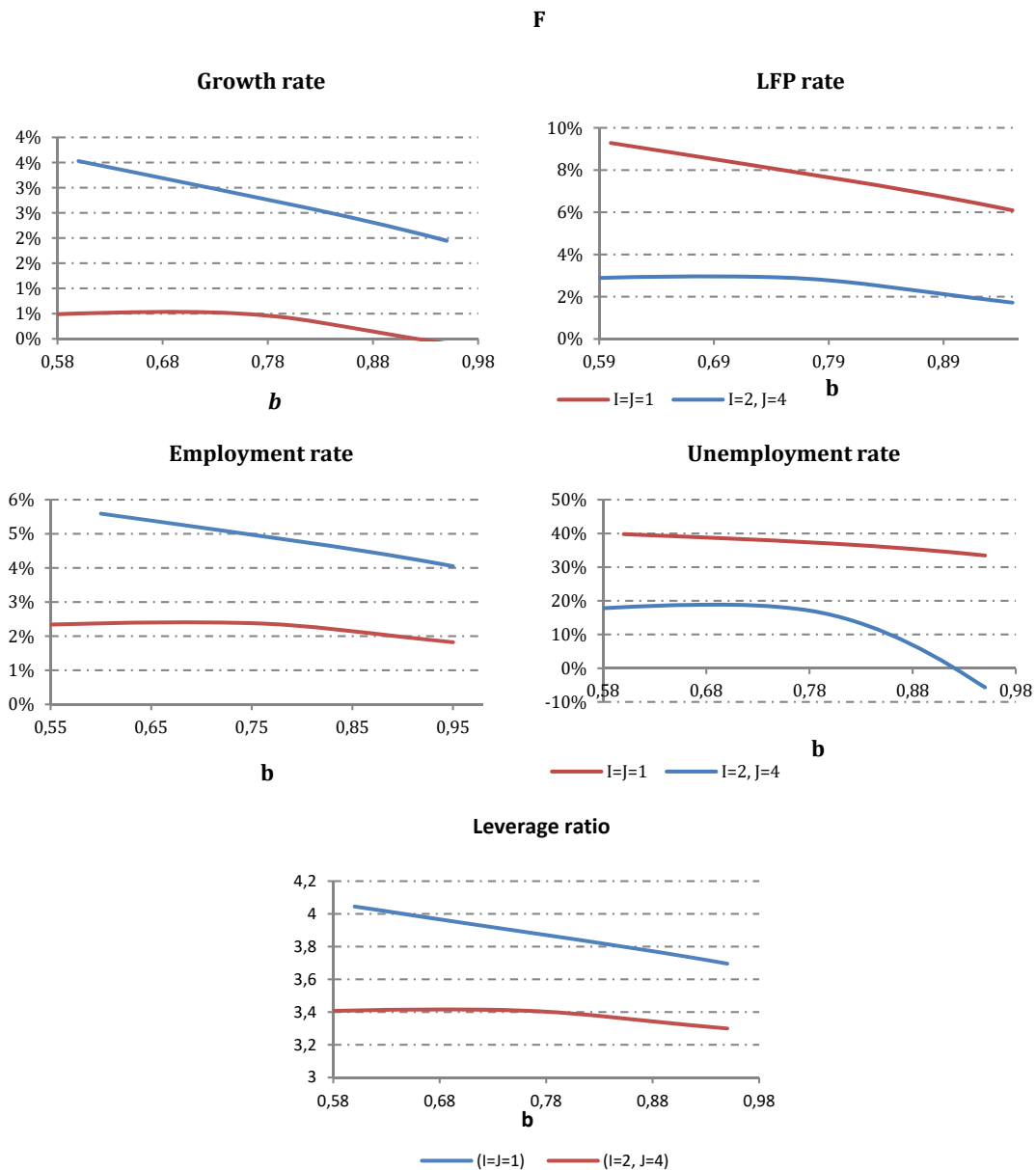


Figure 3.18d: Lucas model: Sensitivity of main economic rates to changes in parameter b .

3.6. An empirical application

Once unemployment and financial frictions have been included in both models, we will verify some empirical implications through estimation procedures provided by Dynare. In particular, we will estimate their parameters for the economies of the United States, Australia, EMU (France, Spain, Germany) and Japan, that is, countries governed by different central banks.

Dynare uses Bayesian estimation procedures, which allows for the estimation of the structural parameters of these economies. Once the parameters of the models have been estimated for each economy, we can resimulate and compare their observed data with their optimal equilibrium. This comparison allows us to establish whether the target inflation chosen by central banks would maximize the long-run economic growth, employment and LFP rates or whether there would be room for improvement by modifying the target rate.

The observed data have been obtained from OECD.org, which have been normalized according to steady-state expressions (Appendix B2 and Appendix C) to obtain the main steady-state variables for each model. Moreover, in order to avoid stochastic singularity, we must consider at least as many shocks or measurement errors as we have observed variables.

3.6.1. Schumpeterian model

The estimation of the Schumpeterian model has been carried out taking into consideration nominal rigidities, unemployment and financial friction; that is, we have used the complete model of this chapter. The following results have been obtained through the Dynare estimation process with eight observed variables:

LFP, unemployment and employment rates, real wage, consumption, real interest rate, long-run economic growth and leverage ratio. We have considered different periods of time for each economy, taking into consideration the stationarity requirement of the observed data.

Once the estimation has been completed and all the parameters of each economy obtained, we have resimulated the model with these values of the parameters in order to obtain the best values of the main macroeconomic variables: growth, employment, LFP and leverage ratio. In this way, we will be able to compare the real or observed situation with the best simulated results for every economy.

Table 3.3 summarizes the main findings of the Schumpeterian model's estimations in the first of the two columns displayed for each country.

Firstly, we obtain a growth loss (1) as a result of not having chosen the optimal trend inflation as target (difference 2). We can observe how this growth loss or difference is higher for economies with a lower observed growth rate, such as Germany, France or Japan. These economies would be those with greater room for growth improvement.

Secondly, we can compare the labor market variables, where simulated values have had to be adjusted in order to be comparable with observed values. The adjusted value has been obtained by multiplying the best simulated value by the ratio between the observed and simulated value corresponding to the observed trend inflation. The leverage ratio has been obtained in the same way.

Table 3.3: Empirical results for Schumpeterian and human capital models

	US		AUS		FRANCE		JAPAN		SPAIN		GERMANY	
	Schumpeter	Human capital	Schumpeter	Human capital	Schumpeter	Human capital	Schumpeter	Human capital	Schumpeter	Human capital	Schumpeter	Human capital
Quarterly observed growth	0.60%	0.68%	0.79%	0.83%	0.28%	0.39%	0.25%	0.25%	0.67%	0.69%	0.31%	0.32%
Annual observed growth	2.43%	2.75%	3.20%	3.36%	1.13%	1.58%	0.99%	1.01%	2.69%	2.80%	1.23%	1.30%
Maximum quarterly growth	0.73%	0.88%	0.81%	1.28%	0.40%	1.04%	0.38%	0.96%	0.69%	0.97%	0.54%	0.98%
Maximum annual growth	2.94%	3.56%	3.26%	5.21%	1.59%	4.22%	1.59%	3.88%	2.79%	3.94%	2.17%	3.97%
Difference (1)	0.51%	0.81%	0.07%	1.85%	0.46%	2.64%	0.60%	2.88%	0.10%	1.14%	0.94%	2.68%
Quarterly observed inflation	0.47%	0.76%	0.68%	0.88%	0.38%	0.36%	-0.23%	-0.21%	0.76%	0.76%	0.25%	0.27%
Quarterly objective inflation	-0.70%	0.00%	-0.80%	0.00%	-0.40%	0.00%	-0.40%	0.00%	-0.60%	0.00%	-0.50%	0.00%
Difference (2)	-1.17%	-0.76%	-1.48%	Q-0.88%	-0.78%	-0.36%	-0.17%	0.21%	-1.36%	-0.76%	-0.75%	-0.27%
Observed leverage ratio	6.754965947		3.85054		5.36622609		7.850810		4.93022		3.1562217	
Best leverage ratio	6.754937141		3.85052		5.3662158		7.85080995		4.93021		3.1561958	
Observed LFP rate	65.56%	65.60%	64.24%	63.50%	56.24%	56.13%	61.13%	61.21%	55.96%	55.58%	58.46%	58.59%
Best LFP rate	65.67%	66.60%	64.41%	64.43%	56.28%	56.28%	61.13%	61.27%	56.06%	56.33%	58.50%	58.67%
Difference (3)	0.11%	0.99%	0.17%	0.93%	0.05%	0.15%	0.00%	0.06%	0.09%	0.75%	0.03%	0.08%
Observed employment rate	61.62%	61.36%	60.35%	59.07%	51.40%	51.06%	58.46%	58.57%	49.31%	48.69%	53.54%	53.80%
Best employment rate	61.72%	62.39%	60.50%	60.03%	51.44%	51.21%	58.46%	58.64%	49.38%	49.42%	53.57%	53.88%
Difference (4)	0.10%	1.04%	0.15%	0.96%	0.04%	0.16%	0.00%	0.06%	0.07%	0.73%	0.03%	0.08%

del	0.0289		0.0296		0.0297		0.0296		0.0296		0.0296	
alp	0.4868	0.3624	0.5330	0.3498	0.5492	0.3574	0.5576	0.357	0.4893	0.3549	0.4050	0.3556
bet	0.9632	0.8396	0.9966	0.84	0.9958	0.8393	0.9802	0.8396	0.9994	0.8401	0.9822	0.8396
eps		4.7963		4.7979		4.7979		4.798		4.7979		4.7978
fi		0.8199		0.8746		0.8586		0.8546		0.8464		0.8627
sig	9.7545	10.8356	9.2820	10.844	9.3208	9.7559	10.4138	9.7533	9.5697	10.8446	10.4849	10.843
v	1.3809	0.8189	1.4377	0.7766	1.6992	0.7781	0.1120	0.785	0.2580	0.7732	2.7512	0.7647
A		0.5668		0.6017		0.5673		0.5672		0.5711		0.5675
gam	1.0223		1.0197		1.0116		1.0063		1.0135		1.0163	
gama	0.9991		0.9835		0.9280		0.9367		0.9642		0.9837	
chi	0.1905		0.1499		0.1860		0.0770		0.1092		0.2012	
land	0.3795		0.1713		0.1132		0.1625		0.2289		0.0220	
nb	0.0003		0.0045		0.0131		0.0073		0.0074		0.0083	
Period of time	Q1-1995 Q4-2013	Q1-1978 Q4-2014	Q4-1995 Q4-2016	Q4-1983 Q4-2014	Q1-2001 Q1-2014	Q1-1998 Q1-2014	Q1-1995 Q4-2013	Q2-1994 Q1-2014	Q1-1998 Q1-2010	Q1-1998 Q1-2010	Q3-1998 Q4-2012	Q2-1998 Q1-2014

Then, regarding the employment and LFP rates, we also notice small losses or differences between observed and best values (differences 3 and 4); nevertheless, these losses are minimal for economies where we observe lower economic growth

rates, since observed trend inflation is very close to the best one (difference 2). Consequently, although France, Japan and Germany are the analyzed countries that have a more significant difference to improve their growth rate, these countries have little room for improving their long-run economic growth through the labor market. In fact, in the case of Japan, we can note that it is very difficult or even impossible to improve the situation of its labor market.

