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Analysis and Modeling of the Forces Exerted on the Cookware in Induction Heating Applications

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ABSTRACT We present a semianalytical model for calculating the forces exerted on cookware in domestic induction heating applications. The developed model is based on the Maxwell's stress tensor and is also based on the existing semianalytic expressions of the electromagnetic fields in planar induction heating systems, which are expressed in terms of Fourier-Bessel series. Taking advantage of the axial symmetry of usual domestic induction heating systems, the flux of the vertical component of the Maxwell's stress tensor is analytically integrated and the vertical force is obtained. The proposed model captures both eddy currents and magnetization that occurs in typical ferromagnetic cookware. The model is verified by means of two-dimensional Finite Element simulations and also is tested by means of measurements of the change of the weight experimented by cookware due to the forces during the heating process.

INDEX TERMS Induction heating, appliances, electromagnetic forces, Maxwell's stress tensor, electromagnetic properties.

I. INTRODUCTION

Induction heating cooktops have progressively gained the interest of users due to its intrinsic advantages, as contactless heat transfer, automatic cookware detection, fast operation and high efficiency, which outperforms the traditional cooking stoves [1]. Currently, induction cooktops are technologically advanced products whose mentioned advantages compensate its extra cost with respect to gas or radiant burners. The research and development efforts conducted in the last years have been focused on the enabling technologies of induction cooktops: magnetics [2]–[7], power electronics converters [8]–[10], digital controllers, modelling and control [11]–[13]. These efforts have been reflected in more accurate control, safer operation and faster appliances.

The high efficiency achieved with induction cookers and ferromagnetic pots is attracting a considerable number of users concerned by environmental aspects. In a domestic induction heating process the heat is generated in the workpiece. This is efficient by nature because neither external heat sources nor heat transmission media are required. For this reason, in current arrangements the efficiency of the energy transfer can be up to 97.5% for the case of ferromagnetic cookware [14]. In the case of non-ferromagnetic materials (copper or aluminum) efficiencies comprised between $60\% - 70\%$ can be reached.

Currently, the approach of manufacturers is moving towards the improvement of the cooking experience of users [15], [16]. In this way, advanced induction heating appliances incorporate innovations, as connectivity to other devices, self-guided helping for preparing recipes, and automatic cooking functionalities. In general, the two last innovations require measurements of the weight and the temperature of the pot during the cooking process. However, due to the inductive nature of heating, the weight measurement is more complex than temperature measurement because the pot is under electromagnetic forces which depend on several aspects as power level, materials and size of the cookware, and the inductor system arrangement. Therefore, accurate weight measurements should take into account the force

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between the cookware and the inductor system. Literature has echoed this issue and some contributions have been presented in order to reduce or optimize the mentioned forces [17], [18].

Usually, the analysis and modeling of domestic induction heating systems has been inspired by other related applications. However, the application of related models to this case is not, in general, straightforward. Industrial induction heating applications have the same nature of domestic induction heating. However, industrial induction heating presents several differences with respect to domestic applications. In industrial applications the coil geometry is usually solenoidal, whereas domestic applications uses flat coils. Moreover, non-ferromagnetic materials are mainly used as workpieces in industrial applications, and the phenomenology associated to the magnetic permeability of the ferromagnetic cookware is not present. The nature of the electromechanical force in cookware an the forces existing in electrical machines is essentially the same. However, domestic induction heating applications presents two distinct characteristics with respect to the electrical machines. The first characteristic is due to the properties of usual cookware materials, which combine both electrical conductivity and magnetic permeability. Consequently, the cookware is simultaneously the path of the magnetic flux the path of the induced currents. However, in electrical machines flux and current paths are usually separated in the core (yoke) and the windings, and eddy currents are tried to be avoided in cores by using different strategies. Second, operating frequencies are usually orders of magnitude higher in induction heating than electrical machines.

This work is aimed at obtained an analytical model of the forces in domestic induction heating applications. For this purpose, models existing in the literature are first reviewed. In general, two main approaches have been followed for calculating forces in electromechanical applications: finite element (FEA) simulations and analytical models. FEA approaches have been extensively used for analyzing both electrical machines [19]–[22] and industrial induction heating applications [23]–[26]. Considering domestic induction applications and FEA approaches, a preliminary analysis of forces on the cookware was presented in [27]. Results pointed that the total force has two components, in opposite directions, whose value depends on the electrical conductivity and the magnetic permeability of the cookware. A FEA approach is also followed in the above mentioned references [17], [18] The first reference proposes a new coil structure for reducing the force and the second reference proposes a coil structure for optimizing the force in a levitating induction cooker. In both works, two-dimensional (2D) finite element simulations are used for obtaining the forces on the cookware.

Analytical or semianalytical models are effective for study the effect of different parameters on the electromagnetic magnitudes of a system. Generally, in analytical approaches the field magnitudes are obtained from the magnetic scalar or vector potentials and, subsequently, parameters as impedance, emf or forces are calculated by integrating or by means of the Maxwell's stress tensor. In general, an analytical model can only be obtained if the system under study presents symmetry because it allows to simplify the problem. In the case of electrical machines, some analytical models for calculating forces were proposed by applying symmetry [28]–[33]. Regarding industrial applications, analytical models were proposed in the past for obtaining the forces in elongated systems with cylindrical symmetry [34], [35]. However, similar models for planar induction systems are not found in the current literature.