Finally, regarding the leverage ratio, we also observe a small difference between the observed and the best values, it being almost impossible to improve the financial situation.

3.6.2. Human capital model

According to sections 3 and 4 of this chapter, if we consider rigid wages, financial friction of the type *costly verification* does not have any impact on the main macroeconomic variables. Consequently, the estimation of the Lucas human capital model has been carried out, taking into consideration nominal rigidities and unemployment, that is, the model of the previous chapter. The following results have been obtained through the Dynare estimation process with seven observed variables: LFP, unemployment and employment rates, real wage, consumption, real interest and long-run economic growth. The samples considered intervals of time periods that guarantee stationarity of the observed data.

As in the Schumpeterian model, once the estimation has been completed for each economy, their models have been resimulated with the estimated parameters in order to obtain the optimal equilibrium (maximum growth, LFP and employment rates).

Table 3.3 also summarizes the main findings of the Lucas human capital model's estimations in the second of the two columns displayed for each country.

Similarly to the Schumpeterian model, we observe a growth loss regardless of the economy (difference 1) as a result of not having chosen the optimal trend inflation as the target rate, which is higher for economies with a lower observed growth rate. We can also compare the labor market variables, where simulated values have been adjusted in the same way as they were for the Schumpeterian model in order to be comparable with observed values. Regarding employment and LFP rates, we also notice a loss (differences 3 and 4), which is minimal for economies where we observe the lowest growth rates, since the observed trend inflation is very close to the best one.

Consequently, in the countries where we observe a lower growth rate and, then, greater room for improvement, it is not possible to boost their economies through labor market variables because they are close to the optimum situation of their labor market.

Finally, if we compare the values of common parameters in the models, we can note a clear difference in α and β . Parameter α , *output elasticity to intermediate goods*, is substantially higher for the Schumpeterian model, which is evident since this model represents an endogenous growth model based on the quality improvement of intermediate goods. With regard to parameter β , *intertemporal discount factor*, this is considerably lower for the human capital model, which also makes sense, since workers devote part of their time to human capital accumulation improving their future earnings and, hence, utility.

On the other hand, if we pay attention to the influence of the financial friction on the economies (Schumpeterian model), the higher the financial distortion, that is, the higher the possibility of diverting assets (λ), the lower the room for growth improvement.

3.7. Conclusions

The models used in Chapter 2 have been enriched in this chapter through the incorporation of the financial sector in order to know the effects of financial market distortions and to explore the empirical implications for six developed countries. The focus is on the long-run relationships of inflation–growth and inflation–labor market variables compared to the conclusions of the previous chapter and the inflation–leverage relationship. The main results have been obtained from simulation and estimation using Dynare.

Firstly, we can conclude that the introduction of financial frictions does not substantially affect the relationship between trend inflation, long-run growth and labor market variables in their main features.

Moreover, in the *financial intermediation model* (Gertler and Karadi, 2011) we find that the trend inflation that minimizes the leverage ratio is the same as that which maximizes long-run economic growth, employment and LFP rates. In this case the economy reaches the maximum growth with the minimum level of indebtedness. On the contrary, in the *costly verification model* the maximum growth, employment and LFP rates are reached with the maximum leverage ratio. The reason for this difference is that in the first case the information asymmetry is in favor of financial intermediaries, whereby the conditions are imposed by the depositors who do best

when growth is maximum. In the second, information asymmetry is in favor of borrowers, whereby the conditions are imposed by the banks, which do best when growth is maximum.

Nevertheless, although in the case of the Schumpeterian model we find a growth loss as a consequence of the introduction of the financial friction (Figure 3.2), we cannot sustain the generality of this result because the financial friction introduced in the human capital model (*costly verification model*) does not have any impact on the main macroeconomic variables analyzed (Figure 3.10). The reason for this difference is the low value of the variance of the idiosyncratic shock. Undoubtedly there is a threshold from where this effect appears, but in our case this value has not been overtaken.

Finally, our exploration of the empirical implications of the models has been performed through Dynare estimation procedures. The conclusions of the Schumpeterian model are that the two countries with more potential increase in the long-run growth are Japan and Germany. The USA and France are situated at an intermediate level of improvement, while Australia and Spain are the two countries with the lowest level of growth gain. In the Lucas human capital model, France is added to the first group, Australia and Spain would be in the intermediate group and the USA in the group with the lowest improvement.

The way to reach these gains would be a change in the trend inflation (inflation target). The single country that should make a positive change in the quarterly inflation rate target is Japan (+0.21%), while the rest of the countries should decrease the target by at least -0.27% (Germany), -0.36% (France), -0.76% (Spain), and -0.88% (the USA and Australia). For these last two countries the gain

in the employment and LFP rates would be around one percentage point, three quarters of a percentage point in Spain and almost near zero in Japan, Germany and France. So, the growth gain in the first three countries would come from the improvement in the LFP rate, while in the case of the last three it would come from a change in the allocation that leads them to an increase in the TFP. In other words, the growth gain would come from the increase in the TFP in the second group, while in the first group the origin would come from the LFP.

Consequently, although the labor markets of the United States, Australia and Spain do show room for improvement, the other three countries show a labor market situation near the optimum, with little room for improvement. Finally, it is important to note that the conclusions obtained are similar for both models.

Bibliography

- [1] Aghion, P. and P. Howitt. 1992. A model of growth through creative destruction. *Econometrica* 60 (2), 323–351.
- [2] Aghion, A. and P. Howitt. 1998: *The Economics of Growth*. Massachusetts. MIT Press.
- [3] Amano, R., S. Ambler and N. Rebei. 2007. The macroeconomic effects of nonzero trend inflation. *Journal of Money, Credit and Banking* 39 (7), 1821–1838.
- [4] Amano, R., K. Moran, S. Murchison and A. Rennison. 2009. Trend inflation, wage and price rigidities, and productivity growth. *Journal of Monetary Economics* 56 (3), 353–364.
- [5] Amano, R., T. Carter and K. Moran. 2012. Inflation and growth: a new Keynesian perspective. *CIRANO-Scientific Publications* 2012s–20.
- [6] Arestis, P., A. D. Luintel and K. B. Luintel. 2008. Financial structure and economic growth. *Journal of Development Economics* 86 (1), 181–200.
- [7] Ascari, G. 2004. Staggered prices and trend inflation: some nuisances. *Review of Economic Dynamics* 7, 642–667.
- [8] Ascari, G. and T. Ropele. 2007. Optimal monetary policy under low trend inflation. *Journal of Monetary Economics* 54, 2568–2583.
- [9] Ascari, G. and A. M. Sbordone. 2014. The macroeconomics of trend inflation. *Journal of Economic Literature* 52 (3), 679–739.
- [10] Auerbach, A. J. 1985. Real determinants of corporate leverage. In B.M. Friedman (Ed.). *Corporate Capital Structures in the United States*. University of Chicago Press, Chicago, 301–324.
- [11] Ball, L. M. (2013) The case for four percent inflation. *Central Bank Review*, 13, pp. 17–31.
- [12] Bastani, H., S. Adjemian, M. Ratto, S. Villemot, G. Perendia, J. Maih and F. Karamé. 2012. *Dynare: Reference Manual*, version 4.
- [13] Basu, S. 1996. Procyclical productivity: increasing returns or cyclical utilization? *Quarterly Journal of Economics* 111, 719–751.

- [14] Basu, S. and J.G. Fernald. 1997. Returns to scale in U.S. production: estimates and implications. *Journal of Political Economy* 105, 249–283.
- [15] Bean, C. and C.A. Pissarides. 1993. Unemployment, consumption and growth. *European Economic Review* 37, 837–859.
- [16] Beck, T., A. Demirgüç-Kunt, R. Levine and V. Maksimovic. 2001. Financial structure and economic development: firm, industry, and country evidence. In Demirgüç-Kunt, A. and R. Levine (Eds.). *Financial Structure and Economic Growth: Perspectives and Lessons*. The MIT Press. Cambridge, Massachusetts.
- [17] Beck, T. and R. Levine. 2002. Stock Markets, Banks and Growth: Panel Evidence. NBER Working Paper Series No. 9082, National Bureau of Economic Research: Cambridge, Mass.
- [18] Bernanke, Ben & Gertler, Mark, 1989. Agency Costs, Net Worth, and Business Fluctuations. *American Economic Association Review*, vol. 79(1), pages 14-31, March.
- [19] Bernanke, B.S., and M. Gertler and S. Gilchrist. 1999. The financial accelerator in a quantitative business cycle framework. *Handbook of Macroeconomics* 1, 1341–1393.
- [20] Billi, R. M. and G.A. Kahn, 2008. What is the optimal inflation rate? *Federal Reserve Bank of Kansas City Economic Review*, issue Q II, pages 5-28.
- [21] Bils, M. and P.J. Klenow. 2004. Some evidence on the importance of sticky prices. *Journal of Political Economy* 112, 947–985.
- [22] Blanchard, O. 2017. Should We Reject the Natural Rate Hypothesis? Working Paper 17–14, Peterson Institute for International Economics.
- [23] Bullard, J. 2014. The rise and fall of labor force participation in the United States. *Federal Reserve Bank of St. Louis Review* First Quarter, 96 (1), 1–12.
- [24] Cecchetti, S.G. 1987. Indexation and incomes policy: a study of wage adjustment in unionized manufacturing. *Journal of Labour Economics* 5, 391–412.
- [25] Chen, B.L., M. Hsu and C.F. Lai. 2016. Relation between growth and unemployment in a model with labor-force participation and adverse labor institutions. *Journal of Macroeconomics* 50, 273–292.

- [26] Christiano, L.J., M. Eichenbaum and C.L. Evans. 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113, 1–45.
- [27] Coibion, O. and Y. Gorodnichenko. 2011. Monetary policy, trend inflation and the great moderation: an alternative interpretation. *American Economic Review* 101 (1), 341–370.
- [28] Coibion, O., Y. Gorodnichenko and J. Wieland. 2012. The optimal inflation rate in New Keynesian models: should central banks raise their inflation targets in light of the ZLB. *Review of Economic Studies* 79, 1371–1406.
- [29] Demirgüç-Kunt, A. and R. Levine. 1996. Stock markets, corporate finance and economic growth: An overview. *World Bank Economic Review* 10 (2), 223–239.
- [30] Demirgüç-Kunt, A. and R. Levine (Ed.). 2001. *Financial Structure and Economic Growth: Perspectives and Lessons. A Cross-Country Comparison of Banks, Markets, and Development*. The MIT Press. Cambridge, Massachusetts.
- [31] Evans, G., S. Honkapohja and P.M. Romer. 1998. Growth cycles. *American Economic Review* 88, 495–515.
- [32] Erceg, C. J., D.W. Henderson, and A.T. Levin. 2000. Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46 (2), 281–313.
- [33] Eriksson, C. 1997. Is there a trade-off between employment and growth? *Oxford Economic Papers* 49, 77–88.
- [34] Feng, S., Y. Hu and R. Moffitt. 2017. Long-run trends in unemployment and labor force participation in urban China. *Journal of Comparative Economics* 45, 304–324.
- [35] Fregert, K. and L. Jonung. 2006. Policy rule evaluation by contract-makers: 100 years of wage contract length in Sweden. *European Commission, Economic Papers* No. 270.
- [36] Friedman, M. 1968. The role of monetary policy. *American Economic Review*, 58 (1), 1–17.
- [37] Friedman, M. 1969. *The Optimum Quantity of Money*. Macmillan. Chicago: Aldine.

- [38] Fernández-Villaverde, J. 2010. Fiscal policy in a model with financial frictions. *The American Economic Review* 100 (2), 35–40.
- [39] Gali, J. 2008. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton: Princeton University Press.
- [40] Gambacorta, L., J. Yang, K. Tsatsaronis. 2014. Financial structure and growth. *BIS Quarterly Review* March, 21–35.
- [41] Gertler, Mark (2009): Financial Market Frictions and Real Activity: Basic Concepts. Lecture 1 Mini-Course: *Incorporating Financial Factors Within Macroeconomic Modelling and Policy Analysis*. New York.
- [42] Gertler, M. and N. Kiyotaki. 2010. Financial intermediation and credit policy in business cycle analysis. In B. M. Friedman and M. Woodford (Ed. 1), *Handbook of Monetary Economics*. North Holland. Amsterdam. 547–599.
- [43] Gertler, Mark and P. Karadi. 2011. A model of unconventional monetary policy. *Journal of Monetary Economics* 58 (1), January, 17–34.
- [44] Goldsmith, Raymond W. 1969. *Financial Structure and Development*. New Haven, CT: Yale University Press.
- [45] Gomme, P. 1993. Money and growth revisited. Measuring the costs of inflation in an endogenous growth model. *Journal of Monetary Economics* 46 (2), 281–313.
- [46] Hornstein, A. and A. L. Wolman. 2005. Trend inflation, firm-specific capital and sticky prices. *Federal Reserve Bank of Richmond Economic Quarterly Review* 91, 57–83.
- [47] Huang, K.X.D. and Z. Liu. 2002. Staggered price-setting, staggered wage-setting, and business cycle persistence. *Journal of Monetary Economics* 49, 405–433.
- [48] Kiley, M.T. 2007. Is moderate-to-high inflation inherently unstable? *International Journal of Central Banking* 3 (2), 173–201.
- [49] King, R.G. and R. Levine, 1993. Finance and growth: Schumpeter might be right. *The Quarterly Journal of Economics* 108, 717–737.
- [50] Krugman, P. 2014. Inflation targets reconsidered. *ECB Forum on Central Banking*. Sintra, Portugal.

- [51] Lang, L., E. Ofek, and R.M. Stulz. 1996. Leverage, investment, and firm growth. *Journal of Financial Economics* 40, 3–29.
- [52] Levin, A. and T. Yun. 2007. Reconsidering the natural rate hypothesis in a New Keynesian framework. *Journal of Monetary Economics* 54 (5), 1344–1365.
- [53] Levine, R. 2002. Bank-based or market-based financial systems: which is better? *Journal of Financial Intermediation* 11(4), 398–428.
- [54] Levine, R. 2003. More on finance and growth: more finance more growth? *Federal Reserve Bank of St. Louis Review* 85 (4), 31–46.
- [55] Lucas, R.E. 1988. On the mechanics of economic development, *Journal of Monetary Economics* 22, 3–42.
- [56] Romer, P. 1986. Increasing returns and long-run growth. *Journal of Political Economy* 94 (5), 1002–1037.
- [57] Romer, P. 1990. Endogenous technological change. *Journal of Political Econom*, 98 (5), S71–S102.
- [58] Saraiva, C. and S. Matthews. 2017. What Powell Needs to Know About U.S. Labor-Force Participation. Bloomberg, 6 December 2017.
- [59] Schmitt-Grohe, S & M. Uribe, 2010. The Optimal Rate of Inflation. In: Benjamin M. Friedman & Michael Woodford (ed.), *Handbook of Monetary Economics*, edition 1, volume 3, chapter 13, pages 653-722 Elsevier.
- [60] Schubert, S.F. and S.J. Turnovsky. 2018. Growth and unemployment: short-run and long-run tradeoffs. *Journal of Economic Dynamics & Control* 91, 172–189.
- [61] Shapiro, C. and J.E. Stiglitz. 1984. Equilibrium unemployment as a worker discipline device. *The American Economic Review* 74 (3), 433–444.
- [62] Sidrausky, M. 1967. Rational choice and patterns of growth in a monetary economy. *American Economic Review* 57 (2), 534–544.
- [63] Smets, F. and R. Wouters. 2007. Shocks and frictions in US business cycles: a Bayesian DSGE approach. *American Economic Review* 97, 586–606.
- [64] Taylor, J.B. 1980. Aggregate dynamics and staggered contracts. *Journal of Political Economy* 88 (1), 1–22.

- [65] Taylor, J.B. 1999. Staggered price and wage setting in macroeconomics. In: J.B. Taylor, , and M. Woodford, (Eds.), *Handbook of Macroeconomics*, vol. 1 (Part B). Elsevier Science B.V., New York, pp. 1009–1050.
- [66] Tesfaselassieb, M.F. and M.H. Wolters. 2018. The impact of growth on unemployment in a low vs. a high inflation environment. *Review of Economic Dynamics* 28, 34–50.
- [67] Van Zandweghe, W. 2012. Interpreting the recent decline in labor force participation. *Economic Review* (First Quarter), Federal Reserve Bank of Kansas City, 5–34.
- [68] Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

Appendix A

Appendix for the first chapter

A.1. Optimal control problem in the human capital model

Wage flexibility

The wage is the same for all types of labor services.

The Hamiltonian for this problem is:

$$\begin{aligned} H_{t+\tau} = & \beta^\tau \left[\log(C_{t+\tau}) - \frac{1}{1+\nu} \int_0^1 (N_{st+\tau})^{1+\nu} ds \right] \\ & + \lambda_{1,t+\tau} \left[D_{t+\tau} + \int_0^1 \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) L_{st+\tau} \left(\frac{W_{st+\tau}^*}{P_{t+\tau}^m} \right) ds + (R_{t+\tau} - \delta)K_{t+\tau} - C_{t+\tau} \right] \\ & + \lambda_{2,t+\tau} \left\{ \int_0^1 \xi (1 - u_{st+\tau}) N_{st+\tau} h_{st+\tau} \right\} \end{aligned}$$

subject to (1.5), (1.12), (1.14), (1.17), (1.18), (1.20), (1.21) and (1.22). The first first-order conditions are given as follows:

$$(A1.1) \quad \frac{\beta^\tau}{C_{t+\tau}} = \lambda_{1,t+\tau}$$

$$(A1.2) \quad \beta^\tau N_{st+\tau}^\nu = \lambda_{1,t+\tau} \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) u_{st+\tau} h_{st+\tau} + \lambda_{2,t+\tau} \xi (1 - u_{st+\tau}) h_{st+\tau} \quad \forall i \in [0,1]$$

$$(A1.3) \quad \lambda_{2,t+\tau} = \frac{\lambda_{1,t+\tau} W_{st+\tau}^*}{\xi P_{t+\tau}} \quad \forall s \in [0,1]$$

$$\begin{aligned} (A1.4) \quad & \lambda_{1,t+\tau+1} - \lambda_{1,t+\tau} \\ & = -\lambda_{1,t+\tau} (R_{t+\tau} - \delta) \\ & \quad - \lambda_{1,t+\tau} \int_0^1 \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) \left(\frac{W_{st+\tau}^*}{P_{t+\tau}^m} \right)^{-\sigma} \left[\frac{(1-\alpha)A}{(\Delta_{w,t+\tau}^i)^{1-\sigma\alpha}} \right]^{\frac{1}{\alpha}} di \end{aligned}$$

$$(A1.5) \quad \lambda_{2,t+\tau+1} - \lambda_{2,t+\tau} = -\lambda_{1,t+\tau} \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) u_{st+\tau} N_{st+\tau} - \lambda_{2,t+\tau} \xi (1 - u_{st+\tau}) N_{st+\tau} \quad \forall s \in [0,1]$$

$$(A1.6) \quad K_{t+\tau+1} = D_{t+\tau} + \int_0^1 \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) L_{i,t+\tau} (W_{i,t+\tau}^*) ds + (1 + R_{t+\tau} - \delta) K_{t+\tau} - C_{t+\tau}$$

$$(A1.7) \quad h_{t+\tau+1} = \left\{ \int_0^1 [1 + \xi (1 - u_{st+\tau}) N_{st+\tau}] \frac{h_{st+\tau}}{h_{t+\tau}} ds \right\} h_{t+\tau}$$

In steady state, from A1:

$$\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g(\lambda_1) = \frac{\beta^{\tau+1} / C_{t+\tau+1}}{\beta^{\tau} / C_{t+\tau}} = \frac{\beta}{1 + g(C)}$$

From A1.3:

$$\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g(\lambda_2) = \frac{\beta}{1 + g(C)}$$

From A1.5:

$$\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} = 1 - \zeta N_{st+\tau} = \frac{\beta}{1 + g(C)}$$

The supply of labor is the same for all i and is constant over time. From this expression, the constant value of N_{ss} in steady state can be obtained

$$N_{ss} = \frac{1}{\zeta} \left(1 - \frac{\beta}{1 + g(C)} \right)$$

From A1.2:

$$\frac{\beta^{\tau+1} N_{st+\tau+1}^v}{\beta^{\tau} N_{st+\tau}^v} = \frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} \frac{h_{st+\tau+1}}{h_{st+\tau}}$$

$$\beta = \frac{h_{st+\tau+1}}{h_{st+\tau}} \frac{\beta}{1 + g(C)} \Rightarrow g(h_s) = g(C)$$

The growth rate of human capital is the same as the consumption growth rate and the same for all s . We can see that from the accumulation process of human capital

$$h_{st+\tau+1} = h_{st+\tau} + \xi (1 - u_{st}) N_{st+\tau} h_{st+\tau}$$

its growth rate is:

$$g(h_s) = g(C) = \xi(1 - u_{st})N_{st+\tau} = \xi(1 - u_{ss})N_{ss}$$

where u_{ss} is the steady-state value for any s . From this expression we can deduce that the value of u is also the same for all type of labor services and is constant over time:

$$u_{ss} = 1 - \frac{g(C)}{\xi N_{ss}}$$

with those expressions the system of equations in steady state is closed.