Fig. 1 illustrates a typical arrangement of a domestic induction system. This system basically has cylindrical symmetry and this characteristic was used in the past to derive a semianalytical model of the fields [36], which allowed to obtain a model of the equivalent impedance [37] and the inductive efficiency [14]. At the interest frequency range (from 20 kHz to 100 kHz) the equivalent impedance consists of the series connection of equivalent resistor and inductor, whose values depend on the characteristics of the system and the operating frequency. The resistance represents the power dissipation in the different elements (cookware, windings, shielding) and the inductance represents the existing magnetic field in the system. The equivalent impedance can be obtained by means of both FEA approaches or analytical models. Ferrite bars improve the coupling between the windings and the cookware and they play a similar role of the core in inductors and transformers. Despite Fig. 1 shows a 3D system, its high symmetry has favoured the analysis by means 2D simulations or analytical solutions because both strategies reduce the development time and the required computational resources. However, the suitability of 2D approaches should be confirmed by means of experiments.

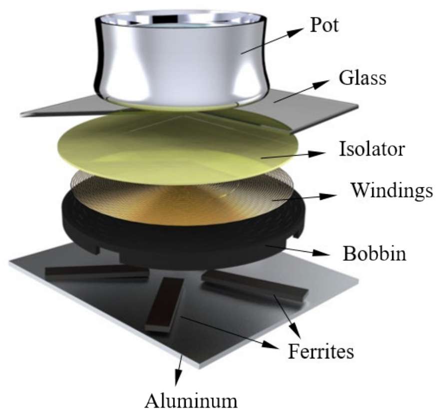

FIGURE 1. Basic induction heating system comprising the planar windings, ferrites, aluminum shielding and vessel.

Considering this previous semianalytical solution, in this paper a model of the force exerted on the cookware heated by induction is contributed with respect to previous works. The model is verified by means of a double test. First, 2D Finite Element (FEA) simulations, and second, experiments where the weight of a cookware during a cooking process is recorded by means of several load cells and a data logger.

The paper is organized as follows. The field solution for a multilayer induction system is adapted to this case en Section II. In Section III the semianalytical model of forces is derived. Section IV presents the FEA verification of the model, and some experimental tests are described in Section V. Finally, some conclusions are summarized in Section VI.

II. MAGNETIC FIELD DERIVATION

The fields involved in the induction system of Fig. 1 can be obtained by particularizing the semianalytical model of a planar coil between two multilayer media [36]. In this model the vector potential was obtained by solving the Poisson's equation and applying the Fourier-Bessel integral transform method. Therefore, the vector potential is expressed as definite integrals which are numerically solved.

The assumptions adopted to obtain the vector potential are as follows:

- Axial symmetry.
- Filamentary currents for the coil.
- Semiinfinite media in the horizontal dimension.
- Linear, homogeneous and isotropic materials. Therefore, saturation of ferromagnetic materials is not considered.
- Magneto-quasi-static approximation at the interest frequency range, from 20 kHz to 100 kHz.

The first four assumptions are requirements for obtaining an analytical solution. The magneto-quasi static approximation is valid if the dimensions of the system are lesser than the wavelength of the field at the higher frequency range. In this case, considering 100 kHz, the corresponding electromagnetic wavelength is about 3 km, whereas the typical dimension of cookers is about tens of centimeters. Linearity of the materials can be assumed if the temperature reached in the cooking process is below the Curie's temperature. The assumptions imposed by the analytical solution will be validated by means of an experimental verification.

The geometrical model for obtaining the fields of the induction system is presented in Fig. 2. In this model, the coil is placed at $z = 0$, and it consists of n concentric filamentary currents of value $\hat{I}e^{j\omega t}$, where $\hat{I}$ is the amplitude and ω the angular frequency. Filamentary currents are placed between two multilayer media representing the air, the ferrite bars, the aluminum shielding and the pot. Ferrite bars and the existing air between them are modeled by means of a lossless flux concentrator layer with the same thickness of the bars and an equivalent magnetic relative permeability. The equivalent relative permeability is lesser than the relative permeability of ferrite bars and it is in between the relative permeability of the ferrite (about 1000) and the relative permeability of the air. The equivalent relative permeability is proportional to the volume occupied by ferrite bars with respect to the volume of a disk whose external radius coincides with the most external face of ferrites. At the interest frequency range the dissipative

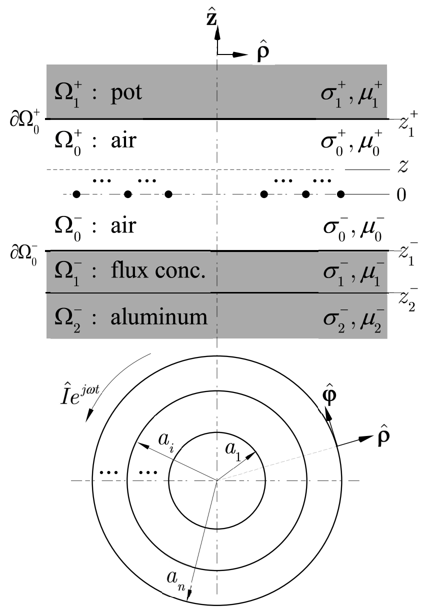

FIGURE 2. Geometry of the resolution model of the induction heating system.

media (i.e. the pot and the aluminum) can be considered as semiinfinite in the vertical direction due to the small skin depth of the fields compared with the usual thickness of these elements. At the lowest frequency of interest (i.e. 20 kHz) the skin depth of a typical ferromagnetic pot ($\sigma = 3 \cdot 10^6$ [S/m], $\mu_r = 250$) is 130 μm, and the skin depth of a typical aluminum ($\sigma = 2 \cdot 10^7$ [S/m], $\mu_r = 1$) is 800 μm. In contrast the thicknesses of these elements in typical applications can range from four or five millimeters up one or two centimeters.

Considering a point placed at $\mathbf{r}(\rho, z)$ of the domain Ω_0^+, the vector potential is the sum of two contributions: the first, $\mathbf{A}_c$, corresponds to the coil in the air, and the second, $\Delta\mathbf{A}_m$, corresponds to the effect of the top and bottom multilayer media. The total magnetic vector potential at any point of the domain Ω_0^+ is:

$$\mathbf{A}(\rho, z) = \mathbf{A}_c + \Delta\mathbf{A}_m \tag{1}$$

These contributions are, respectively [36]:

$$\mathbf{A}_c(\rho, z) = \frac{\mu_0 \hat{I}}{2} \int_0^\infty e^{-\beta z} J_1(\beta\rho)\, G(\beta a)\, d\beta \tag{2}$$

$$\Delta\mathbf{A}_m(\rho, z) = \frac{\mu_0 \hat{I}}{2} \int_0^\infty \Phi_A(z, \beta)\, J_1(\beta\rho)\, G(\beta a)\, d\beta \tag{3}$$

where β is the integration variable of the Fourier-Bessel transform, J_1 is the Bessel function of first class and order 1. The function $G(\beta a_i)$ of (2) and (3) accounts for radii of the filamentary turns, and is defined as follows:

$$G(\beta a) = \sum_{i=1}^{n} J_1(\beta a_i) \tag{4}$$

The function $\Phi_A(z, \beta)$ includes the dependency of the vector potential with respect to the top and bottom multilayer properties and thicknesses. This function is defined as follows:

$$\Phi_A(\beta, z) = \frac{\phi_t\left(e^{-\beta d_t} + \phi_b e^{-\beta(d_t - 2d_b)}\right)e^{-\beta(d_t - z)}}{1 - \phi_t\phi_b e^{-2\beta(d_t - d_b)}} + \frac{\phi_b\left(e^{-\beta d_b} + \phi_t e^{-\beta(2d_t - d_b)}\right)e^{-\beta(z - d_b)}}{1 - \phi_t\phi_b e^{-2\beta(d_t - d_b)}} \quad (5)$$

In this equation the following characteristic distances are used:

$$d_t = z_1^+$$
$$d_b = z_1^- \quad (6)$$

Functions ϕ_t and ϕ_b of (5) are characteristic of the properties and arrangement of the top and bottom multilayer media. In order to define these functions, the following parameter is required:

$$\eta_k^{\pm} = \left(\beta^2 + j\omega\sigma_k^{\pm}\mu_k^{\pm}\right)^{\frac{1}{2}} \quad (7)$$

where $\sigma_k^{\pm}$, $\mu_k^{\pm}$ are the electrical conductivity and magnetic permeability of the k^{th}−layer as it is shown in Fig. 2. Moreover, the thickness of the flux concentrator layer is defined as $t_f = z_1^- - z_2^-$. According to these definitions and the results of [36], for this particular case:

$$\phi_t = \frac{\beta\mu_{r1}^+ - \eta_1^+}{\beta\mu_{r1}^+ + \eta_1^+} \quad (8)$$

and

$$\phi_b = \frac{\left(\mu_{r1}^- - 1\right)\left(\beta + \mu_{r1}^-\eta_2^-\right)e^{-\beta t_f} + \left(\mu_{r1}^- + 1\right)\left(\beta - \mu_{r1}^-\eta_2^-\right)e^{\beta t_f}}{\left(\mu_{r1}^- + 1\right)\left(\beta + \mu_{r1}^-\eta_2^-\right)e^{-\beta t_f} + \left(\mu_{r1}^- - 1\right)\left(\beta - \mu_{r1}^-\eta_2^-\right)e^{\beta t_f}} \quad (9)$$