Sticky wages

Note that the first-order condition for $u_{st+\tau}$ in (A1.3) implies that the real wage at time $t + \tau$ has to be the same across all individuals. However, since the nominal wage is expressed in terms of effective labor, the re-optimized real wage should be constant at the steady state, and therefore the nominal re-optimized wage grows at the same rate as the aggregate price. This implies that when the trend inflation is different from zero, there will be variations in the real wage across individuals. Obviously, this contradicts (A1.3). Then the previous problem is not valid with wage rigidity.

The Hamiltonian for this situation is:

$$\begin{aligned} H_{t+\tau} = & \beta^\tau \left[\log(C_{t+\tau}) - \frac{1}{1+\nu} \int_0^1 (N_{st+\tau})^{1+\nu} ds \right] \\ & + \lambda_{1,t+\tau} \left[D_{t+\tau} + \int_0^1 \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) L_{i,t+\tau} \left(\frac{W_{st+\tau}^*}{P_{t+\tau}^m} \right) ds + (R_{t+\tau} - \delta)K_{t+\tau} - C_{t+\tau} \right] \\ & + \sum_{q=1}^{J-1} \lambda_{2,t+\tau}^{1q} \left\{ \int_{\frac{q-1}{J}}^{\frac{q}{J}} \xi(1 - u_{i,t+\tau}^{1q}) N_{st+\tau}^{1q} h_{st+\tau}^{1q} ds \right\} + \dots + \lambda_{2,t+\tau}^0 \left\{ \int_{\frac{J-1}{J}}^1 \xi(1 - \right. \\ & \left. u_{st+\tau}^0) N_{st+\tau}^0 h_{st+\tau}^0 ds \right\} \quad q=1, 2, \dots, J-1 \end{aligned}$$

subject to (1.5), (1.12), (1.14), (1.17), (1.18), (1.20), (1.21) and (1.22). The first-order conditions are given as follows:

$$(A1.8) \quad \frac{\beta^\tau}{C_{t+\tau}} = \lambda_{1,t+\tau}$$

$$(A1.9) \quad \beta^\tau N_{st+\tau}^\nu = \lambda_{1,t+\tau} \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) u_{st+\tau} h_{st+\tau} + \lambda_{2,t+\tau} \xi(1 - u_{st+\tau}) h_{st+\tau} \quad \forall s \in [0,1]$$

$$(A1.10.1) \quad \lambda_{2,t+\tau}^0 = \frac{\lambda_{1,t+\tau}}{\xi} \frac{W_{st+\tau}^0}{P_{t+\tau}} \quad \forall s \in \left[\frac{J-1}{J}, 1 \right]$$

$$(A1.10.2), (A1.10.3), \dots, (A1.10.J) \quad \lambda_{2,t+\tau}^{1q} = \frac{\lambda_{1,t+\tau}}{\xi} \frac{W_{st+\tau}^{1q}}{P_{t+\tau}} \quad \forall s \in \left[\frac{s-1}{J}, \frac{s}{J} \right]$$

$$q = 1, 2, \dots, J-1$$

$$(A1.11) \quad \lambda_{1,t+\tau+1} - \lambda_{1,t+\tau} \\ = -\lambda_{1,t+\tau}(R_{t+\tau} - \delta) \\ - \lambda_{1,t+\tau} \int_0^1 \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) \left(\frac{W_{st+\tau}^*}{P_{t+\tau}^m} \right)^{-\sigma} \left[\frac{(1-\alpha)A}{(\Delta_{w,t+\tau}^i)^{1-\sigma\alpha}} \right]^{\frac{1}{\alpha}} ds$$

$$(A1.12.1) \quad \lambda_{2,t+\tau+1}^0 - \lambda_{2,t+\tau}^0 \\ = -\lambda_{1,t+\tau} \left(\frac{W_{st+\tau}^0}{P_{t+\tau}} \right) u_{st+\tau}^0 N_{st+\tau}^0 - \lambda_{2,t+\tau}^0 \xi (1 - u_{st+\tau}^0) N_{st+\tau}^0 \quad \forall s \in \left[\frac{J-1}{J}, 1 \right]$$

$$(A1.12.2), (A1.12.3), \dots, (A1.12.J) \quad \lambda_{2,t+\tau+1}^{1q} - \lambda_{2,t+\tau}^{1q} \\ = -\lambda_{1,t+\tau} \left(\frac{W_{st+\tau}^{1q}}{P_{t+\tau}} \right) u_{st+\tau}^{1q} N_{st+\tau}^{1q} - \lambda_{2,t+\tau}^{1q} \xi (1 - u_{st+\tau}^{1q}) N_{st+\tau}^{1q} \quad \forall s \\ \in \left[\frac{q-1}{J}, \frac{q}{J} \right] \quad q = 1, 2, \dots, J-1$$

$$(A1.13) \quad K_{t+\tau+1} = D_{t+\tau} + \int_0^1 \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) L_{st+\tau}(W_{st+\tau}^*) di + (1 + R_{t+\tau} - \delta)K_{t+\tau} - C_{t+\tau}$$

$$(A1.14) \quad h_{t+\tau+1} = \left\{ \int_0^1 [1 + \xi(1 - u_{st+\tau})N_{st+\tau}] \frac{h_{st+\tau}}{h_{t+\tau}} di \right\} h_{t+\tau}$$

In steady state, from A1.8:

$$\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g(\lambda_1) = \frac{\beta^{\tau+1}/C_{t+\tau+1}}{\beta^{\tau}/C_{t+\tau}} = \frac{\beta}{1 + g(C)}$$

From A1.10.2–A1.10.J (which represent labor services that do not change wages):

$$\frac{\lambda_{2,t+\tau+1}^{1q}}{\lambda_{2,t+\tau}^{1q}} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} \frac{1}{\Pi} = 1 + g(\lambda_2^1) = \frac{\beta/\Pi}{1 + g(C)} \quad q = 1, 2, \dots, J-1$$

From A1.10.1 (which represents labor services that change wages):

$$\frac{\lambda_{2,t+\tau+1}^0}{\lambda_{2,t+\tau}^0} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} \Pi^{J-1} = 1 + g(\lambda_2^0) = \frac{\beta \Pi^{J-1}}{1 + g(C)}$$

As a consequence, there will be two values of N . From A1.12:

$$\frac{\lambda_{2,t+\tau+1}^{1q}}{\lambda_{2,t+\tau}^{1q}} = 1 - \zeta N_{it+\tau}^{1q} = 1 - \zeta N_{st+\tau}^{1q} = \frac{\beta/\Pi}{1+g(C)} \Rightarrow N^{1q} = \frac{1}{\zeta} \left(1 - \frac{\beta/\Pi}{1+g(C)} \right) \quad q = 1, 2, \dots, J-2$$

$$\frac{\lambda_{2,t+\tau+1}^0}{\lambda_{2,t+\tau}^0} = 1 - \zeta N_{st+\tau}^0 = \frac{\beta \Pi^{J-1}}{1 + g(C)} \Rightarrow N^0 = \frac{1}{\zeta} \left(1 - \frac{\beta \Pi^{J-1}}{1 + g(C)} \right)$$

From A1.9:

$$\frac{\beta^{\tau+1} N_{st+\tau+1}^v}{\beta^\tau N_{st+\tau}^v} = \frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} \frac{h_{st+\tau+1}}{h_{st+\tau}}$$

$$\beta = \frac{h_{st+\tau+1}}{h_{st+\tau}} \frac{\beta/\Pi}{1 + g(C)} \Rightarrow g(C) = \frac{1 + g(h^1)}{\Pi} - 1 \quad s = 2, 3, \dots, J-1$$

$$\beta \frac{N^0}{N^1} = \frac{h_{st+\tau+1}}{h_{st+\tau}} \frac{\beta \Pi^{J-1}}{1 + g(C)} \Rightarrow g(C) = \left(1 + g(h^0) \right) \frac{\Pi^{J-1}}{\frac{N^0}{N^1}} - 1 \quad s = 0$$

$$\beta \frac{N^1}{N^0} = \frac{h_{st+\tau+1}}{h_{st+\tau}} \frac{\beta/\Pi}{1 + g(C)} \Rightarrow g(C) = \frac{(1 + g(h^{01}))}{\Pi \frac{N^1}{N^0}} - 1 \quad s = 1$$

As a consequence, there will also be three expressions of u in steady state:

$$u^1 = \frac{[2 - (1 + g(C))\Pi] - \frac{\beta/\Pi}{1 + g(C)}}{1 - \frac{\beta/\Pi}{1 + g(C)}} \quad s = 2, 3, \dots, J-1$$

$$u^0 = \frac{\left[2 - \frac{(1 + g(C)) N^0}{\Pi^{J-1} \frac{N^1}{N^0}} \right] - \frac{\beta \Pi^{J-1}}{1 + g(C)}}{1 - \frac{\beta \Pi^{J-1}}{1 + g(C)}} \quad s = 0$$

$$u^{01} = \frac{\left[2 - (1 + g(C)) \Pi \frac{N^1}{N^0} \right] - \frac{\beta/\Pi}{1 + g(C)}}{1 - \frac{\beta/\Pi}{1 + g(C)}} \quad s = 1$$

A.2. Steady-state systems of equations

A2.1. Model of physical capital externality

$$g = 1 + I^{n,k} \quad (\text{A2.1.1})$$

$$I^k = I^{n,k} + \delta \quad (\text{A2.1.2})$$

$$r^q = \alpha P^i Y^{i,k} + 1 - \delta \quad (\text{A2.1.3})$$

$$r^q = R \quad (\text{A2.1.4})$$

$$L^{1-\alpha} = Y^{i,k} \quad (\text{A2.1.5})$$

$$Y^{i,k} = \Delta^P Y^k \quad (\text{A2.1.6})$$

$$\Delta^P = \frac{1}{I} \frac{P^*}{P} \sum_{\tau=0}^{I-1} \Pi^{-\varepsilon \tau} \quad (\text{A2.1.7})$$

$$Y^k = C^k + I^k \quad (\text{A2.1.8})$$

$$\beta \frac{R^{st}}{g\Pi} = 1 \quad (\text{A2.1.9})$$

$$R^{st} = R\Pi \quad (\text{A2.1.10})$$

$$\frac{P^*}{P} = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{\tau=0}^{I-1} (\beta \Pi^\varepsilon)^\tau P^i}{\sum_{\tau=0}^{I-1} (\beta \Pi^{\varepsilon-1})^\tau} \quad \frac{P_{-s}^*}{P} = \frac{1}{\Pi^s} \frac{P^*}{P} \quad s = 1, 2, \dots, I-1 \quad (\text{A2.1.11})$$

$$L = \frac{(1-\alpha) Y^{i,k}}{\Delta_W^K} \quad (\text{A2.1.12})$$

$$\frac{P^*}{P} = \left(\frac{1}{I} \sum_{\tau=0}^{I-1} \Pi^{(\varepsilon-1)\tau} \right)^{\frac{1}{\varepsilon-1}} \quad (\text{A2.1.13})$$

$$\Delta_W^K = \frac{W^{*k}}{P P^i} \left[\frac{1}{J} \sum_{\tau=0}^{J-1} (\Pi^\tau)^{1-\sigma} \right]^{1/1-\sigma} \quad (\text{A2.1.14})$$

$$\frac{W^{*k}}{P} = \left(\frac{\sigma}{\sigma-1} \frac{C^K \sum_{\tau=0}^{J-1} \beta^\tau L^{(1+\nu)} P^i \sigma^{(1+\nu)} (g^\tau \Delta_W^K)^{\sigma(1+\nu)} \Pi^{\sigma(1+\nu)\tau}}{\sum_{\tau=0}^{J-1} \beta^\tau P^i \sigma (g^\tau \Delta_W^K)^\sigma L \Pi^{(\sigma-1)\tau}} \right)^{1/1+\sigma \nu} \quad (\text{A2.1.15})$$

$$\frac{W_{-s}^{*k}}{P} = \frac{1}{(\Pi g)^s} \frac{W^{*k}}{P} \quad s = 1, 2, \dots, J-1$$