Considering (1), the electrical and magnetic field components are obtained by means of:

$$E_\varphi(\rho, z) = -j\omega A_\varphi = -\frac{j\omega\mu_0\hat{I}}{2}\int_0^\infty \left[e^{-\beta z} + \Phi_A(\beta, z)\right]J_1(\beta\rho)\,G(\beta a)\,d\beta \quad (10)$$

$$H_z(\rho, z) = \frac{1}{\mu_0}\frac{1}{\rho}\partial_\rho\left(\rho A_\varphi\right) = \frac{\hat{I}}{2}\int_0^\infty \beta\left[e^{-\beta z} + \Phi_A(\beta, z)\right]J_0(\beta\rho)\,G(\beta a)\,d\beta \quad (11)$$

and

$$H_\rho(\rho, z) = \frac{1}{\mu_0}\partial_z A_\varphi = -\frac{\hat{I}}{2}\int_0^\infty \beta\left[e^{-\beta z} - \Phi_B(\beta, z)\right]J_1(\beta\rho)\,G(\beta a)\,d\beta \quad (12)$$

where

$$\Phi_B(\beta, z) = \frac{\phi_t\left(e^{-\beta d_t} + \phi_b e^{-\beta(d_t - 2d_b)}\right)e^{-\beta(d_t - z)}}{1 - \phi_t\phi_b e^{-2\beta(d_t - d_b)}} - \frac{\phi_b\left(e^{-\beta d_b} + \phi_t e^{-\beta(2d_t - d_b)}\right)e^{-\beta(z - d_b)}}{1 - \phi_t\phi_b e^{-2\beta(d_t - d_b)}} \quad (13)$$

The precedent fields are used for calculating the force in the cookware. Moreover, these fields can be also used for obtaining the delivered power by means of the Poynting's vector, $\mathbf{S} = \mathbf{E} \times \mathbf{H}$, which represents the electromagnetic energy density flowing through a surface. In this case, the power density at the bottom of the cookware is:

$$p(\rho, z) = E_\varphi(\rho, z) \times H_\rho(\rho, z) \quad (14)$$

The power density is not uniformly distributed at the bottom of the cookware and this distribution is determined by the geometry of planar coils with equally-spaced turns. Fig. 3 shows the typical power density distribution at the bottom of the pot for the mentioned case of and equally-spaced turn coil. This fact is also reflected in the distribution of the vertical force.

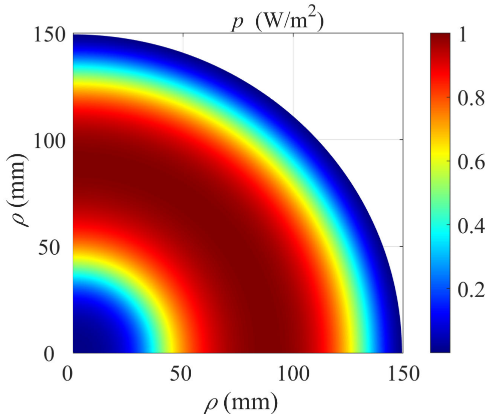

FIGURE 3. Power density distribution at the bottom of the cookware.

III. ANALYTICAL CALCULATION OF VERTICAL FORCES ON THE COOKWARE

Skin depth of varying fields in a material ($\delta = \sqrt{2/\omega\sigma\mu}$) is determinant in the induction heating process. The inductive heating of ferromagnetic cookware is mainly due to intense induced currents, which are confined at the bottom surface in a layer of tens of microns. Additionally, magnetic hysteresis and its corresponding dissipation also occurs in the layer penetrated by the fields. Consequently, the induction heating of ferromagnetic materials combines both current density and magnetization, and these phenomena interact with the field created by the coil generating two opposite forces: repulsive force caused by the interaction between the driven and the induced current, and attractive force due to magnetization of the material. In this case, the Maxwell's stress tensor is chosen for calculating the net force. This option shows some advantages, for example, current densities are not involved in calculations and only fields $\mathbf{E}$ and $\mathbf{H}$ are required.

In this case the Maxwell 's stress tensor is a 3 × 3 rank-2 tensor whose ij^{th} element is given by:

$$\mathbb{T}_{ij}(\rho, z) = \varepsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij}E^2 \right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij}B^2 \right) \quad i, j = \rho, z, \varphi \tag{15}$$

where δ_{ij} is the Kronecker delta. Considering in this case sinusoidal sources, the average value of the $\mathbb{T}_{zz}$ in a point of the domain Ω_0^+ is:

$$\mathbb{T}_{zz\_avg}(\rho, z) = \frac{1}{4}\left[\varepsilon_0 \left(-E_\varphi E_\varphi^* \right) + \mu_0 \left(H_z H_z^* - H_r H_r^* \right) \right] \tag{16}$$

where the asterisk denotes a complex conjugated magnitude. The following expression connects the stress tensor and the force density:

$$\mathbf{f} + \underbrace{\varepsilon_0 \mu_0 \frac{\partial \mathbf{S}}{\partial t}}_{0} = \nabla \cdot \mathbb{T} \tag{17}$$

where the term dependent of the Poynting's vector is neglected due to adopted magneto-quasi-static approximation. The force is obtained by integrating the force density at the interest domain volume, or alternatively, applying the divergence theorem, the force is obtained by integrating the stress tensor at the bounding surface. This surface is denoted as $\partial\Omega_0^+$ in Fig. 2. Therefore, the average vertical force exerted on the cookware is:

$$F_{z\_avg} = \int_{\Omega_0^+} f_{z\_avg} dv = \int_{\partial\Omega_0^+} \mathbb{T}_{zz\_avg} ds \tag{18}$$