A2.2. Schumpeterian model

$$\frac{P^*}{P} = \frac{1}{\alpha} \frac{\sum_{\tau=0}^{I-1} (\beta \Pi^{1/1-\alpha})^\tau}{\sum_{\tau=0}^{I-1} (\beta \Pi^{\alpha/1-\alpha})^\tau} \quad (\text{A2.2.1})$$

$$\frac{P_{-s}^*}{P} = \frac{1}{\Pi^s} \frac{P^*}{P} \quad s = 1, 2, \dots, I-1$$

$$g = \left[\chi \alpha^{\frac{1}{1-\alpha} L} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1 \right) \right]^{\frac{\chi}{1-\chi}} (\gamma - 1) + 1 \quad (\text{A2.2.2})$$

$$L = \frac{(1-\alpha)}{\Delta_W^Y} \quad (\text{A2.2.3})$$

$$\Delta_W^Y = \frac{W^*}{PY} \left[\frac{1}{J} \sum_{\tau=0}^{J-1} \left(\frac{1}{\Pi g} \right)^{\tau(1-\sigma)} \right]^{\frac{1}{1-\sigma}} \quad (\text{A2.2.4})$$

$$\frac{W^*}{PY} = \left(\frac{\sigma(1-\alpha)^v}{\sigma-1} \frac{C}{Y} \frac{\sum_{\tau=0}^{J-1} \beta^\tau (g^\tau \Delta_W^Y)^{(\sigma-1)(1+v)} \Pi^{\sigma(1+v)\tau} g^{(1+v)\tau}}{\sum_{\tau=0}^{J-1} \beta^\tau (g^\tau \Delta_W^Y)^{(\sigma-1)\tau} \Pi^{(\sigma-1)\tau}} \right)^{\frac{1}{1+\sigma v}} \quad (\text{A2.2.5})$$

$$\frac{W_{-s}^*}{PY} = \frac{1}{(\Pi g)^s} \frac{W^*}{P} \quad s = 1, 2, \dots, J-1$$

$$\frac{C}{Y} = 1 - \alpha^{\frac{1}{1-\alpha} L} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \frac{A}{Y} - \left[\chi \alpha^{\frac{1}{1-\alpha} L} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1 \right) \right]^{\frac{1}{1-\chi}} \frac{A}{Y} \quad (\text{A2.2.6})$$

$$\frac{A}{Y} = \frac{1}{\left(\alpha^{\frac{1}{1-\alpha}} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \right)^\alpha L} \quad (\text{A2.2.7})$$

$$\frac{R}{\Pi} = g \left(\frac{1}{\beta} \right) \quad (\text{A2.2.8})$$

A2.3. Technological change model

$$L = \frac{1 - \alpha}{\Delta_W^Y} \quad (\text{A2.3.1})$$

$$\frac{P^*}{P} = \frac{\epsilon}{\epsilon - 1} \left(\frac{\sum_{\tau=0}^{I-1} \left[\beta g^{\frac{(1-\epsilon)(1-\alpha)}{\alpha}} \Pi^\epsilon \right]^\tau}{\sum_{\tau=0}^{I-1} \left[\beta g^{\frac{(1-\epsilon)(1-\alpha)}{\alpha}} \Pi^{(\epsilon-1)} \right]^\tau} \right) \quad (\text{A2.3.2})$$

$$\frac{P_{-s}^*}{P} = \frac{1}{\Pi^s} \frac{P^*}{P} \quad s = 1, 2, \dots, I - 1$$

$$\Delta^P = \frac{P^*}{P} \left[\frac{1}{I} \sum_{\tau=0}^{I-1} \left(\frac{1}{\Pi} \right)^{(1-\epsilon)\tau} \right]^{\frac{1}{1-\epsilon}} \quad (\text{A2.3.3})$$

$$\frac{W^*}{PY} = \left(\frac{\sigma(1-\alpha)^{\sigma\nu}}{\sigma-1} \frac{C}{Y} \frac{\sum_{\tau=0}^{J-1} (\beta \Pi^{\sigma(1+\nu)} g^{\sigma(1+\nu)})^\tau L^{\nu(1-\sigma)}}{\sum_{\tau=0}^{J-1} (\beta (\Pi g)^{\sigma-1})^\tau} \right)^{\frac{1}{1+\sigma\nu}} \quad (\text{A2.3.4})$$

$$\frac{W_{-s}^*}{P} = \frac{1}{(\Pi g)^s} \frac{W^*}{PY} \quad s = 1, 2, \dots, J - 1$$

$$\Delta_W^Y = \frac{W^*}{YP} \left[\frac{1}{J} \sum_{\tau=0}^{J-1} \left(\frac{1}{\Pi g} \right)^{(1-\sigma)\tau} \right]^{\frac{1}{1-\sigma}} \quad (\text{A2.3.5})$$

$$\eta = \left(\frac{\beta}{g - \beta} \right) \left(\frac{1}{(\Delta^P)^{\sigma-1}} \right)^{\frac{1-\epsilon(1-\alpha)}{(\epsilon-1)(1-\alpha)}} \alpha^{\frac{1}{1-\alpha}} L \left[\frac{1}{I} \sum_{\tau=0}^{I-1} \left(\frac{P_{-\tau}^*}{P} \right)^{-\epsilon} \left(\frac{P_{-\tau}^*}{P} - 1 \right) \right] \quad (\text{A2.3.6})$$

$$\frac{C}{Y} = 1 - \alpha (\Delta^P)^{\epsilon-1} \left(\frac{P^*}{P} \right)^{-\epsilon} \left(\frac{1}{I} \sum_{\tau=0}^{I-1} \Pi^{\epsilon\tau} \right) - \frac{\eta(g-1)}{1+\rho} \left(\frac{\Delta^P}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} \frac{1}{L} \quad (\text{A2.3.7})$$

A2.4. Human capital model

$$\frac{W^*}{P} = \left[\left(\frac{\sigma}{\sigma-1} \right) \left(\left(\frac{\varepsilon}{\varepsilon-1} \right) \frac{\Delta_W^{1-\alpha\sigma}}{(1-\alpha)A} \right)^{\frac{1}{\alpha}} \frac{C}{K} \frac{\sum_{\tau=0}^{J-1} \beta^\tau N_\tau^{1+v}}{\sum_{\tau=0}^{J-1} \beta^\tau \Pi^{(\sigma-1)\tau}} \right]^{\frac{1}{1-\sigma}} \quad (\text{A2.4.1})$$

$$N_\tau = N^1 \text{ for } \tau = 0, 1, 2, \dots, J-2 \quad N_\tau = N^0 \text{ for } \tau = J-1$$

$$\frac{W_{-s}^*}{P} = \frac{1}{(\Pi)^s} \frac{W^*}{P} \quad s = 1, 2, \dots, J-1$$

$$\Delta_W = \frac{W^*}{P} \left[\frac{1}{J} \sum_{\tau=0}^{J-1} \left(\frac{1}{\Pi} \right)^{(1-\sigma)\tau} \right]^{\frac{1}{1-\sigma}} \quad (\text{A2.4.2})$$

$$\frac{C}{K} = \frac{A^{\frac{1}{\alpha}}}{\Delta^P} \left[\left(\frac{\varepsilon-1}{\varepsilon} \right) \frac{1-\alpha}{\Delta_W} \right]^{\frac{1-\alpha}{\alpha}} - g(C) - \delta \quad (\text{A2.4.3})$$

$$1 + g(C) = \frac{\beta}{(1+\delta) - \left[A \left(\frac{\varepsilon-1}{\varepsilon} \right) \right]^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\Delta_W} \right)^{\frac{1-\alpha}{\alpha}}} \quad (\text{A2.4.4})$$

$$N^1 = \frac{1}{\zeta} \left(1 - \frac{\beta/\Pi}{1+g(C)} \right) \quad (\text{A2.4.5})$$

$$N^0 = \frac{1}{\zeta} \left(1 - \frac{\beta \Pi^{J-1}}{1+g(C)} \right) \quad (\text{A2.4.6})$$

$$u^1 = \frac{[2 - (1+g(C))\Pi] - \frac{\beta/\Pi}{1+g(C)}}{1 - \frac{\beta/\Pi}{1+g(C)}} \quad (\text{A2.4.7})$$

$$u^0 = \frac{\left[2 - \frac{(1+g(C))N^0}{\Pi^{J-1}} \frac{N^1}{N^1} \right] - \frac{\beta \Pi^{J-1}}{1+g(C)}}{1 - \frac{\beta \Pi^{J-1}}{1+g(C)}} \quad (\text{A2.4.8})$$

$$u^{01} = \frac{\left[2 - (1+g(C))\Pi \frac{N^1}{N^0} \right] - \frac{\beta/\Pi}{1+g(C)}}{1 - \frac{\beta/\Pi}{1+g(C)}}$$

$$\frac{P^*}{P} = \frac{\sum_{\tau=0}^{J-1} (\beta \Pi^\varepsilon)^\tau}{\sum_{\tau=0}^{J-1} (\beta \Pi^{\varepsilon-1})^\tau} \quad (\text{A2.4.9})$$

$$\frac{P_s^*}{P} = \frac{1}{\Pi^s} \frac{P^*}{P} \quad s = 1, 2, \dots, l-1$$

$$\Delta^P = \frac{P^*}{P} \frac{1}{l} \left[\sum_{\tau=0}^{l-1} \left(\frac{1}{\Pi^\tau} \right)^{-\varepsilon} \right] \quad (\text{A2.4.10})$$

Appendix B

Appendix for the second chapter

B.1. Optimal control problem in the human capital model

Wage flexibility

The wage is the same for all types of labor services.

The Hamiltonian for this problem is:

$$\begin{aligned} H_{t+\tau} = & \beta^\tau \left[\log(C_{t+\tau}) - \frac{1}{1+\nu} \int_0^1 (N_{st+\tau})^{1+\nu} ds \right] \\ & + \lambda_{1,t+\tau} \left[D_{t+\tau} + (1-d_t) \int_0^1 \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) N_{st+\tau} ds + (R_{t+\tau} - \delta) K_{t+\tau} - C_{t+\tau} \right] \\ & + \lambda_{2,t+\tau} \left\{ \int_0^1 \xi (1 - u_{st+\tau} (1 - d_t)) N_{st+\tau} h_{st+\tau} \right\} \end{aligned}$$

subject to (1.5), (1.12), (1.14), (1.17), (1.18), (1.20), (1.21) and (1.22). The first-order conditions are given as follows:

$$(B1.1) \quad \frac{\beta^\tau}{C_{t+\tau}} = \lambda_{1,t+\tau}$$

$$(B1.2) \quad \beta^\tau N_{st+\tau}^\nu = \lambda_{1,t+\tau} (1 - d_t) \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) + \lambda_{2,t+\tau} \xi (1 - u_{st+\tau} (1 - d_t)) h_{st+\tau} \quad \forall i \\ \in [0,1]$$

$$(B1.3) \quad \lambda_{2,t+\tau} = \frac{\lambda_{1,t+\tau} W_{st+\tau}^*}{\xi P_{t+\tau}} \quad \forall s \in [0,1]$$

$$(B1.4) \quad \lambda_{1,t+\tau+1} - \lambda_{1,t+\tau} \\ = -\lambda_{1,t+\tau} (R_{t+\tau} - \delta) \\ - \lambda_{1,t+\tau} \int_0^1 (1 - d_t) \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) \left(\frac{W_{st+\tau}^*}{P_{t+\tau}^m} \right)^{-\sigma} \left[\frac{(1-\alpha)A}{(\Delta_{w,t+\tau}^i)^{1-\sigma\alpha}} \right]^{\frac{1}{\alpha}} di$$

$$\begin{aligned}
\text{(B1.5)} \quad & \lambda_{2,t+\tau+1} - \lambda_{2,t+\tau} \\
& = -\lambda_{1,t+\tau}(1-d_t) \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) u_{st+\tau} N_{st+\tau} \\
& \quad - \lambda_{2,t+\tau} \xi(1-u_{st+\tau}(1-d_t)) N_{st+\tau} \quad \forall s \in [0,1]
\end{aligned}$$

$$\begin{aligned}
\text{(B1.6)} \quad & K_{t+\tau+1} \\
& = D_{t+\tau} + \int_0^1 (1-d_t) \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) N_{i,t+\tau} (W_{i,t+\tau}^*) ds + (1+R_{t+\tau}-\delta)K_{t+\tau}
\end{aligned}$$

$$\text{(B1.7)} \quad h_{t+\tau+1} = \left\{ \int_0^1 [1 + \xi(1-u_{st+\tau}(1-d_t)) N_{st+\tau}] \frac{h_{st+\tau}}{h_{t+\tau}} ds \right\} h_{t+\tau}$$

In steady state, from B1:

$$\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g(\lambda_1) = \frac{\beta^{\tau+1} / C_{t+\tau+1}}{\beta^{\tau} / C_{t+\tau}} = \frac{\beta}{1 + g(C)}$$

From B1.3:

$$\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g(\lambda_2) = \frac{\beta}{1 + g(C)}$$

From B1.5:

$$\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} = 1 - \xi(1-d_t) N_{st+\tau} = \frac{\beta}{1 + g(C)}$$

The supply of labor is the same for all i and is constant over time. From this expression, the constant value of N_{ss} in steady state can be obtained

$$N_{ss} = \frac{1}{\xi(1-d)} \left(1 - \frac{\beta}{1 + g(C)} \right)$$

From B1.2:

$$\begin{aligned}
& \frac{\beta^{\tau+1} N_{st+\tau+1}^v}{\beta^{\tau} N_{st+\tau}^v} = \frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} \frac{h_{st+\tau+1}}{h_{st+\tau}} \\
& \beta = \frac{h_{st+\tau+1}}{h_{st+\tau}} \frac{\beta}{1 + g(C)} \Rightarrow g(h_s) = g(C)
\end{aligned}$$

The growth rate of human capital is the same as the consumption growth rate and the same for all s . We can see that from the accumulation process of human capital

$$h_{st+\tau+1} = h_{st+\tau} + \xi(1 - (1 - d_t)u_{st})N_{st+\tau}h_{st+\tau}$$

its growth rate is:

$$g(h_s) = g(C) = \xi(1 - (1 - d_t)u_{st})N_{st+\tau}$$

where u_{ss} is the steady-state value for any s . From this expression we can deduce that the value of u is also the same for all types of labor services and is constant over time:

$$u_{ss} = \frac{1}{1 - d_t} \left(1 - \frac{g(C)}{\xi N_{ss}} \right)$$

With those expressions the system of equations in steady state is closed.

Sticky wages

Note that the first-order condition for $u_{st+\tau}$ in (B1.3) implies that the real wage at time $t + \tau$ has to be the same across all individuals. However, since the nominal wage is expressed in terms of effective labor, the re-optimized real wage should be constant at the steady state, and therefore the nominal re-optimized wage grows at the same rate as the aggregate price. This implies that when the trend inflation is different from zero, there will be variations in the real wage across individuals. Obviously, this contradicts (B1.3). Then the previous problem is not valid with wage stickiness.

The Hamiltonian for this situation is:

$$\begin{aligned} H_{t+\tau} = & \beta^\tau \left[\log(C_{t+\tau}) - \frac{1}{1 + \nu} \int_0^1 (N_{st+\tau})^{1+\nu} ds \right] \\ & + \lambda_{1,t+\tau} \left[D_{t+\tau} + (1 - d_t) \int_0^1 \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) L_{i,t+\tau} \left(\frac{W_{st+\tau}^*}{P_{t+\tau}^m} \right) ds + (R_{t+\tau} - \delta)K_{t+\tau} \right. \\ & \left. - C_{t+\tau} \right] \end{aligned}$$

$$\begin{aligned}
& + \sum_{q=1}^{J-1} \lambda_{2,t+\tau}^{1q} \left\{ \int_{\frac{q-1}{J}}^{\frac{q}{J}} \xi(1 - u_{i,t+\tau}^{1q}(1 - d_t)) N_{st+\tau}^{1q} h_{st+\tau}^{1q} ds \right\} + \dots \\
& + \lambda_{2,t+\tau}^0 \left\{ \int_{\frac{J-1}{J}}^1 \xi(1 - u_{st+\tau}^0(1 - d_t)) N_{st+\tau}^0 h_{st+\tau}^0 ds \right\}
\end{aligned}$$

q=1, 2, ,J-1

subject to (1.5), (1.12), (1.14), (1.17), (1.18), (1.20), (1.21) and (1.22). The first-order conditions are given as follows:

$$(B1.8) \quad \frac{\beta^\tau}{C_{t+\tau}} = \lambda_{1,t+\tau}$$

$$\begin{aligned}
(B1.9) \quad & \beta^\tau N_{st+\tau}^v \\
& = \lambda_{1,t+\tau}(1 - d_t) \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) u_{st+\tau} h_{st+\tau} + \lambda_{2,t+\tau} \xi(1 \\
& - u_{st+\tau}(1 - d_t)) h_{st+\tau} \quad \forall s \in [0,1]
\end{aligned}$$

$$(B1.10.1) \quad \lambda_{2,t+\tau}^0 = \frac{\lambda_{1,t+\tau}}{\xi} \frac{W_{st+\tau}^0}{P_{t+\tau}} \quad \forall s \in \left[\frac{J-1}{J}, 1 \right]$$

$$(B1.10.2), (B1.10.3), \dots, (B1.10.J) \quad \lambda_{2,t+\tau}^{1q} = \frac{\lambda_{1,t+\tau}}{\xi} \frac{W_{st+\tau}^{1q}}{P_{t+\tau}} \quad \forall s \in \left[\frac{s-1}{J}, \frac{s}{J} \right]$$

$$q = 1, 2, \dots, J-1$$

$$\begin{aligned}
(B1.11) \quad & \lambda_{1,t+\tau+1} - \lambda_{1,t+\tau} \\
& = -\lambda_{1,t+\tau}(R_{t+\tau} - \delta) \\
& - \lambda_{1,t+\tau}(1 - d_t) \int_0^1 \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) \left(\frac{W_{st+\tau}^*}{P_{t+\tau}^i} \right)^{-\sigma} \left[\frac{(1-\alpha)A}{(\Delta_{w,t+\tau}^i)^{1-\sigma\alpha}} \right]^{\frac{1}{\alpha}} ds
\end{aligned}$$

$$\begin{aligned}
(B1.12.1) \quad & \lambda_{2,t+\tau+1}^0 - \lambda_{2,t+\tau}^0 \\
& = -\lambda_{1,t+\tau}(1 - d_t) \left(\frac{W_{st+\tau}^0}{P_{t+\tau}} \right) u_{st+\tau}^0 N_{st+\tau}^0 - \lambda_{2,t+\tau}^0 \xi(1 \\
& - u_{st+\tau}^0(1 - d_t)) N_{st+\tau}^0 \quad \forall s \in \left[\frac{J-1}{J}, 1 \right]
\end{aligned}$$

$$\begin{aligned}
& (B1.12.2), (B1.12.3), \dots, (B1.12.J) \quad \lambda_{2,t+\tau+1}^{1q} - \lambda_{2,t+\tau}^{1q} \\
& = -\lambda_{1,t+\tau}(1-d_t) \left(\frac{W_{st+\tau}^{1q}}{P_{t+\tau}} \right) u_{st+\tau}^{1q} N_{st+\tau}^{1q} - \lambda_{2,t+\tau}^{jq} \xi(1 \\
& - u_{st+\tau}^{1q}(1-d_t)) N_{st+\tau}^{1q} \quad \forall s \quad \in \left[\frac{q-1}{J}, \frac{q}{J} \right] \quad q = 1, 2, \dots, J-1
\end{aligned}$$

$$\begin{aligned}
(B1.13) \quad K_{t+\tau+1} \\
& = D_{t+\tau} + (1-d_t) \int_0^1 \left(\frac{W_{st+\tau}^*}{P_{t+\tau}} \right) L_{st+\tau}(W_{st+\tau}^*) di + (1+R_{t+\tau}-\delta)K_{t+\tau} \\
& - C_{t+\tau}
\end{aligned}$$

$$(B1.14) \quad h_{t+\tau+1} = \left\{ \int_0^1 [1 + \xi(1 - u_{st+\tau}(1-d_t))N_{st+\tau}] \frac{h_{st+\tau}}{h_{t+\tau}} di \right\} h_{t+\tau}$$