Considering (16), the force calculation involves the surface integrals of the product of a field and its conjugate at the points $(\rho, z = d_t)$. Therefore the aspect of these integrals is:

$$2\pi \int_0^\infty M_\alpha(\rho, d_t) M_\alpha^*(\rho, d_t) \cdot \rho \cdot d\rho \tag{19}$$

where M_α can be E_φ, H_z or H_ρ. Taking into account the field definitions of (10), (11) and (12), the conjugate applies only to functions $\Phi_A(\beta, d_t)$ and $\Phi_B(\beta, d_t)$. Moreover, the Fourier-Bessel integration variables β and β' are adopted for a field and its conjugate, respectively. Additionally, considering the so called *closure equations* of integrals involving products of Bessel functions [38]:

$$\int_0^\infty \rho J_n(\rho\beta) J_n(\rho\beta') d\rho = \frac{1}{\beta}\delta(\beta - \beta') \quad n = 0, 1 \tag{20}$$

where $\delta(\beta - \beta')$ is the Dirac delta function, the following results are obtained:

$$2\pi \int_0^\infty E_\varphi(\rho, d_t) E_\varphi^*(\rho, d_t) \rho d\rho = -\pi \mu_0^2 \omega^2 \frac{\hat{I}^2}{2} \times \int_0^\infty \frac{1}{\beta}\left[e^{-\beta d_t} + \Phi_A(\beta, d_t) \right] \times \left[e^{-\beta d_t} + \Phi_A^*(\beta, d_t) \right] G^2(\beta a) d\beta \tag{21}$$

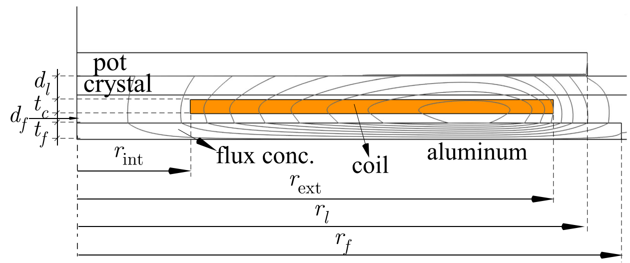

FIGURE 4. 2D simulation of the induction heating system. The current density of the coil and flux density lines are represented for illustrative purposes.

$$2\pi \int_0^\infty H_z(\rho, d_t) H_z^*(\rho, d_t) \rho d\rho = \pi \frac{\hat{I}^2}{2} \times \int_0^\infty \beta \left[e^{-\beta d_t} + \Phi_A(\beta, d_t) \right] \times \left[e^{-\beta d_t} + \Phi_A^*(\beta, d_t) \right] G^2(\beta a) d\beta \tag{22}$$

$$2\pi \int_0^\infty H_\rho(\rho, d_t) H_\rho^*(\rho, d_t) \rho d\rho = \pi \frac{\hat{I}^2}{2} \times \int_0^\infty \beta \left[e^{-\beta d_t} - \Phi_B(\beta, d_t) \right] \times \left[e^{-\beta d_t} - \Phi_B^*(\beta, d_t) \right] G^2(\beta a) d\beta \tag{23}$$

Expressions (21), (22), (23) only involve an integral, as occurs in the case of the equations (10), (11), (12) of the fields. At the interest frequency range (21) is much smaller than (22) and (23). Moreover, at sinusoidal regime it holds that $\hat{I}^2 / 2 = I_{rms}^2$.

IV. FINITE ELEMENT VALIDATION

In order to validate the derived models of fields and force, (10), (11) and (12), (21), (22) and (23) are implemented in MATLAB and results are compared with 2D axisymmetric FEA simulations by means of COMSOL. An image of the simulated system is shown in Fig. 4, where the current density of the coil and some magnetic flux density lines are also shown for illustrative purposes. This structure basically corresponds to the geometrical model of Fig. 2, except for the following aspects:

- The coil of filamentary currents is replaced by an ideal rectangular cross-sectional constant current density of value $J_i = n\hat{I} / h_i(r_{ext} - r_{int})$.
- Dissipative media (aluminum and pot) are considered as an Impedance Boundary Condition (IBC) for the resolution. In the case of the aluminum, the domain is directly replaced by the IBC condition. However, the domain of the pot is part of the resolution system due to its finite radius, and the IBC condition is applied to the bottom boundary of the domain.

The first assumption is essentially valid in the case of coils with high density of turns, as occurs in commercial applications. The second is valid if the skin depth of fields is lesser than the thickness of the domain, which is the case of both cookware and aluminum, as was previously discussed.

The dimensions and parameters the geometry of both semi-analytical and FEA models are listed in Table 1 and Table 2, respectively.

TABLE 1. Parameters of the analytical model.