In steady state, from B1.8:

$$\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g(\lambda_1) = \frac{\beta^{\tau+1}/C_{t+\tau+1}}{\beta^{\tau}/C_{t+\tau}} = \frac{\beta}{1+g(C)}$$

From B1.10.2–B1.10.J (which represent labor services that do not change wages):

$$\frac{\lambda_{2,t+\tau+1}^{1q}}{\lambda_{2,t+\tau}^{1q}} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} \frac{1}{\Pi} = 1 + g(\lambda_2^1) = \frac{\beta/\Pi}{1+g(C)} \quad q = 1, 2, \dots, J-1$$

From B1.10.1 (which represents labor services that change wages):

$$\frac{\lambda_{2,t+\tau+1}^0}{\lambda_{2,t+\tau}^0} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} \Pi^{J-1} = 1 + g(\lambda_2^0) = \frac{\beta \Pi^{J-1}}{1+g(C)}$$

As a consequence, there will be two values of N . From B1.12:

$$\begin{aligned}
\frac{\lambda_{2,t+\tau+1}^{1q}}{\lambda_{2,t+\tau}^{1q}} = 1 - \xi(1-d)N_{it+\tau}^{1q} = 1 - \xi(1-d)N_{st+\tau}^{1q} = \frac{\beta/\Pi}{1+g(C)} \Rightarrow N^{1q} = \frac{1}{\xi(1-d)} \left(1 - \right. \\
\left. \frac{\beta/\Pi}{1+g(C)} \right) \quad q = 1, 2, \dots, J-2
\end{aligned}$$

$$\frac{\lambda_{2,t+\tau+1}^0}{\lambda_{2,t+\tau}^0} = 1 - \xi N_{st+\tau}^0 = \frac{\beta \Pi^{J-1}}{1+g(C)} \Rightarrow N^0 = \frac{1}{\xi(1-d)} \left(1 - \frac{\beta \Pi^{J-1}}{1+g(C)} \right)$$

From B1.9:

$$\frac{\beta^{\tau+1} N_{st+\tau+1}^v}{\beta^\tau N_{st+\tau}^v} = \frac{\lambda_{2,t+\tau+1} h_{st+\tau+1}}{\lambda_{2,t+\tau} h_{st+\tau}}$$

$$\beta = \frac{h_{st+\tau+1}}{h_{st+\tau}} \frac{\beta/\Pi}{1+g(C)} \Rightarrow g(C) = \frac{1+g(h^1)}{\Pi} - 1 \quad s = 2, 3, \dots, J-1$$

$$\beta \frac{N^0}{N^1} = \frac{h_{st+\tau+1}}{h_{st+\tau}} \frac{\beta \Pi^{J-1}}{1+g(C)} \Rightarrow g(C) = (1+g(h^0)) \frac{\Pi^{J-1}}{\frac{N^0}{N^1}} - 1 \quad s = 0$$

$$\beta \frac{N^1}{N^0} = \frac{h_{st+\tau+1}}{h_{st+\tau}} \frac{\beta/\Pi}{1+g(C)} \Rightarrow g(C) = \frac{(1+g(h^{01}))}{\Pi \frac{N^1}{N^0}} - 1 \quad s = 1$$

As a consequence, there will also be three expressions of u in steady state:

$$u^1 = \frac{[2 - (1+g(C))\Pi] - \frac{\beta/\Pi}{1+g(C)}}{1 - \frac{\beta/\Pi}{1+g(C)}} \quad s = 2, 3, \dots, J-1$$

$$u^0 = \frac{\left[2 - \frac{(1+g(C)) N^0}{\Pi^{J-1} N^1}\right] - \frac{\beta \Pi^{J-1}}{1+g(C)}}{1 - \frac{\beta \Pi^{J-1}}{1+g(C)}} \quad s = 0$$

$$u^{01} = \frac{\left[2 - (1+g(C))\Pi \frac{N^1}{N^0}\right] - \frac{\beta/\Pi}{1+g(C)}}{1 - \frac{\beta/\Pi}{1+g(C)}} \quad s = 1$$

B.2. Steady-state systems of equations

B2.1. Schumpeterian model

$$\frac{P^*}{P} = \frac{1}{\alpha} \frac{\sum_{\tau=0}^{I-1} (\beta \Pi^{1/1-\alpha})^\tau}{\sum_{\tau=0}^{I-1} (\beta \Pi^{\alpha/1-\alpha})^\tau} \quad (\text{B2.1.1})$$

$$\frac{P_{-s}^*}{P} = \frac{1}{\Pi^s} \frac{P^*}{P} \quad s = 1, 2, \dots, I-1 \quad (\text{B2.1.2})$$

$$g = \left[\chi \alpha^{\frac{1}{1-\alpha} L} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1 \right) \right]^{\frac{\chi}{1-\chi}} (\gamma - 1) + 1 \quad (\text{B2.1.3})$$

$$L = \frac{(1-\alpha)}{\Delta_W^Y} \quad (\text{B2.1.4})$$

$$L_{-s} = \left(\frac{(1-\alpha)L^{1-\sigma/\sigma}}{w_{-s}} \right)^\sigma \quad (\text{B2.1.5})$$

$$LL = \binom{1}{J} \sum_{s=0}^{J-1} L_{-s} \quad s = 0, 1, 2, \dots, J-1 \quad (\text{B2.1.6})$$

$$w = \frac{e\Delta_b \left[(4(R-1) + b\Delta_b + q\Delta_q) - \frac{q\Delta_q a\Delta_a}{(4(R-1) + a\Delta_a + b\Delta_b)} \right] + \frac{q\Delta_q (4(R-1) + b\Delta_b)}{(4(R-1) + a\Delta_a + b\Delta_b)} z\Delta_a}{\Delta_w^b \left[(4(R-1) + b\Delta_b + q\Delta_q) - \frac{q\Delta_q a\Delta_a}{(4(R-1) + a\Delta_a + b\Delta_b)} \right] - \Delta_w^{bq} (4(R-1) + b\Delta_b)} \quad (\text{B2.1.7})$$

$$\Delta_W^Y = \frac{w}{Y} \left[\frac{1}{J} \sum_{\tau=0}^{J-1} \left(\frac{1}{\Pi g} \right)^{\tau(1-\sigma)} \right]^{\frac{1}{1-\sigma}} \quad (\text{B2.1.8})$$

$$\left(\frac{w}{Y} \right)_{-s} = \frac{1}{(\Pi g)^s} \left(\frac{w}{Y} \right) \quad s = 0, 1, 2, \dots, J-1 \quad (\text{B2.1.9})$$

$$N_{-s} = \left(\frac{1}{C} (1-d)w_{-s} \right)^{1/\nu} \quad s = 0, 1, 2, \dots, J-1 \quad (\text{B2.1.10})$$

$$N = \binom{1}{J} \sum_{s=0}^{J-1} N_{-s} \quad (\text{B2.2.11})$$

$$d = \frac{N - LL}{N} \quad (\text{B2.1.12})$$

$$\frac{C}{Y} = 1 - \alpha^{\frac{1}{1-\alpha} L} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \frac{A}{Y} - \left[\chi \alpha^{\frac{1}{1-\alpha} L} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1 \right) \right]^{\frac{1}{1-\chi}} \frac{A}{Y} \quad (\text{B2.1.13})$$

$$\frac{A}{Y} = \frac{1}{\left(\alpha^{\frac{1}{1-\alpha}} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \right)^\alpha L} \quad (\text{B2.1.14})$$

$$\frac{R}{\Pi} = g\left(\frac{1}{\beta}\right) \quad (\text{B2.1.15})$$

B2.2. Human capital model

$$w = \frac{e\Delta_b \left[(4r + b\Delta_b + q\Delta_q) - \frac{q\Delta_q a\Delta_a}{(4r + a\Delta_a + b\Delta_b)} \right] + \frac{q\Delta_q (4r + b\Delta_b)}{(4r + a\Delta_a + b\Delta_b)} z\Delta_a}{\Delta_w^b \left[(4r + b\Delta_b + q\Delta_q) - \frac{q\Delta_q a\Delta_a}{(4r + a\Delta_a + b\Delta_b)} \right] - \Delta_w^{bq} (4r + b\Delta_b)} \quad (\text{B2.2.1})$$

$$w_{-s} = \frac{1}{(\Pi g)^s} w \quad s = 1, 2, \dots, J-1 \quad (\text{B2.2.2})$$

$$\Delta_w = w \left[\frac{1}{J} \sum_{\tau=0}^{J-1} \left(\frac{1}{\Pi} \right)^{(1-\sigma)\tau} \right]^{\frac{1}{1-\sigma}} \quad (\text{B2.2.3})$$

$$\frac{C}{\bar{K}} = \frac{A^{\frac{1}{\alpha}}}{\Delta^P} \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{1 - \alpha}{\Delta_w} \right]^{\frac{1-\alpha}{\alpha}} - g(C) - \delta \quad (\text{B2.2.4})$$

$$1 + g(C) = \frac{\beta}{(1 + \delta) - \left[A \left(\frac{\varepsilon - 1}{\varepsilon} \right) \right]^{\frac{1}{\alpha}} \left(\frac{1 - \alpha}{\Delta_w} \right)^{\frac{1-\alpha}{\alpha}}} \quad (\text{B2.2.5})$$

$$N^0 = \frac{1}{\xi} \left(1 - \frac{\beta \Pi^{J-1}}{1 + g(C)} \right) \quad s = J-1 \quad (\text{B2.2.6})$$

$$N^1 = \frac{1}{\xi} \left(1 - \frac{\beta/\Pi}{1 + g(C)} \right) \quad s = 0, 1, 2, \dots, J-2 \quad (\text{B2.2.7})$$

$$N = \left(\frac{1}{J} \right) (N^0 + (J-1)N^1) \quad (\text{B2.2.8})$$

$$u^1 = \frac{[2 - (1 + g(C))\Pi] - \frac{\beta/\Pi}{1 + g(C)}}{1 - \frac{\beta/\Pi}{1 + g(C)}} \quad s = 2, \dots, J-1 \quad (\text{B2.2.9})$$

$$u^0 = \frac{\left[2 - \frac{(1 + g(C))N^0}{\Pi^{J-1}} \frac{N^1}{N^1} \right] - \frac{\beta \Pi^{J-1}}{1 + g(C)}}{1 - \frac{\beta \Pi^{J-1}}{1 + g(C)}} \quad s = 0 \quad (\text{B2.2.10})$$

$$u^{01} = \frac{\left[2 - (1 + g(C))\Pi \frac{N^1}{N^0} \right] - \frac{\beta/\Pi}{1 + g(C)}}{1 - \frac{\beta/\Pi}{1 + g(C)}} \quad s = 1 \quad (\text{B2.2.11})$$