Description	Symbol	Value	Units
Number of turns	n	17	[-]
Internal radius	a_1	25	[mm]
External radius	a_{17}	105	[mm]
Coordinate of the top media	z_1^+	6.5	[mm]
Coordinate of the bottom media	z_1^-	-3.5	[mm]
Flux concentrator thickness	t_f	3.5	[mm]

TABLE 2. Parameters of the FEA model.

Description	Symbol	Value	Units
Internal radius	r_{int}	25	[mm]
External radius	r_{ext}	105	[mm]
Distance coil-pot	d_l	5	[mm]
Coil thickness	t_c	3	[mm]
Distance coil-flux concentrator	d_f	2	[mm]
Pot radius	r_l	150	[mm]
Flux concentrator radius	r_f	170	[mm]

The material properties and current level required for the validation are set according to the experimental tests of the next section. Two different materials, Material A and Material B are used for comparing calculated and simulated results. These materials are typically used in the available cookware and represents two cases of ferromagnetic materials with different resistivity. The properties of these materials, and the others involved in the system are presented in Table 3. As it is shown in this Table, both materials have similar permeability. In order to improve the thermal stability of the system, the cookware is filled with water in the experiments. For this reason, the properties are estimated for the boiling water condition, i.e. $T = 100\,°\text{C}$. On the other hand, the equivalent relative magnetic permeability of the flux concentrator disk is obtained by comparing the measured inductance of the planar coil in the air with respect to the case of the ferrite bars [14].

TABLE 3. Properties of materials.

Description	Symbol	Value	Units
Material A conductivity	σ	$1 \cdot 10^7$	[S/m]
Material A rel. permeability	μ_r	250	[-]
Material B conductivity	σ	$3 \cdot 10^6$	[S/m]
Material B rel. permeability	μ_r	250	[-]
Flux concentrator conductivity	σ	0	[S/m]
Flux concentrator rel. permeability	μ_r	5	[-]
Aluminum conductivity	σ	$2 \cdot 10^7$	[S/m]
Aluminum. rel. permeability	μ_r	1	[-]

The rms current for materials A and B is set to 30 and 28 A, respectively. These currents correspond to the rated power for this inductor size with ferromagnetic materials, i.e. $P_o = 2200$ W. The current for the Material A is higher than the current for the Material B because its equivalent impedance is lesser, corresponding to its lesser resistivity.

Calculated and simulated results of fields E_ϕ, H_r, H_z are shown in Fig. 5, Fig. 6, Fig. 7. These magnitudes are evaluated at the bottom surface of the pot and they are represented with respect to the radial coordinate. As it can be observed, the agreement between calculated and simulated results is, in general, good. Discrepancies could be caused for the different model of the coil adopted between both analytical and FEA resolutions.

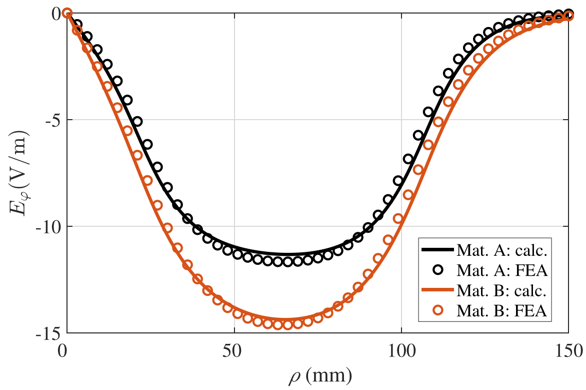

FIGURE 5. Calculated and simulated electrical field with respect to the radial coordinate of the bottom boundary of the cookware.

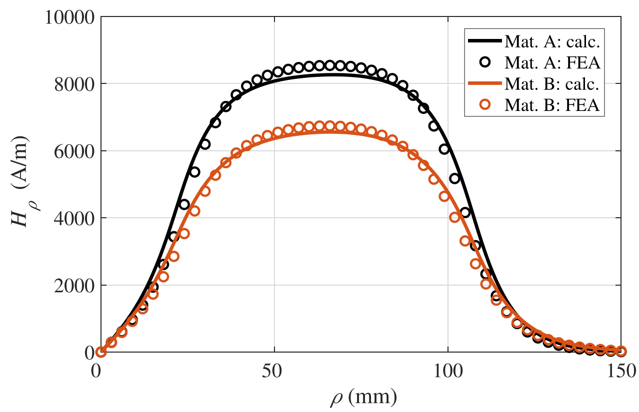

FIGURE 6. Calculated and simulated radial component of the magnetic field with respect to the radial coordinate of the bottom boundary of the cookware.

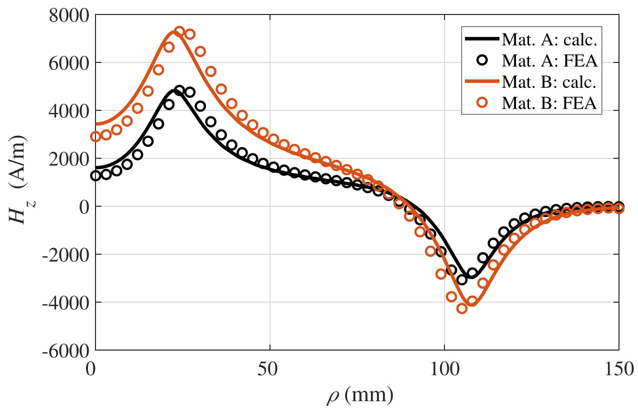

FIGURE 7. **Calculated and simulated vertical component of the magnetic field with respect to the radial coordinate of the bottom boundary of the cookware.**

The magnitude $\mathbb{T}_{zz\_avg}$ is evaluated by means of (16) and the results are compared with FEA simulations in Fig. 8. This magnitude has two signs corresponding to the different directions of the force, which depends on the radial coordinate. The negative sign corresponds to repulsive force. The global force is obtained by means of (18) and (21), (22) and (23) and results are presented in Table 4.

TABLE 4. **Calculated and simulated global force.**

	Material A	Material B	Units
Calculated	-0.556	-0.265	[N]
Simulated	-0.581	-0.271	[N]

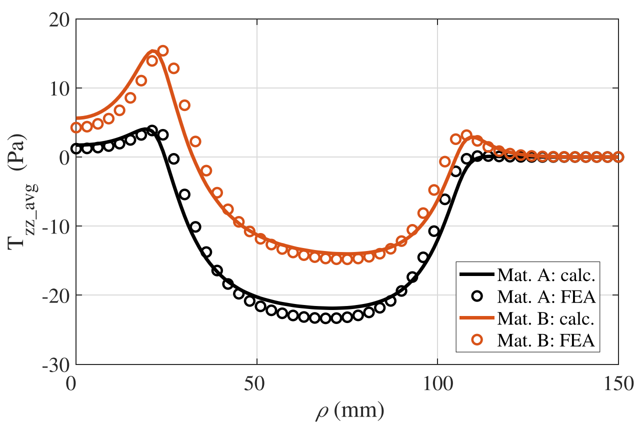

FIGURE 8. **Calculated and simulated vertical component of the average stress tensor with respect to the radial coordinate of the bottom boundary of the cookware.**

The model also captures the dependency on the force with respect to the frequency. This dependency reflects the fact that induced currents are dependent of the frequency and the material properties. This result is relevant because the in usual arrangements the power supplied to the cookware is controlled by means of the frequency. Simulated and calculated forces with respect to frequency are shown in Fig. 9. As it can be observed, at low frequencies the net force is attractive because the magnetization effect is predominant. However, at the higher frequency range, eddy currents are predominant and the resultant force is repulsive. In general, the observed agreement is good, in concordance with the results obtained for the fields.

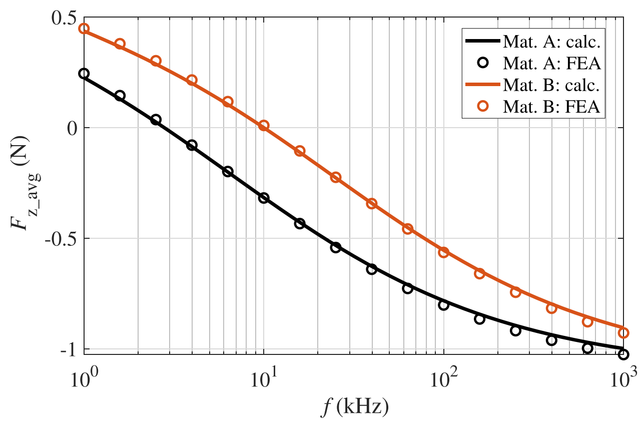

FIGURE 9. **Calculated and simulated vertical force with respect to the frequency.**

Is also worth to compare the computational cost required for obtaining Fig. 5, Fig. 6, Fig. 7 with the developed model and a FEA simulation. The same platform was used in this comparison and it consisted of a basic computer with an Intel^{TM} Core i7 at 3.6GHz and 32 GB of RAM memory. The analytical model was conveniently parameterized and implemented in Matlab and the used FEA tool was COMSOL. The CPU time required to calculate the results shown in the precedent figures was 0.055 s whereas the time required by the FEA tool (considering a mesh with 7059 elements) was 2.8 s. Apart form these results, in the case of systematic analysis with respect to geometrical parameters, the FEA tool requires redrawn each new geometry whereas the analytical model just needs to change some parameters.

V. EXPERIMENTAL TESTS

Some experimental tests are carried out with the purpose of qualitatively verify the developed model and simulations. Rather than a systematic verification, the objective of experiments is to measure the force in typical cooking experiences and to compare the measurements with the developed semi-analytical model and simulations.

The experimental setup is based on a commercial induction heating appliance and commercial pots, whose properties are shown in the previous section. The appliance is connected to the mains and its nominal rated power for ferromagnetic pots is $P_o = 2200$ W. The pot-cooker set is resting on three load cells (Fig. 10) whose measurements are captured by means of a Yokogawa MW100 data logger. An image of the experimental setup is shown in Fig. 11.