$$\frac{P^*}{P} = \frac{\sum_{\tau=0}^{J-1} (\beta \Pi^\varepsilon)^\tau}{\sum_{\tau=0}^{J-1} (\beta \Pi^{\varepsilon-1})^\tau} \quad (\text{B2.2.12})$$

$$\frac{P_{-s}^*}{P} = \frac{1}{\Pi^s} \frac{P^*}{P} \quad s = 1, 2, \dots, I - 1 \quad (\text{B2.2.13})$$

$$\Delta_P = \frac{P^*}{P} \frac{1}{I} \left[\sum_{\tau=0}^{I-1} \left(\frac{1}{\Pi^\tau} \right)^{-\varepsilon} \right] \quad (\text{B2.2.14})$$

$$R = 2 + \delta - \frac{\beta}{1 + g(C)} \quad (\text{B2.2.15})$$

$$L^0 = (1 - d)u^0 N^0 \quad (\text{B2.2.16})$$

$$L^1 = (1 - d)u^1 N^1 \quad (\text{B2.2.17})$$

$$L^{01} = (1 - d)u^{01} N^1 \quad (\text{B2.2.18})$$

$$LL = \binom{1}{J} (L^0 + (J - 2)L^1 + L^{01}) \quad (\text{B2.2.19})$$

$$d = \frac{N - LL}{N} \quad (\text{B2.2.20})$$

Appendix C

Appendix for the third chapter

C.1. Steady-state system of equations. Schumpeterian model

$$\frac{P^*}{P} = \frac{1}{\alpha} \frac{\sum_{\tau=0}^{l-1} (\beta(\Pi)^{1/1-\alpha})^\tau}{\sum_{\tau=0}^{l-1} (\beta(\Pi)^{\alpha/1-\alpha})^\tau} \quad (C1.1)$$

$$\frac{P_{-s}^*}{P} = \frac{1}{\Pi^s} \frac{P^*}{P} \quad s = 1, 2, \dots, l-1 \quad (C1.2)$$

$$g = \left[\frac{\chi}{(1 + R_t^k)} \alpha^{\frac{1}{1-\alpha}} L \frac{1}{l} \sum_{s=0}^{l-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P_{-s}^*}{P} - 1 \right) \right]^{\frac{\chi}{1-\chi}} (\gamma - 1) + 1 \quad (C1.3)$$

$$L_t = \frac{(1 - \alpha)}{(1 + R_t^k)} \frac{Y_t}{\Delta_t^w} \quad (C1.4)$$

$$L_{-st} = \frac{(1 - \alpha)}{(1 + R_t^k)} \frac{L_t^{\frac{1-\sigma}{\sigma}}}{W_{-st}} \quad (C1.5)$$

$$LL = \frac{1}{J} \sum_{s=0}^{J-1} L_{-s} \quad s = 0, 1, 2, \dots, J-1 \quad (C1.6)$$

$$w = \frac{e\Delta_b \left[(4(R-1) + b\Delta_b + q\Delta_q) - \frac{q\Delta_q a\Delta_a}{(4(R-1) + a\Delta_a + b\Delta_b)} \right] + \frac{q\Delta_q (4(R-1) + b\Delta_b)}{(4(R-1) + a\Delta_a + b\Delta_b)} z\Delta_a}{\Delta_w^b \left[(4(R-1) + b\Delta_b + q\Delta_q) - \frac{q\Delta_q a\Delta_a}{(4(R-1) + a\Delta_a + b\Delta_b)} \right] - \Delta_w^{bq} (4(R-1) + b\Delta_b)} \quad (C1.7)$$

$$\Delta_w^y = \frac{w}{Y} \left[\frac{1}{J} \sum_{\tau=0}^{J-1} \left(\frac{1}{\Pi g} \right)^{\tau(1-\sigma)} \right]^{\frac{1}{1-\sigma}} \quad (C1.8)$$

$$\left(\frac{w}{Y} \right)_{-s} = \frac{1}{(\Pi g)^s} \left(\frac{w}{Y} \right) \quad s = 0, 1, 2, \dots, J-1 \quad (C1.9)$$

$$N_{st} = \left(\frac{1}{C_t} (1 - d_{st}) w_{st} \right)^{1/\nu} \quad (C1.10)$$

$$N = \sum_{s=0}^{J-1} N_{-s} \quad (C1.11)$$

$$d = \frac{N - LL}{N} \quad (C1.12)$$

$$\frac{C}{Y} = 1 - \left(\frac{\alpha}{1 + R^k}\right)^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_s^*}{P}\right)^{-\frac{1}{1-\alpha}} \frac{A}{Y} - \left[\frac{\chi}{(1 + R_t^k)} \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_s^*}{P}\right)^{-\frac{1}{1-\alpha}} \left(\frac{P_s^*}{P} - 1\right) \right]^{\frac{1}{1-\chi}} \frac{A}{Y} \quad (C1.13)$$

$$\frac{A}{Y} = \frac{1}{\left(\left(\frac{\alpha}{1 + R^k}\right)^{\frac{1}{1-\alpha}} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_s^*}{P}\right)^{-\frac{1}{1-\alpha}} \right)^\alpha L} \quad (C1.14)$$

$$\frac{R}{\Pi} = g\left(\frac{1}{\beta}\right) \quad (C1.15)$$

$$v_{t+1} = [v - (1 - \gamma)(R^k - R)] + \frac{1}{\gamma} \beta G(S) \quad (C1.16)$$

$$\eta_{t+1} = [\eta - (1 - \gamma)R] + \frac{1}{\gamma} \beta G(T) \quad (C1.17)$$

$$\emptyset = \frac{\eta}{\lambda - v} \quad (C1.18)$$

$$G(T) = \frac{T_{jt+1}}{T_{jt}} = (R^k - R)\emptyset + R \quad (C1.19)$$

$$G(T) = G(S) \quad (C1.20)$$

$$G(T) = [\gamma(R^k - R)\emptyset + R] + \psi R\emptyset \quad (C1.21)$$

$$\emptyset \frac{F}{Y} = \frac{A}{Y} \left[\frac{\chi}{(1 + R_t^k)} \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_s^*}{P}\right)^{-\frac{1}{1-\alpha}} \left(\frac{P_s^*}{P} - 1\right) \right]^{\frac{1}{1-\chi}} \quad (C1.22)$$

C.2. Steady-state system of equations. Human capital model

$$w = \frac{e\Delta_b \left[(4R + b\Delta_b + q\Delta_q) - \frac{q\Delta_q a\Delta_a}{(4R + a\Delta_a + b\Delta_b)} \right] + \frac{q\Delta_q (4R + b\Delta_b)}{(4R + a\Delta_a + b\Delta_b)} z\Delta_a}{\Delta_w^b \left[(4R + b\Delta_b + q\Delta_q) - \frac{q\Delta_q a\Delta_a}{(4R + a\Delta_a + b\Delta_b)} \right] - \Delta_w^{bq} (4R + b\Delta_b)} \quad (C2.1)$$

$$w_{-s} = \frac{1}{(\Pi)^s} w \quad s = 1, 2, \dots, J-1 \quad (C2.2)$$

$$\Delta_w = w \left[\frac{1}{J} \sum_{\tau=0}^{J-1} \left(\frac{1}{\Pi} \right)^{(1-\sigma)\tau} \right]^{\frac{1}{1-\sigma}} \quad (C2.3)$$

$$\frac{C}{K} = A^{\frac{1}{\alpha}} \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{1 - \alpha}{(1 + R_t^k)} \frac{1}{\Delta_t^w} \right]^{\frac{1-\alpha}{\alpha}} - g(C) - \delta \quad (C2.4)$$

$$1 + g(C) = \frac{\beta}{(1 + \delta) - \left[\frac{A}{(1 + R_t^k)} \left(\frac{\varepsilon - 1}{\varepsilon} \right) \right]^{\frac{1}{\alpha}} \left[\frac{(1 - \alpha)}{\Delta_t^w} \right]^{\frac{1-\alpha}{\alpha}}} \quad (C2.5)$$

$$N^0 = \frac{1}{\xi} \left(1 - \frac{\beta \Pi^{J-1}}{1 + g(C)} \right) \quad s = J-1 \quad (C2.6)$$

$$N^1 = \frac{1}{\xi} \left(1 - \frac{\beta/\Pi}{1 + g(C)} \right) \quad s = 0, 1, 2, \dots, J-2 \quad (C2.7)$$

$$N = (1/J)(N^0 + (J-1)N^1) \quad (C2.8)$$

$$u^1 = \frac{[2 - (1 + g(C))\Pi] - \frac{\beta/\Pi}{1 + g(C)}}{1 - \frac{\beta/\Pi}{1 + g(C)}} \quad s = 2, \dots, J-1 \quad (C2.9)$$

$$u^0 = \frac{\left[2 - \frac{(1 + g(C))N^0}{\Pi^{J-1}} \frac{N^1}{N^1} \right] - \frac{\beta \Pi^{J-1}}{1 + g(C)}}{1 - \frac{\beta \Pi^{J-1}}{1 + g(C)}} \quad s = 0 \quad (C2.10)$$

$$u^{01} = \frac{\left[2 - (1 + g(C))\Pi \frac{N^1}{N^0} \right] - \frac{\beta/\Pi}{1 + g(C)}}{1 - \frac{\beta/\Pi}{1 + g(C)}} \quad s = 1 \quad (C2.11)$$

$$\frac{P^*}{P} = \frac{\sum_{\tau=0}^{I-1} (\beta \Pi^\varepsilon)^\tau}{\sum_{\tau=0}^{I-1} (\beta \Pi^{\varepsilon-1})^\tau} \quad (\text{C2.12})$$

$$\frac{P_{-s}^*}{P} = \frac{1}{\Pi^s} \frac{P^*}{P} \quad s = 1, 2, \dots, I-1 \quad (\text{C2.13})$$

$$\Delta_P = \frac{P^*}{P} \frac{1}{I} \left[\sum_{\tau=0}^{I-1} \left(\frac{1}{\Pi^\tau} \right)^{-\varepsilon} \right] \quad (\text{C2.14})$$

$$R = 2 + \delta - \frac{\beta}{1 + g(C)} \quad (\text{C2.15})$$

$$(1 + R^k) = [1 + R(1 + prima)] \quad (\text{C2.16})$$

$$prima = \frac{\mu G(w^*)}{[\Gamma(w^*) - G(w^*)]} R^* \quad (\text{C2.17})$$

$$L^0 = (1 - d)u^0 N^0 \quad (\text{C2.18})$$

$$L^1 = (1 - d)u^1 N^1 \quad (\text{C2.19})$$

$$L^{01} = (1 - d)u^{01} N^1 \quad (\text{C2.20})$$

$$LL = \left(\frac{1}{J}\right)(L^0 + (J - 2)L^1 + L^{01}) \quad (\text{C2.21})$$

$$d = \frac{N - LL}{N} \quad (\text{C2.22})$$