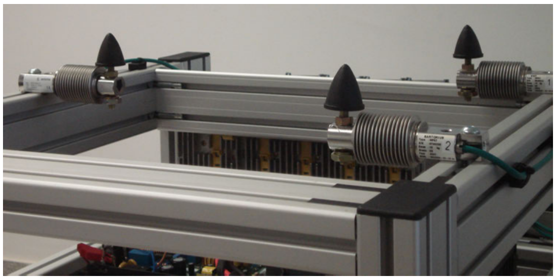

FIGURE 10. Image of the experimental weighting system based on three load cells.

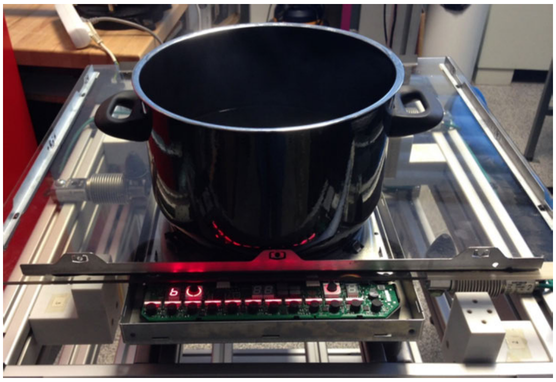

FIGURE 11. Image of the experimental setup.

Experiments are described as follows. Water is added in the pot until the weight of the set is 78.48 N, (8 kg). This water is preheated just before the boiling point and, at this moment, the heating is turned off and the weight is started to be recorded. Few seconds after, the heating at nominal power is turned on and the weight experiments a sharp change. This change can be observed if the heating is turned-on turned-off several times. When the boiling regime is reached, an smooth drop of the weight is observed.

The weight recorded during this process is presented in Fig. 12 for the Material A and Fig. 13 for the Material B. As it is shown in these figures, the measured change of the weigh was 0.510 N and 0.192 N. Comparing with the results of the Table 4 an acceptable agreement is also observed.

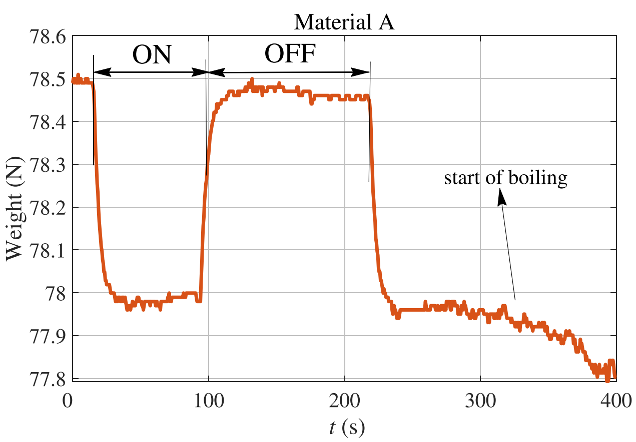

FIGURE 12. Experimental results corresponding to the Material A.

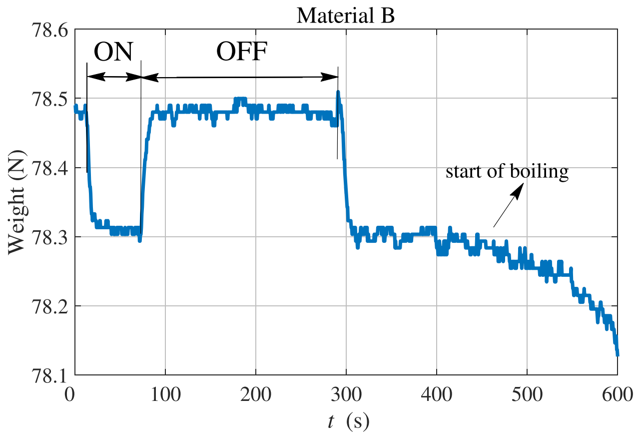

FIGURE 13. Experimental results corresponding to the Material B.

VI. CONCLUSION

In this work, a two-dimensional semianalytical solution for calculating the forces on cookware heated by domestic induction heating appliances is presented. The model is original for this application and is based on the analytical solution of the fields generated by a planar coil in a stratified media. The forces are obtained by integrating the Maxwell's stress tensor, which is obtained by means of the mentioned fields. This approach captures the combination of electromechanical and dissipative regimes of the cookware in induction heating applications. This regime is specific of this application and is not found in other electromechanical applications, as electrical machines, where usually magnetic and electrical paths are different. The model includes the dependencies with respect to the excitation current, the system geometry, the operating frequency and the properties of the materials of the system. The analytical model is implemented in Matlab and results are compared with finite element simulations. In general, both calculated and simulated results show good agreement. However, the analytical model takes less computation time and also reduces the development time required by finite element tools for implementing the analysis of the influence of the system geometry on the forces.

Additionally, the model is also checked by means of experimental results. Forces on the cookware are reflected in a change of the weight, and this change could interfere with the automatic cooking functionalities of modern appliances. Weighting experiments are carried out and measurements also shows good agreement with numerical results for two different materials.

Summarizing, this work contributes an original analytical model for an application with a rich physical insight, different from other electromechanical systems. The observed advantages of the analytical model with respect to finite element simulations, as computational time savings and flexibility for systematic parameterized analysis, makes it suitable for analyzing the considered systems.

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